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Physics of the Hypothetical Gauge Boson Z'

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Contents

Chapter 1

Introduction

The Standard Model (SM) is today the theoretical framework describing the fundamental particles and their interactions which form all the structures of the universe. The concept of the standard model is based on quantum mechanics and special relativity, which are incorporated into quantum field theory. The Standard Model is based on gauge theory, which is a special case of quantum field theory. For example, in quantum mechanics, we know that the energy is transmitted as a discrete quanta. On this basic, the standard model determined the energy that transmitted between particle by gauge bosons which carry the force and transmitted them between different particles. This model achieved great success. But despite its success, the standard model is incomplete. It does not describe gravitational interactions and many questions remain unanswered[1, 5]. What is the dark matter? why we have only three generations of elementary particles. How can we explain the asymmetry of matter and antimatter in the universe?

There are many physical phenomena that have not yet been discovered and we need to extend the standard model and a new physics with a scale higher than the electroweak scale (TeV). The standard model remains an effective theory at low energy.

Several beyond Standard Model theories (BSM) predict the existence of a hypothetical gauge boson denoted "Z'" (gauge boson). The Z' is more massive than the SM Z boson. This boson appears in many theories beyond the standard model. Its properties such as the coupling and mass are arbitrary and different according to the BSM model. There are many Z' models such as the sequential model (Z_{SSM}), Extra Dimensions Models (Z_K), the Left-Right symmetric models of GUT (Z_{LR}) ...etc. The mass range of Z' is from the TeV scale (electroweak scale) to planck scale M_P. The way to know if the massive gauge boson Z' actually exists is to look for it at large Hadron collide[2, 3].

In this work, we study the top quark pair production at LHC in order to test the prediction of new physics beyond the Standard Model. Since the top quark pair is the heaviest particle in the Standard Model, which has a short lifetime and a mass close to the electroweak symmetry breaking scale. The measurement of their decay properties such as the cross section, branching ratio and the kinematic distributions... allows us to observe any deviations of SM and any contribution of new physics that can be observed at LHC[4]. The calculation of the cross section at leading order of the top quark pair production can give us new prediction about the new physics contribution in the process but it is not an accurate prediction. There is a more precision prediction represented in the NLO calculation, the QCD correction of higher order (NLO) can give us a more accurate prediction, where these corrections contributes to more physical observables. We can also consider other approximations, such as the parton shower which in some case more accurate than fLO and fNLO (LO+PS, NLO+PS), and can provide nice description to what happen at detectors.

In chapter (2), we present an introduction to the Standard Model and the theory of weak interaction and electroweak unification. We discuss in more detail the Higgs mechanism that is responsible for spontaneous symmetry breaking and which give mass to the particles in this theory. Finally, we mention success and failure of SM.

In chapter (3), we study the physics of the hypothetical gauge bosons Z' that appear in several theories beyond the Standard Model. We first discuss the importance of studying such boson and its possible production at LHC. We mention some BSM models that predict Z' gauge boson, and then we present a genaral parametrisation of the neutral current lagrangian beyond the Standard Model. Finally , we study one of the simplest model predicting the existence of Z' gauge boson, which is called $U(B-L)$ model.

In chapter (4), we study the production of top quark pair beyond the Standard Model at the LHC. We present the analytical calculation of the LO cross section for the reaction $pp \to t\bar{t}$, we use the Hip program to perform the analytical calculations, and MadGraph and MadAnalysis to perform the numerical calculation of the hadronic cross section and the differential distributions.

In chapter (5), we present the NLO QCD corrections to the top pair production at the LHC in a general BSM model including Z'. We first introduce the factorization theorem, the virtual and the real correction to the partonic cross section, the substraction method and then we present the numerical calculation of the hadronic cross section in term of both the Z' mass and the non physical scale μ (factorisation and renormlaisation scales). We conclude this chapter by giving the differential distribution in many observables in the approximations fLO(fixed leading order), fNLO(fixed next-to-leading order), LO+PS(LO matched to parton shower) and NLO+PS(NLO matched to parton shower).

CHAPTER 2

Standard Model

2.1 Introduction to the Standard model

The Standard Model (SM) of particle physics is the most successful mathematical model for describing physical phenomena observed in experiments. It is a theory of matter, describing the elementary particles of matter and their interaction in nature. This model involves three of the four fundamental forces: electromagnetic, weak and strong forces. The fourth force, gravity, is not included in the model.

In the Standard Model, The particle interactions are expressed within the framework of quantum field theory. SM is a gauge theory based on the gauge symmetry group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. Where $SU(3)_c$ is the color group for the strong interactions. The electromagnetic and weak interactions are described by a singlet theory called "electroweak theory ", which is based on the combination of the two gauge group $SU(2)_L \otimes U(1)_Y$. $SU(2)_L$ and $U(1)_Y$ are the isospin and hypercharge group, respectively.

2.1.1 Elementary Particles and Interactions of the SM

The Standard Model classifies all known particles into two types according to their spin: fermions and bosons. Fermions are the elementary particles that constitute matter. These particles have a half integer-spin and satisfy Fermi-Dirac statistics, which are divided into two subgroups: leptons and quarks [1, 6, 7, 8].

The Quarks are particles carried the electric and color charges, as they are described by three color charges responsible for the interaction between quarks and gluons (strong interaction): red, blue, and green. Quarks do not exist in a free state, and we cannot observe them directly, but as an assemblage of quarks called Hadron. The table (2.1) shows some properties of SM Quark .

Gen	Quark	Mass	Q	S	C	$\tilde{B}$	T
1^{st}	u	1.7-3.1 MeV	$\frac{2}{3}$	0	0	0	0
	d	4.1-5.7 MeV	-$\frac{1}{3}$	0	0	0	0
2^{nd}	c	1.15-1.35 GeV	$\frac{2}{3}$	0	1	0	0
	s	80-130 MeV	-$\frac{1}{3}$	-1	0	0	0
3^{rd}	t	172-174 GeV	$\frac{2}{3}$	0	0	0	1
	b	4-5 Gev	-$\frac{1}{3}$	0	0	-1	0

Table 2.1: properties of SM Quark

The leptons are divided into two types: charged (massive) leptons have electric charge and participate in both electromagnetic and weak interactions, and neutral leptons (known as neutrinos) associate for each charged lepton. The neutrinos interact only via the weak interactions. The leptons do not interact via the strong interactions because they do not have a color charge. The table (2.2) shows some properties of SM Lepton .

Gen	lepton	Mass (MeV)	Q	L_e	L_μ	L_τ
1^{st}	e^-	0.511	-1	1	0	0
	ν_e	¡ 2 $\times 10^{-16}$	0	1	0	0
2^{nd}	μ^-	105.658	-1	0	1	0
	ν_μ	¡ 0.19	0	0	1	0
3^{rd}	τ	1777	-1	0	0	1
	ν_τ	¡ 18.2	0	0	0	-1

Table 2.2: properties of SM Lepton

Bosons are particles with integer spins, satisfying Bose-Einstein statistics (we can find two bosons in the same quantum state). There are Vector bosons (spin 1) that transmit forces, called ”gauge bosons ”. The gauge bosons are defined as mediator of the fundamental interactions, in which the properties of these gauge bosons and the way in which they interact with the particles determines the nature of the fundamental interactions.

In the SM, there are 12 fermions (6 quarks and 6 leptons), 12 gauge bosons ($W^\pm$, Z^0, photon γ and 8 gluons g) and one scalar boson called ”Higgs boson ” gives mass to these particles. For each particle there are an antiparticle with the same mass, spin , live time but has an opposite electric charge.

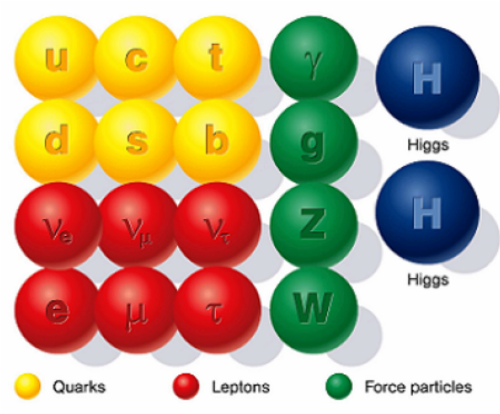


Figure 2.1: Standard Model Particles

2.1.2 Fundamental Interactions

There are four known fundamental interactions,

- **Electromagnetic interaction:** The electromagnetic interaction is a unification of the electric and magnetic forces, which is responsible for electric and magnetic effects. It is mediated by massless boson called photon γ and acts on all electrically charged particles (like leptons and quarks). It is described by quantum electrodynamics (QED).

- **Strong interaction:** The strong interaction is described by quantum chromodynamics (QCD). This interaction is responsible for the cohesion of quarks within the nucleus, it acts on particles with a color charge and it is mediated by the gluons.

- **Weak interaction:** The weak interaction is responsible for radioactive beta decay (β^+ , β^-) and neutrino interactions, it is mediated by the charged $W^\pm$ and the neutral Z^0 gauge bosons. It is described by the Standard Model.

- **Gravitational interaction:** The gravitational interaction is the weakest interaction, it acts on all massive particles and negligible for the nuclear particles.

2.1.3 Fermi model

In 1932, Fermi proposed the first model to describe the structure of the weak interaction in the β-decay of nuclei [6, 9, 10, 11]. This model suggested that the weak interaction has the form of current-current interaction as shown in the figure (2.2),

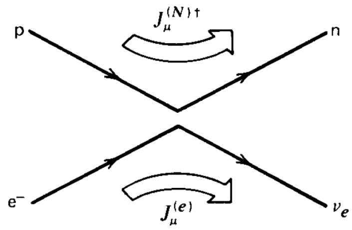


Figure 2.2: the diagram of β decay for $p \longrightarrow n + e^+ + \nu_e$ process [6]

The figure (2.2) represents the proton decay process in the Fermi model. The proton is transformed into a neutron via emission of electron-neutrino pair. In this case, the amplitude is of the form:

$$\mathcal{M} = G_F J_n^\mu J_{e\mu} \tag{2.1}$$

where G_F is the Fermi coupling constant, J_n^μ and $J_{e\mu}$ are the baryonic and leptonic currents, they are given by,

$$J_n^\mu = \bar{u}_p \gamma^\mu u_n \qquad , \qquad J_e^\mu = \bar{u}_{\nu_e} \gamma^\mu u_e \tag{2.2}$$

In this model, the weak interaction current is of the form vector-vector as in the electromagnetic and strong interaction. The discovery of parity violation in the weak interaction (In 1956) led to the necessity to change the form of vector-vector weak current (γ_μ) to agree with the experiment in which the current in the weak interaction must be violating the parity and the charge conjugate. It was found that the most convenient choice is the vector-axial vector ($\gamma_\mu - \gamma_\mu \gamma_5$) form. The amplitude in eq (2.1) becomes :

$$\mathcal{M} = G_F \left[\bar{u}_p \gamma^\mu (1 - \gamma^5) u_n\right] \left[\bar{u}_{\nu_e} \gamma^\mu (1 - \gamma^5) u_e\right] \tag{2.3}$$

The Fermi Model has been improved by adding an intermediate gauge boson (massive) W in order to solve the problem of the Fermi coupling constant (which has dimension $G_F = -2$). However, this theory still available only at low energy. At high energy, the theory violates the unity condition. Therefore, the Fermi model is not a complete model for describing weak interactions.

Parity Violation

The first suggestion of parity violation in weak interaction was proposed by Lee and Young in the K^+ decay. The K^+ meson has two different decay modes in the final state. One decays into two pions ($\theta \to \pi^+\pi^0$) with parity ($\eta_P = +1$) and the other into three pions ($\tau \to \pi^+\pi^+\pi^-$) with parity ($\eta_P =$ -1). θ and τ are two particles have the same properties, but different decay modes and opposite parities . In order to solve this problem (known as $\theta - \tau$ puzzle), Lee and Yang (in 1956) proposed that the θ and τ are identical particles, and the parity is not conserved in weak interaction [6, 10]. In 1957, the violation of parity was confirmed by WU experiment of β decay, in which WU and her collaborators studied the β decay of polarized colbalt-60 nuclei:

$$^{60}Co \longrightarrow ^{60} Ni^* + e^- + \bar{\nu_e} \tag{2.4}$$

Parity would be conserved only if, in the decay of the nuclei ^{60}Co, the electron emitted in a direction opposite to the nucleus spin. But the mirror image of the decay shows that the spin and momentum of electron in the same direction. This result implies that the symmetry of mirror is violated and the parity is not conserved in the weak interaction.

2.2 Glashow-Weinberg-Salam Model

The Glashow-Weinberg-Salam (GWS) model is the first successful model, introducing an accurate theoretical description of the weak interaction, which is consistent with most existing experimental data. It has been experimentally proved by the discovery of the Z and $W^{\pm}$ gauge bosons in proton-antiproton collisions [6, 9, 10, 12].

2.2.1 GWS lagrangian

The model describes the electromagnetic and weak interactions within a unified theory known as "electroweak theory". It is a gauge theory based on the non-Abelian gauge group:

$$\mathcal{G}_{GWS} = SU(2)_L \otimes U(1)_Y \tag{2.5}$$

we can classify fermions according to the helicity operator into two types :

$$\psi = \psi_L + \psi_R = P_L\psi + P_R\psi \tag{2.6}$$

the ψ_L is the left handed fermion and ψ_R is the right handed fermion. The P_L and P_R are the chiral projections operator, given by:

$$P_L = \frac{1}{2}(1-\gamma_5) \quad , \quad P_R = \frac{1}{2}(1+\gamma_5) \tag{2.7}$$

We have seen in experimental observations for the weak interaction that the parity is not conserved. This fact indicates that only left handed fermions component participate in the weak interaction. In this model, the left handed leptonss are represented by an $SU(2)$ doublet, while right handed leptons are represented by an $SU(2)$ singlets (no right handed neutrinos):

$$L = \begin{pmatrix} e^- \\ \nu_e \end{pmatrix}_L , \quad \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}_L , \quad \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}_L ; \qquad R = e_R \ , \ \mu_R \ , \ \tau_R$$

There are two current forming the weak interaction: the neutral and charged current, the charged current are given by:

$$J^+_\mu = \frac{1}{2}\bar{\nu}_e\gamma_\mu(1-\gamma_5)e = \frac{1}{2}\bar{\nu}_{eL}\gamma_\mu e_L \tag{2.8}$$

$$J^-_\mu = \frac{1}{2}\bar{e}(1+\gamma_5)\gamma_\mu\nu_e = \frac{1}{2}\bar{e}_L\gamma_\mu\nu_{eL} \tag{2.9}$$

where $J^{\pm}_\mu$ correspond to the gauge bosons $W^{\pm}$. We can write the charged current in the weak isospin space:

$$J^+_\mu = \bar{L}\gamma_\mu\sigma^+L \tag{2.10}$$

$$J^-_\mu = \bar{L}\gamma_\mu\sigma^-L \tag{2.11}$$

where $\sigma^{\pm} = \frac{\sigma^1 \pm i\sigma^2}{2}$ are defined in term of Pauli matrices σ^i:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad , \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{2.12}$$

the neutral current is defined as:

$$\begin{aligned} J^3_\mu &= \bar{L}\gamma_\mu\frac{\sigma^3}{2}L \\ &= \frac{1}{2}\bar{\nu_{e_L}}\gamma_\mu\nu_{e_L} - \frac{1}{2}\bar{e}_L\gamma_\mu e_L \end{aligned} \tag{2.13}$$

J^3_μ correspond to the neutral boson A^3_μ.

There are two charged current and one neutral current, we can write them as:

$$J^i_\mu = \frac{1}{2}\bar{L}\gamma_\mu \sigma_i L \tag{2.14}$$

In this model, we consider the electromagnetic interaction associates with the $U(1)_{EM}$ group and we defined a conserved quantity " hypercharge Y " in term of weak isospin and electric charge as:

$$Y = 2(Q - T^3) \tag{2.15}$$

The quantum numbers of SM particles are defined in table (2.3),

Field	l_L	l_R	ν_L	u_L	d_L	u_R	d_R	ϕ^+	ϕ^0
T_3	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
Y	-1	-2	-1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{4}{3}$	$-\frac{2}{3}$	1	1
Q	-1	-1	0	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	1	0

Table 2.3: quantum number Q , Y and T_3 for Standard Model particle

The invariant fermions lagrangian under the electroweak gauge group $\mathcal{G}_{GWS}$ is defined as:

$$\mathcal{L}_F = \bar{R}i\gamma_\mu(\partial^\mu + ig'B^\mu)R + \bar{L}i\gamma_\mu(\partial^\mu - \frac{\tau}{2}igA_\mu - ig'B^\mu)L \tag{2.16}$$

where g and g' are the gauge coupling constants corresponding to $SU(2)_L$ and $U(l)_Y$. The covariant derivative is given by,

$$D_\mu = \partial^\mu - \frac{\tau}{2}igA_\mu - ig'\frac{Y}{2}B^\mu \tag{2.17}$$

where the hypercharge $Y = -1$ for L and $Y = -2$ for R. We notice here that the fermion cannot have a mass term because the gauge invariance $SU(2)_L \otimes U(1)_Y$ will be violated.

The lagrangian describe the gauge field is given by,

$$\mathcal{L}_{gauge} = -\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \tag{2.18}$$

The mass term of gauge bosons violate the gauge invariance, so in order to generate the mass of gauge bosons and fermions we must break the symmetry of $SU(2)_L \otimes U(1)_Y$.

2.2.2 Higgs mechanism

We have seen that the gauge bosons in the electroweak theory can not acquire masses since the mass term violates the gauge invariance of the theory. However, the experimental evidence found that Z and W are massive gauge bosons. This fact indicates that the electroweak symmetry is broken $SU(2)_L \otimes U_Y(1)$, in order to solve this problem. P. Higgs and others proposed "the Higgs mechanism ", which is based on a fundamental concept known as "spontaneous symmetry breaking" [1, 6, 10].

In this mechanism, the vacuum was filled with a new scalar field called the "Higgs field", which requires the existence of a zero-spin Higgs boson that interact with the standard model particles and gives them masses.

The standard model Higgs field can be expressed as an $SU(2)$ doublet with $Y = 1$ in the term of two complex fields as:

$$\phi = \begin{pmatrix} \phi^\dagger \\ \phi \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \tag{2.19}$$

The Higgs Lagrangian is given by:

$$\mathcal{L}_{Higgs} = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi^\dagger \phi)^2 \tag{2.20}$$

where $V(\phi^\dagger\phi)$ is the Higgs potential, is given by:

$$V(\phi^\dagger \phi) = -\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2 \tag{2.21}$$

we minimize the potential to find the ground state of the field (vacuum):

$$\frac{\partial V}{\partial \phi} = 0 \qquad \rightarrow \qquad \phi(\mu^2 + \lambda \phi^2) = 0. \tag{2.22}$$

We have two different cases: (i) The first case corresponds to $\mu^2 > 0$ and $\lambda > 0$. In this case, the minimum of the potential corresponds to the value $\phi_0 = 0$ (null), so the electroweak symmetry is not broken. (ii) The second case corresponds to $\mu^2 < 0$ and $\lambda > 0$. In this case, the potential V have two minimum values which corresponds to $\phi^+\phi = \pm\frac{-\mu^2}{\lambda} = v^2$, where v is the vacuum expectation value of ϕ. The minimum value of ϕ is:

$$\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{2.23}$$

and the Higgs potential can be parametrized as,

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \tag{2.24}$$

We can choose just two components of field to represent the Higgs field, see figure (2.3).

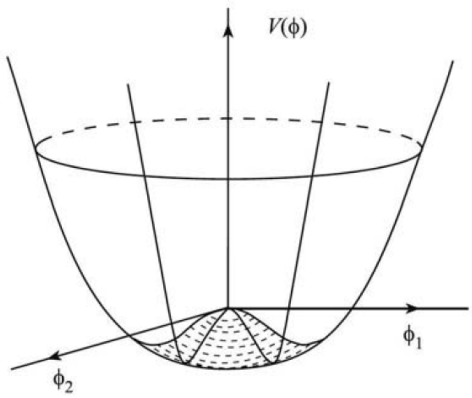


Figure 2.3: Higgs potential V for $\mu^2 < 0$ and $\lambda > 0$ [13]

In this case the symmetry $SU(2)_L \otimes U_Y(1)$ is broken to $U_{em}(1)$. In order to conserve the invariance under $SU(2)_L \otimes U_Y(1)$ transformation, we define the covariant derivative for the Higgs doublet is given by:

$$D_\mu \phi = (\partial_\mu + igT^a W_\mu^a + i\frac{g'}{2} B_\mu)\phi \tag{2.25}$$

2.2.3 Gauge Bosons masses

In order the obtain the mass of the gauge bosons, we have to substitute the complex field in (2.24) in the covariant derivative. The mass term then becomes,

$$\begin{aligned} \mathcal{L}_{mass} &= (D_\mu \phi)^\dagger (D^\mu \phi) \\ &= \left[(\partial_\mu + igT^a W_\mu^a + i\frac{g'}{2} B_\mu)\phi \right]^2 \end{aligned} \tag{2.26}$$

To simplify we use the relation,

$$T^a W^a_\mu = \frac{1}{2}\sigma^a W^a_\mu = \frac{1}{2}\begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix}$$

we obtain

$$\begin{aligned}\mathcal{L}_{mass} &= \frac{1}{2}\left[\begin{pmatrix} \partial_\mu - \frac{i}{2}(gW^3_\mu + g'B_\mu) & -i\frac{g}{2}(W^1_\mu - iW^2_\mu) \\ -i\frac{g}{2}(W^1_\mu + iW^2_\mu) & \partial_\mu + \frac{i}{2}(gW^3_\mu + g'B_\mu) \end{pmatrix}\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + H(x)) \end{pmatrix}\right]^2 \\ &= \frac{1}{2}(\partial_\mu H)^2 + \frac{1}{8}g^2(v+H)^2(W^1_\mu - iW^2_\mu)^2 + \frac{1}{8}(v+H)^2(gW^\mu_3 - g'B_\mu)^2 \end{aligned} \tag{2.27}$$

From this relation, we have

$$(\frac{1}{2}gv)^2 W^+_\mu W^{-\mu} + \frac{1}{8}v^2(gW^\mu_3 - g'B_\mu)^2 \tag{2.28}$$

where

$$W^\pm_\mu = \frac{(W^1_\mu \pm W^2_\mu)}{\sqrt{2}}$$

the first term in eq. (2.28) is of the form $M^2_W W^+_\mu W^{-\mu}$, so the mass of W is given by,

$$M_W = \frac{1}{2}vg \tag{2.29}$$

we can write the second term in eq. (2.28) as:

$$\begin{aligned}\frac{1}{8}v^2(gW^\mu_3 - g'B_\mu)^2 &= \frac{1}{8}v^2\left[g^2(W^\mu_3)^2 - 2gg'W^\mu_3 B_\mu + g'^2 B^2_\mu\right] \\ &= \frac{1}{8}v^2\begin{pmatrix} W^\mu_3 & B^\mu \end{pmatrix}\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W_{\mu 3} \\ B_\mu \end{pmatrix}\end{aligned} \tag{2.30}$$

Now, in order to obtain the mass term of the physical field A_μ and Z_μ, we diagonalize the matrix in eq. (2.30) by using the transformation:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} W_{3\mu} \\ B_\mu \end{pmatrix}$$

we get

$$\mathcal{L}^{A,Z}_{mass} = \frac{1}{8}v^2\begin{pmatrix} Z_\mu & A_\mu \end{pmatrix}\begin{pmatrix} g^2 + g'^2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix} \tag{2.31}$$

$$= \frac{1}{8}v^2(g^2 + g'^2)Z_\mu Z^\mu + (0)A_\mu A^\mu \tag{2.32}$$

The mass terms for the gauge boson Z and the photon A is of the form:

$$\mathcal{L}^{A,Z}_{mass} = \frac{1}{2}M^2_Z Z_\mu Z^\mu + \frac{1}{2}M^2_A A_\mu A^\mu \tag{2.33}$$

From the relation (2.32), we find the normalized A_μ and Z_μ fields and the masses are given by:

$$\begin{aligned} A_\mu &= \frac{g'W_{3\mu} + gB_\mu}{\sqrt{g^2 + g'^2}}, \qquad \textit{with} \quad \textit{mass} \qquad M_A = 0 \\ Z_\mu &= \frac{gW_{3\mu} - g'B_\mu}{\sqrt{g^2 + g'^2}}, \qquad \textit{with} \quad \textit{mass} \qquad M_Z = \frac{v}{2}\sqrt{g^2 + g'^2} \end{aligned} \tag{2.34}$$

we have

$$tan\theta_W = \frac{g'}{g}, \qquad \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}, \qquad \cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}} \tag{2.35}$$

where the photon and Z fields can be expressed in term of the weak mixing angle θ_W and the orginal gauge fields as:

$$A_\mu = B_\mu \cos\theta_W + W_\mu^3 \sin_{\theta_W} \tag{2.36}$$
$$Z_\mu = -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W \tag{2.37}$$

we have

$$\frac{M_W}{M_Z} = cos\theta_W \tag{2.38}$$

So the mass of the Z and W are related by the mixing angle θ_W.

2.2.4 Higgs Bosons mass

The Higgs boson mass is obtained from the Higgs potential after spontaneous symmetry breaking:

$$V(\phi^\dagger\phi) = \lambda v H^3 + \frac{\lambda}{4}H^4 + \frac{1}{2}(2\mu^2)H^2 - \frac{\mu^2 v^2}{4} \tag{2.39}$$

the terms H^3 and H^4 describe the self-interactions of the Higgs boson, the mass of the Higgs boson are obtained from the H^2 term:

$$M_H = \sqrt{2\mu^2} \tag{2.40}$$

We can write the scalar lagrangian after spontaneous symmetry breaking as,

$$\begin{aligned}\mathcal{L}_S &= (D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi^\dagger\phi)\\ &= \frac{1}{2}\partial_\mu H\partial^\mu H - \frac{1}{2}M_H^2H^2 - \lambda v H^3 - \frac{\lambda}{4}H^4 + \frac{\mu^2v^2}{4} + M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu\\ &+ \frac{1}{8}H^2 Z_\mu Z^\mu(g^2+g'^2) + \frac{g^2}{4}H^2 W_\mu^+ W^{\mu-} + \frac{g^2}{4}HvW_\mu^+W^{\mu-} + \frac{1}{8}HvZ_\mu Z^\mu(g^2+g'^2)\end{aligned} \tag{2.41}$$

the last term describes the interaction of the Higgs bosons with the gauge bosons.

2.2.5 Yukawa intercation

The interaction of the fermions with the Higgs field is described by the Yukawa Lagrangian. This interaction gives mass to the fermions (leptons and quarks) after spontaneous symmetry breaking. The Yukawa interaction term for leptons is written as:

$$\mathcal{L}_Y^l = G_e(\bar{L}\phi e_R + \overline{e_R}\phi^\dagger L) \tag{2.42}$$

where L and e_R are the lepton doublet and singlet, respectively, ϕ is the Higgs doublet and G_e is an arbitrary coupling.

We express the Yukawa term in terms of the Higgs field after SSB, we get:

$$\begin{aligned}\mathcal{L}_Y^l &= G_e\left[\begin{pmatrix}\overline{\nu_e} & \overline{e}\end{pmatrix}_L \begin{pmatrix}0\\ \frac{v+H}{\sqrt{2}}\end{pmatrix} e_R + \overline{e_R}\begin{pmatrix}0 & \frac{v+H}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L\right]\\ &= G_e\frac{v+H}{\sqrt{2}}\left[\begin{pmatrix}\overline{\nu_e} & \overline{e}\end{pmatrix}_L\begin{pmatrix}0\\1\end{pmatrix}e_R + \bar{e}_R\begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L\right]\\ &= \frac{G_e v}{\sqrt{2}}(\overline{e_L}e_R + \overline{e_R}e_L) + \frac{G_e}{\sqrt{2}}(\overline{e_L}e_R + \overline{e_R}e_L)H\end{aligned} \tag{2.43}$$

we have

$$\bar{e}e = \overline{e_L}e_R + \overline{e_R}e_L \tag{2.44}$$

we can write (2.43) as,

$$\mathcal{L}_Y^l = \frac{G_e v}{\sqrt{2}}\bar{e}e + \frac{G_e}{\sqrt{2}}\bar{e}eH \tag{2.45}$$

The mass of electron is obtained from the first term, it is given by,

$$m_e = \frac{G_e v}{\sqrt{2}} \tag{2.46}$$

The same thing can be said for the other generations.

The second term describes the interaction of the electron and Higgs boson with a coupling proportional to the mass:

$$G_e = \sqrt{2}\frac{m_e}{v} \tag{2.47}$$

Since Yukawa coupling is arbitrary, the mass value is not specified. The neutrino remains massless ($m_\nu = 0$) because there are no right handed neutrino ν_R in the theory, therefore, the neutrino can not interact with the Higgs doublet.

For the quarks, the interaction term is more complicated because the two components of quark doublet are massive. So in order to generate the mass of quark we need a Higgs doublet:

$$\phi_c = -i\tau_2\phi^* = \begin{pmatrix} -\phi^{0*} \\ \phi^- \end{pmatrix} \tag{2.48}$$

ϕ_c is the conjugate doublet Higgs with Y = -1 .

$$\phi_c \to \begin{pmatrix} -\frac{v+H}{2} \\ 0 \end{pmatrix} \tag{2.49}$$

The Yukawa term is written as:

$$\begin{aligned}
\mathcal{L}_Y^q &= g_d\bar{Q}_L\phi d_R + g_u\bar{Q}_L\phi_c u_R + h.c \\
\mathcal{L}_Y^q &= g_d\frac{v+H}{\sqrt{2}}\left[\,\begin{pmatrix}\bar{u} & \bar{d}\end{pmatrix}_L \begin{pmatrix}0\\1\end{pmatrix} d_R + g_u \begin{pmatrix}\bar{u} & \bar{d}\end{pmatrix}_L \begin{pmatrix}-1\\0\end{pmatrix} u_R\right] \\
&= g_d\frac{v}{\sqrt{2}}\bar{d}_L d_R - g_d\frac{H}{\sqrt{2}}\bar{u}_L u_R
\end{aligned} \tag{2.50}$$

We can write the Yukawa interaction term as,

$$\mathcal{L}_Y^Q = m_d\bar{d}_L d_R + m_u\bar{u}_L u_R + \frac{m_d}{v}\bar{d}_L d_R H + \frac{m_u}{v}\bar{u}_L u_R H \tag{2.51}$$

The mass of the u and d quark is obtained from the first two term, we have

$$m_{u,d} = \frac{g_{u,d}v}{\sqrt{2}} \tag{2.52}$$

It is proportional to the coupling, this means that the H couples most strongly to the heaviest fermions. the last two terms describe the interaction of u and d quarks with Higgs boson H.

2.3 Standard Model Lagrangian

The most general renormalizable lagrangian of the standard model is given by:

$$\mathcal{L} = \mathcal{L}_{Gauge} + \mathcal{L}_{Higgs} + \mathcal{L}_{yukawa} + \mathcal{L}_{fermion} \tag{2.53}$$

where $\mathcal{L}_G$ describes the gauge sector (gauge field), $\mathcal{L}_{Higgs}$ describes the scalar (or Higgs field) Lagrangian, $\mathcal{L}_{yukawa}$ is the Yukawa interaction lagrangian and $\mathcal{L}_{fermion}$ is the fermion sector (quarks and leptons).

2.3.1 Gauge Sector

The Lagrangian that describe gauge field is given by:

$$\mathcal{L}_{gauge} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \tag{2.54}$$

each gauge field is associated with a local gauge symmetry. The $B_{\mu\nu}$, $W^a_{\mu\nu}$ and $G^a_{\mu\nu}$ are the $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ gauge fields, where

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + gf^{abc}G^b_\mu G^c_\nu, \quad a = 1, ...8 \tag{2.55}$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\varepsilon^{abc}W^b_\mu W^c_\mu, \quad a = 1, ...3 \tag{2.56}$$

$$B^{\mu\nu} = \partial_\mu B^\nu - \partial_\nu B\mu \tag{2.57}$$

with f^{abc} and ε^{abc} are the structure constants of the color group $SU(3)_C$ and $SU(2)$, respectively. We have,

$$[T^a, T^b] = if^{abc}T^c \quad , \quad [I^i, I^j] = i\varepsilon^{ijk}I^k \tag{2.58}$$

T^a is the generator of color group $SU(3)_C$, where I_3 is the weak charge.

2.3.2 Higgs Sector

In the standard model, the Higgs Lagrangian is described as:

$$\mathcal{L}_{Higgs} = (D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi^\dagger\phi)^2 \tag{2.59}$$

where ϕ is an $SU(2)_L$ doublet with spin 0 and hypercharge $Y = -1$, it is given by:

$$\phi = \begin{pmatrix} \phi^\dagger \\ \phi \end{pmatrix} \tag{2.60}$$

the first term corresponds to the dynamical interactions of the Higgs field and the last term represent the Higgs potential associate with the scalar field. The potential is given by,

$$V(\phi^\dagger\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \tag{2.61}$$

where μ and λ are free parameters .

2.3.3 Fermion sector

The fermion field lagrangian describes the interactions between the fermion and gauge boson fields, it is given by:

$$\mathcal{L}_{Fermion} = \sum_{quarks} i\bar{q}\gamma^\mu D_\mu q + \sum_{\psi_L} i\bar{\psi_L}\gamma^\mu D_\mu\psi_L + \sum_{\psi_R} i\bar{\psi_R}\gamma^\mu D_\mu\psi_R \tag{2.62}$$

where ψ_L and ψ_R are the left and right chirality, where the covariant derivative for each fermion state is given by,

$$D_\mu\psi_L = (\partial_\mu - ig'W^a_\mu T^a)\psi_L \quad , \quad D_\mu\psi_R = (\partial_\mu + ig'YB_\mu)\psi_R \tag{2.63}$$

2.3.4 Yukawa sector

$\mathcal{L}_{Yukawa}$ is the lagrangian that describe the interaction between fermions and Higgs doublet (after spontaneous symmetry breaking gives masses to the elementary fermions), it is given by:

$$\mathcal{L}_{Yukawa} = -Y_l \bar{L}\phi l_R - Y_d \bar{Q}\phi d_R - Y_u \bar{Q}\tilde{\phi} u_R + hc. \tag{2.64}$$

where ϕ is the Higgs doublet and ϕ_c is the conjugate Higgs doublet, it is given by:

$$\phi_c = -i\tau_2\phi^* = \begin{pmatrix} -\phi^{0*} \\ \phi^- \end{pmatrix} \tag{2.65}$$

2.4 Successes and Failures of SM

The Standard Model successfully explained many physical phenomena observed at high energy in particle accelerators. Most of the experimental data were in agreement with the predictions of the theory. The existence of the neutral current predicting by the theory was discovered by Gargamalle experiment. This is within the experiment that proved the validity of the electroweak theory. Then, the neutral Z and W vector bosons of the weak interactions were discovered (in 1983) in the CERN UA(1) and UA(2) experiment in the $Sp\bar{p}S$ proton antiproton collider. The discovery of the third charged lepton and quark: τ lepton in (1975) and b quark(1977) at SLAC. Then , the discovery of top quark in 1995 in which these experiment confirmed the existence of third family of quark and lepton as predicted in the standard model. All particles predicted by the theory were found in accelerators, and the last particle discovered was the Higgs boson in 2012 [15, 16].

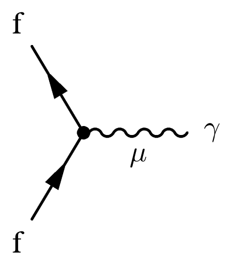
Despite all the success of this theory, it remains incomplete and failed to explain many phenomena:

- The Standard Model does not contain the fourth fundamental force ”gravity ”.
- In the Standard Model, neutrinos cannot have mass (no right handed neutrino), but recent experiments have shown that neutrino should have massive.
- The Standard Model contains all particles that consist the visible matter in the universe. But it does not have any viable candidate for the dark matter that constitutes most of the matter in the universe.
- The hierarchy problem that usually occurs when a physical parameter such as the coupling or mass have different values from its values measured in the experiment.

There are lot of question still unswered in the standard model

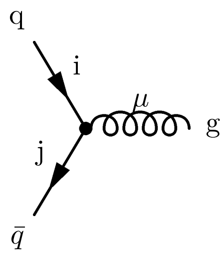
2.5 SM Feynman Rules

- **QED**

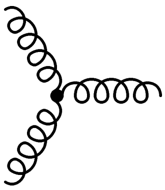


$-ie\gamma_\mu$

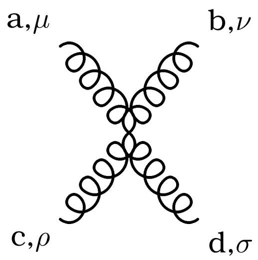
- **QCD**



$$-\,ig_s(T^a)_{ji}\gamma_\mu$$



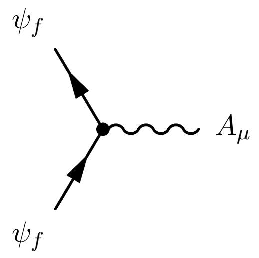
$$-\,g_s f^{abc}(g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu)$$



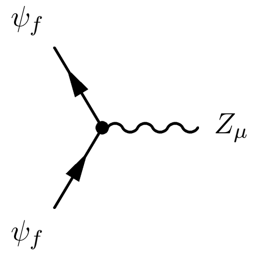
$$\begin{aligned}-\,ig_s^2\Big[&f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\rho})\\ &+f^{ace}f^{bed}(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\sigma}g^{\nu\rho})\\ &+f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\rho}g^{\nu\sigma})\Big]\end{aligned}$$

- **Weak**

  - **Neutral current vertex**



$$-ieQ_f\gamma_\mu$$

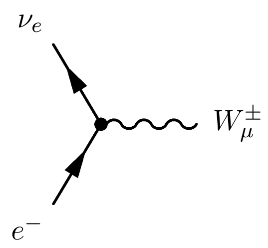


$$\frac{-ie}{\sin\theta_W\cos\theta_W}\gamma_\mu\Big(a_L^f\frac{1-\gamma_5}{2}+a_R^f\frac{1+\gamma_5}{2}\Big)$$

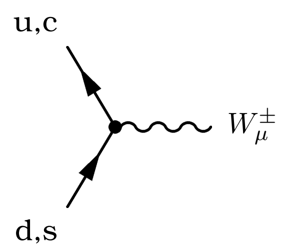
with

$$\begin{aligned}
&a_L^f=-\frac{1}{2}+\sin^2\theta_W \quad , \quad a_R^f=\sin^2\theta_W \quad \text{for} \quad f=e^-,\mu^-,\tau^-\\
&a_L^f=\frac{1}{2}-\frac{2}{3}\sin^2\theta_W \quad , \quad a_R^f=-\frac{2}{3}\sin^2\theta_W \quad \text{for} \quad f=u,c,t\\
&a_L^f=-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W \quad , \quad a_R^f=\frac{1}{3}\sin^2\theta_W \quad \text{for} \quad f=d,s,b\\
&a_L^f=\frac{1}{2} \quad , \quad a_R^f=0 \quad \text{for massless neutrinos}
\end{aligned}$$

- **Charged current vertex**



$$-i\frac{g}{\sqrt{2}}\gamma_\mu(\frac{1-\gamma_5}{2})$$



$$-i\frac{g}{\sqrt{2}}\cos(\theta_C)\gamma_\mu(\frac{1-\gamma_5}{2})$$

Chapter 3

Physics of Z' Gauge Boson

3.1 What is and why Z' gauge boson?

The Standard Model (SM) successfully describes several physical phenomena in nature, in which it represents the most convenient theoretical model with experimental results. However, this model fails to explain some observations and experimental results, such as the origin of dark matter, neutrino masses and flavor mixing ... etc.

In order to understand the physical phenomena that proved by present experiments and cannot be explained by the Standard Model, we need to search for new theories Beyond the Standard Model (BSM) to explain these results.

The physics beyond the standard model contains several models based on the enlarging of the gauge group of the Standard Model which leads to new particles called exotic particles or hypothetical particles, which might be extra gauge bosons, extra fermions, extra Higgs bosons, extra neutrino ...etc. In this work, we focus on the physics of one of the hypothetical gauge bosons which is the Z' Boson [2, 5, 17, 18].

The Z' Boson is predicted by many theories beyond the Standard Model like GUT theories, Little Higgs inspired theories, ... etc. In our studies, we focus on the extension of the Standard Model symmetry gauge group with an additional $U(1)$ group which brings to the existence of the Z' boson. Such models solve many of the SM problems like the hierarchy problems and quadratic divergences on the Higgs mass. Since the new heavy gauge boson Z' is a prediction of several theories beyond the standard model, the search for such boson is very important to give us information for these theories and prove them.

The existence of Z' gauge boson is related to the existence of new abelian gauge group $U'(1)$ within the extension of the SM gauge group, i.e. the simplest quantum field theory containing such gauge boson is based on the gauge groupe $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U'(1)$, where the Z' boson is the gauge field associated to $U'(1)$.

The Z' gauge boson is a hypothetical particle of spin 1, it is massive, neutral and colorless. It is a new force carrier that mediates the neutral current interaction beyond the standard model which has similar properties to standard model Z boson but it is more heavier than Z boson. This particle appears in several theories beyond standard model such as grand unification (GUT), supersymmetry and extra dimensions theories. In addition, Z' has a very short life time, so this particle needs a high energy to be produced on hadron collider or it can be produced through high precision experiment at low energy.

3.2 BSM Models with Z'

There are many BSM models which predict the existence of Z' gauge boson. The Z' gauge boson physics different from to another by its couplings to the SM particles and the $U'(1)$ breaking scale. The existence of Z' is related to extra gauge factor $U(1)'$ in which there are many possibilities for the extension of the standard model gauge symmetry group that may include one or more of this additional factor. Therefore, the new unified gauge group (as the simplest one $G_{SM} \otimes U(1)'$) spontaneously breaks at some scale leading to new neutral gauge bosons such Z'. In this section, we present briefly some of those modes[2, 3, 5, 17, 18].

3.2.1 The sequential model

The feature of this model is that the couplings of Z' gauge boson to the standard model fermions (leptons and quarks) are the same as as the SM Z boson coupling. So, one has just to add a copy of Z' interactions exactly the same as the one of the Z boson. For this, we can consider the canonical sequential model as a suitable reference to describe the other model.

3.2.2 Grand unified theories model (GUT)

The grand unified theories are theories which aim to unify the three fundamental forces (strong, weak and electromagnitic) at high energy within a single theory based on a simple gauge group G_{GUT} and a unified coupling g_{GUT}. Therefore, the complicating standard model gauge group extend to be simple at higher energy where the extended symmetry can be broken at lower energy to retain standard model gauge group, see the figure (3.2.2).

$$G_{GUT}$$

$$\Downarrow$$

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

	Low energy		$\Rightarrow$	High energy
$SU_c(3)\otimes$	$SU_L(2)\otimes$	$U_Y(1)$	$\Rightarrow$	G_{GUT} (*or* G^n + discrete symmetry)
gluons	W, Z	photon	$\Rightarrow$	gauge bosons
quarks	leptons		$\Rightarrow$	fermions
g_3	g_2	g_1	$\Rightarrow$	g_{GUT}

The prediction of Z' in this theories needs to a gauge group larger than $SU(5)$ (larger than 4 rank) so that Z' appears in $SO(10)$, E_6 .. .etc. There are several type of the GUT models according to the construction of the extanded gauge group, we mention here some of them: E_6 models and Left-Right symmetry models.

- The E_6 unified model breaks to $E_6 \to SO(10) \times U(1)_\psi \to SU(5) \times U(1)_\chi \times U(1)_\psi$. These models predict the existence of tree type of the new heavy boson Z': Z'_ψ coming from breaking the symmetry E_6 to $SO(10) \times U(1)_\psi$, Z'_χ from breaking the symmetry $SO(10) \times U(1)_\psi \to SU(5) \times U(1)_\chi \times U(1)_\psi$ and Z'_η from breaking to $SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\eta$.

3.2.3 Left – Right symmetric models

The LR symmetric models based on the gauge symmetry group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ which comes from the symmetry breaking of the groups $SO(10)$ or E_6. In this model, the ratio of the Right g_R and Left g_L gauge coupling is: $k = g_R/g_L = 1$. This model contain one additional neutral gauge boson Z_{LR} and two new right handed charged gauge boson, denoted $W_R^\pm$.

We notice that there are BSM models other than GUT which predict those new gauge bosons, we mension for example Extra Dimensions Models, Supersymmetric models and *Stückelberg* models.

3.2.4 Extra Dimensions Models

In these models, the four dimensions of our world are extended by adding an additional space or time to unify the gravitational and electromagnitic forces, Kaluza klein excitation is the simplest case that contain a single extra dimension propagate in a bulk (1cm) of radius R as shown in the figure(3.1). The gauge bosons, the Higgs boson or the graviton can propagate freely in this small dimension or remain confined to the brane (in our usual three space dimensions).

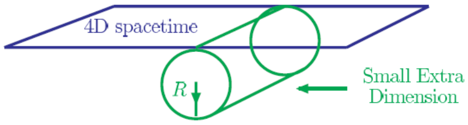


Figure 3.1: Kaluza-Klein picture [5]

3.2.5 *Stückelberg* models

We can use the *Stückelberg* mechanism to make a massive gauge boson without any vev or a higgs boson. This mechanism has recently been used in $U(1)'$ extension of the SM or the MSSM.

3.3 Z' at the LHC

We are looking for new neutral gauge boson Z' in the LHC in which the proton collisions at high energy may produce new particles and hence the Z' can be produced in "proton-proton" collisions, provided that its mass must be smaller than the center of mass energy of colliding protons (and at least Z' two times lighter).

One of the main discovery channel of Z' at the LHC is the dilepton production through Drell Yan mechanism $(p\bar{p})pp \to Z' \to l^+l^-$ where $l = e, \mu$ that shown in the figure (3.2). The annihilation of a quark and anti quark to product a pairs of lepton in final state through the exchange of the virtual γ, Z and the new Z' bosons. This channel allow us to determine the Z' mass, the width $\Gamma_{Z'}$ and the leptonic cross section $\sigma^l_{Z'} = \sigma_{Z'} B_l$ (B_l is the branching ratio into leptons).[2, 23]

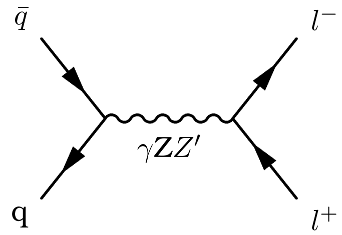


Figure 3.2: Feynman diagram for the Drell-Yan di-lepton production (l = e, μ).

The decay into leptons makes Z' a main search in LHC. In the dilepton resonance , the production of Z' through two dominant channels:

- dimuon channel $q\bar{q} \to Z' \to \mu^+\mu^-$: the process shown in the figure (3.3) represent the production of Z' through its couplings to $b\bar{b}$ quark and then it decays into muon pairs.

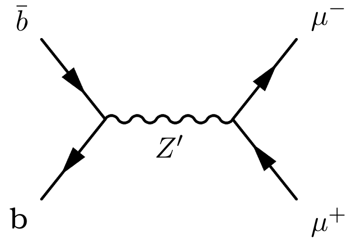


Figure 3.3: dimuon production from Z' via $b\bar{b}$ fusion at the LHC.

In many models, the main production of Z' is via $b - b$ fusion either from gluon splitting or sea quark annihilation, and than it decays into muons, muon neutrinos, bottom, or strange quarks and also into top quarks and dark matter are all possible when kinetically allowed. So searching for Z' in dimuon, dijet($pp \to Z'j$ or $pp \to Z'jj$) or $t\bar{t}$ resonance[21, 22, 25].

- elecron-positron channel: the production of Z' through annihilation of quark anti-quark and its decay to electron-positoon, the process are shown in the figure (3.4):

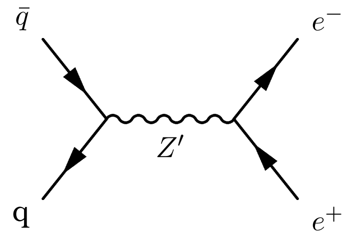


Figure 3.4: Z' decay in e^+e^- channel

In the sequantial model, the Z'_{SSM} mass are determined to be greater than 4.05 TeV in leptonic channels. There are other channels such as the diboson channel [24], in which Z' can be produced in the process $pp \to Z'X \to W^+W^-X$ via its couplings to quarks. This reaction is not the main discovery channels but it allow us to understand the origin of Z' [18, 19, 20].

3.4 Standard Model with additional $U(1)'$ Factor

The extensions of the standard model symmetry group may involve more than one additional $U(1)'$ factor, in general we have to multiply the SM gauge group by $U(1)'^n$, and the new model will be based on

$$SU_L(2) \otimes U_Y(1) \otimes U(1)'^n \qquad\qquad n \geq 1 \tag{3.1}$$

In our case, we want to look for just one extra gauge boson so we need only a one $U(1)'$, then one has to fix $n = 1$. The lagrangien density of the new model in this case is given by,

$$\mathcal{L}_{NC} = \mathcal{L}_{NC}^{SM} + \mathcal{L}_{NC}^{Z\prime} \tag{3.2}$$

where $\mathcal{L}_{NC}^{SM}$ is the usual SM neutral current lagrangian while $\mathcal{L}_{NC}^{Z'}$ is the new neutral current lagrangian associate with $U(1)'$ gauge symmetry.

The neutral current of the new model contains three parts associated respectively to the photon, the Z boson and to Z' gauge boson. The neutral current lagrangian can be written as,

$$\mathcal{L}_{NC} = -eJ^{\mu}_{em}A_{\mu} - g_1 J^{\mu}_1 Z_{\mu} - g_2 J^{\mu}_2 Z'_{\mu} \tag{3.3}$$

where g_1 is the SM gauge couplings of the $U(1)_Y$ factor which is given by

$$g_1 = g/\cos\theta_W = e/\cos\theta_W \tag{3.4}$$

and g_2 is the gauge couplings of the extra gauge groupe $U(1)'$ factor (it is a free parameter). The A_μ and Z_μ are the SM gauge field while Z'_μ is the new gauge field associated the new group. The current J^μ for each term is given by:

$$\begin{aligned} J^{\mu}_{em} &= \sum_i q_i \bar{f}_i \gamma^{\mu} f_i \\ J^{\mu}_{1,2} &= \sum_i \bar{f}_i \gamma^{\mu} (\zeta^{1,2}_L(i) P_L + \zeta^{1,2}_R(i) P_R) f_i \end{aligned} \tag{3.5}$$

where $\zeta^1_{L,R}(i)$ are the SM chiral couplings, there are given by,

$$\begin{aligned} \zeta^1_L(i) &= T^i_3 - \sin^2\theta_W Q_i \\ \zeta^1_R(i) &= -\sin^2\theta_W Q_i \end{aligned} \tag{3.6}$$

where θ_W, T_3^i and Q_i are the Weinberg angle, the third component of the isospin and the electric charge of fermion f_i,respectively. $P_{L,R} = (1 \mp \gamma_5)/2$ is the left and right chiral projectors. The $\zeta^2_{L,R}(i)$ are The new chiral couplings related to new gauge boson Z'.

In terms of these parameters, the neutral current lagrangian of the new gauge boson is given by,

$$\begin{aligned} \mathcal{L}_{NC}^{Z\prime} &= -g_2 J_2^\mu Z^0_{2,\mu} \\ &= -\frac{g_W}{\cos\theta_W}\sum_i \left[\bar{f}_i\gamma^\mu(\zeta_L^{Z'}(i)P_L + \zeta_R^{Z'}(i)P_R)f_i\right] Z'_\mu \end{aligned} \tag{3.7}$$

$\mathcal{L}_{NC}^{Z\prime}$ describes the neutral current interaction of the new gauge boson Z' with the standard model fermions f_i[2, 4, 20, 26].

3.5 Effective Z' and W' model

In this section, we present a phenomenological model (effective theory) which allows as to study the physics of Z' and W'. The two extra gauge boson are predicted by many BSM model especially those extended by an extra $SU(2)$ gauge groupe as the Pati-Salam model and right-left symmetric models, see [4, 27].

The Z' and W' are the new neutral and charged gauge bosons, respectively, which emerge after the symmetry breaking of an enlarging gauge symmetry group of the standard model. In additional, these bosons mediate new neutral and charged interactions that is described by the lagrangian,

$$\mathcal{L}_{CC}^{W\prime} = -\frac{g}{\sqrt{2}}[\sum_{i,j}\bar{u_i}V_{i,j}^{CKM}\gamma^\mu(\kappa_L^q P_L + \kappa_R^q P_R)d_j + \sum_i \bar{\nu}_{l_i}\gamma^\mu\kappa_L^l P_L l_i^-]W'^+_\mu \ + h.c \tag{3.8}$$

$$\mathcal{L}_{NC}^{Z\prime} = -\frac{g}{\cos\theta_W}\sum_i \left[\sum_{q=u,d}\bar{q}_i\gamma^\mu(\zeta_L^q P_L + \zeta_R^q P_R)q_i + \sum_{f=l,\nu_l}\bar{f}_i\gamma^\mu(\zeta_L^f P_L + \zeta_R^f P_R)f_i\right] Z'_\mu + h.c \tag{3.9}$$

$\mathcal{L}_{NC}^{Z'}$ and $\mathcal{L}_{CC}^{W'}$ are the additional neutral and charged current Lagrangian that describe the interaction of the SM fermions (quarks and leptons) with the new gauge bosons (Z' and W'). Where $V_{i,j}^{CKM}$ is the Cabbibo-Kobayashi-Maskawa (CKM) matrix, i and j are the flavor indices. $\zeta^f_{R,L}$,$\zeta^q_{R,L}$ and $\kappa^{q,l}_{L,R}$ are arbitrary complex couplings which are different for quarks and leptons, with ζ^ν_R = $\kappa^l_R = 0$ (There is no right-handed neutrino in SM). We can write $\zeta^f_{R,L}$ as,

$$\zeta^f_{R,L} = \zeta^f_{Z_{SSM}}(g_V^f \pm g_A^f) \tag{3.10}$$

where g_V^f and g_A^f are vector and axial couplings, in standard model given by,

$$\begin{aligned} g_V^f &= \frac{1}{2}T_L^{3,f} - Q^f \\ g_A^f &= -\frac{1}{2}T_L^{3,f} \end{aligned} \tag{3.11}$$

We can use the sequential standard model as a convenient reference to describe the other model. In this model, the Z_{SSM} couplings to the standard model fermions are the same as in Z_{SM} so in this case, we have: $\zeta^f_{Z_{SSM}} = 1, \quad \kappa^q_R = 0$, $\quad$ and $\kappa^{q,l}_L = 1$. So,

$$\zeta_L^f = \frac{\kappa_R^f \cos\theta_W \tan^2\theta_W}{\sqrt{1 - \frac{tan^2\theta_W}{(\kappa_R^f)^2}(\kappa_R^f)^2}}[T_L^{3,f} - Q^f] \tag{3.12}$$

$$\zeta_R^f = \frac{\kappa_R^f \cos\theta_W}{\sqrt{1 - \frac{tan^2\theta_W}{(\kappa_R^f)^2}}}[T_R^{3,f} - \frac{1}{(\kappa_R^f)^2}\tan^2\theta_W Q^f] \tag{3.13}$$

3.6 Flavor Changing Neutral Current Effect (FCNC)

In the standard model, the neutral current interaction mediated by γ and Z don't change the flavor at tree level due to GIM mechanism. In particular, the chiral couplings $\zeta_{L,R}(i)$ of the standard model (Shown in (3.6)) are diagonal matrices. The flavor changing neutral current effect (FCNC) occurs when the matrices are non diagonal (if $\theta_c \neq 0$) [2, 26, 28]. However, there are some models predict the new gauge boson Z' in which they mediate the flavor changing neutral current interaction at tree level. That means that the couplings of Z' to standard model fermions (quarks and leptons) are non diagonal matrices or we can say that Z' have a non universal couplings in this case. We can write the general form of the neutral current interaction mediated by the Z' gauge boson in Eq (3.5) as:

$$J^{\mu}_{Z'} = \bar{f}^0_L \gamma^{\mu} \zeta^{Z'}_{fL} f^0_L + \bar{f}^0_L \gamma^{\mu} \zeta^{Z'}_{fL} f^0_L \tag{3.14}$$

where f^0_L and f^0_R are column vectors which represent the weak eigenstates of the left and right chiral fermions, respectively. We can write $f^0_{L,R}$ in the base of mass eigenstates $f_{L,R}$ as:

$$f^0_L = V^{f\dagger}_L f_L, \qquad f^0_R = V^{f\dagger}_R f_R \tag{3.15}$$

where $V^f_{L,R}$ are unitary.

We write the current $J^{\mu}_{Z'}$ in term of mass basis in Eq (3.15), we obtain:

$$J^{\mu}_{Z'} = \bar{f}_L \gamma^{\mu} V^f_L \zeta^{Z'}_{fL} V^{f\dagger}_L f_L + \bar{f}_R \gamma^{\mu} V^f_R \zeta^{Z'}_{fR} V^{f\dagger}_R f_R \tag{3.16}$$

where $V_{L,R} = V^f_{L,R} V^{f\dagger}_{L,R}$ is the flavor mixing matrix.

We can define two types of flavor mixing matrix:

- the CKM matrix is the quark mixing matrix which represents the mixing of quark flavor (or mixing of two different flavor of quark): $V_{CKM} = V^u_L V^{d\dagger}_L$.
- the PMNS matrix is the lepton matrix which represent the mixing of lepton flavor (or mixing of two different flavor of lepton): $V_{PMNS} = V^{\nu}_L V^{e\dagger}_L$.

and V is defined as a rotation matrix which is given by the form:

$$V = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$

we can write the Eq (3.16) as:

$$J^{\mu}_{Z'} = \bar{f}_L \gamma^{\mu} B^{Z'}_{fL} f_L + \bar{f}_R \gamma^{\mu} V_R \zeta^{Z'}_{fR} f_R \tag{3.17}$$

where

$$B^{Z'}_{fL} = V^f_L \zeta^{Z'}_{fL} V^{f\dagger}_L, \qquad B^{Z'}_{fR} = V^f_R \zeta^{Z'}_{fR} V^{f\dagger}_R \tag{3.18}$$

in the case where $\zeta^{Z'}_{fL,R}$ are diagonal and $B^{Z'}_{fL,R} = \zeta^{Z'}_{fL,R}$, the Z' couplings are family universal. However, the $B^{Z'}_{fL,R}$ not necessarily diagonal. In some models of the additional gauge boson Z', the couplings could be non diagonal so that the couplings will be familly non universal such as in supersymmetric models. Therefore, the neutral current interaction mediated by the new heavy gauge boson Z' can change flavor in which there is no reason prevent flavor changing in quark or leptonic sector at tree level.

We can write the neutral current lagrangian in Eq (3.7) in this case as:

$$\mathcal{L}_{Z'} = \bar{q}\gamma^{\mu}[g^L_{qq'} P_L + g^R_{qq'} P_R] q' Z'_{\mu} + \bar{l}\gamma^{\mu}[g^L_{ll'} P_L + g^R_{ll'} P_R] l' Z'_{\mu} + h.c \tag{3.19}$$

where $g^{R,L}_{qq'}, g^{R,L}_{ll'}$ are the right and left chiral couplings to different types of quarks and leptons,respectively.

We mention here an important process mediated by Z' and allow to change flavor which is the meson mixing. The meson mixing is one of the most important FCNC processes at tree level in which such processes are not allowed in standard model. The processes are shown in figure (3.5) in which the mixing between a meson $\mathcal{M}$ and its conjugate state $\bar{\mathcal{M}}$ are allowed at tree level, where $\mathcal{M}$ = K,D,B_d or B_s meson .

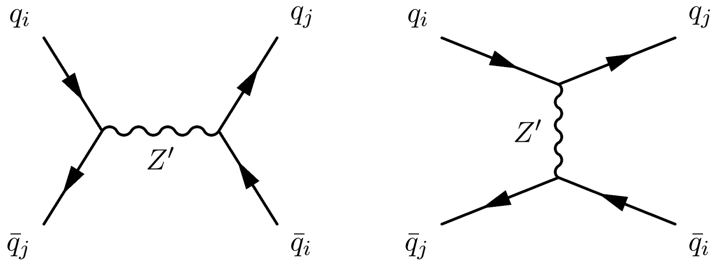


Figure 3.5: meason mixing via Z' at tree level

3.7 The Minimal U_{B-L} Model

The minimal U_{B-L} model is one of the simplest extensions of the standard model based on the enlarging gauge symmetry group:

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L} \tag{3.20}$$

where $U(1)_{B-L}$ is an extra gauge factor added to SM gauge symmetry group, the generator of this group correspond to the leptonic and baryonic quantum number (B - L)[2, 23], see also [30, 36].

In general, all standard model particles have charges under $U(1)_{B-L}$, the charges of different particles are shown in the table (3.1).

ψ	q_L	u_R	d_R	l_L	e_R	H	ν_R	χ
$SU(3)_C$	3	3	3	1	1	1	1	1
$SU(2)_L$	2	1	1	2	1	2	1	1
Y	$\frac{1}{6}$	$\frac{2}{3}$	-$\frac{1}{3}$	-$\frac{1}{2}$	-1	$\frac{1}{2}$	0	0
B - L	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	-1	-1	0	-1	2

Table 3.1: The $U(1)_{B-L}$ charge assignments of SM particles.

This model contains three extra particles in addition to standard model particles:

- Three generation of right-handed neutrino (ν_R): two are added to cancel the gauge anomalies and one with zero U(B - L) charge to give neutrino masses and mixings through see saw mechanism.
- A complex neutral scalar singlet χ to break the new additional symmetry $U(1)_{B-L}$ and gives mass to the new gauge boson Z'.
- The new heavy gauge bosons Z': in this model it is denoted by "Z_{B-L}" in which it is associate with the conserved quantities baryon and lepton number. The charge are given by: $Q_{BL} \propto T_{BL}$, where $T_{BL} = (B-L)/2$.

The gauge boson Z_{B-L} can be coupled only to fermions that have the same B - L quantum number so that the $Z-Z'$ mixing is vanished at tree level. Furthermore, the Left and Right handed fermions have the same $B-L$ quantum number.

$$g^R_{Z'} = g^L_{Z'} \tag{3.21}$$

In this case, the couplings of Z_{B-L} to Dirac fermions are vectorial couplings (the axial couplings are equal to zero):

$$\begin{aligned} g^V_{Z'} &= \frac{g^R_{Z'} + g^L_{Z'}}{2} = (B-L)g'_1 \\ g^A_{Z'} &= \frac{g^R_{Z'} - g^L_{Z'}}{2} = 0 \end{aligned} \tag{3.22}$$

But the Majorana neutrinos (the neutrinos after see-saw mechanism) have pure axial couplings to the Z_{B-L}.

3.7.1 The $U(1)_{B-L}$ Model Lagrangian

The total lagrangian of the model is given by,

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_f + \mathcal{L}_Y + \mathcal{L}_S \tag{3.23}$$

where each term will be discussed in the next sections.

Yang-Mills sector

$\mathcal{L}_{YM}$ represent the Yang-Mills Lagrangian for the Z' boson is of the form :

$$\mathcal{L}_{YM} = \mathcal{L}^{SM}_{YM} + \mathcal{L}^{Abel}_{YM} \tag{3.24}$$

where $\mathcal{L}^{SM}_{YM}$ is the usual SM lagrangian while $\mathcal{L}^{Abel}_{YM}$ is given by :

$$\mathcal{L}^{Abel}_{YM} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}F'^{\mu\nu}F'_{\mu\nu} \tag{3.25}$$

where

$$F'_{\mu\nu} = \partial_\mu F'_\nu - \partial_\nu F'_\mu \tag{3.26}$$

$$F_{\mu\nu} = \partial_\mu F_\nu - \partial_\nu F_\mu \tag{3.27}$$

F_ν is the usual SM gauge field associate to $U(1)_Y$ gauge symmerty group , while F'_ν is is the new gauge field.

In this case, we can write the covariant derivative as:

$$D_\mu = \partial_\mu + ig_s T^\alpha G^\alpha_\mu + igT^a W^a_\mu + ig_1 Y B_\mu + ig_E Y^E B'_\mu \tag{3.28}$$

where gs, g and g_1 are the couplings related to SM gauge field G^α_μ , W^a_μ and B'_μ, respectively. T^α, T^a and Y are the generators of the group. We introduce here an effective charge Y^E and effective coupling g_E, with

$$g_E Y^E = \tilde{g}Y + g'_1 Y_{B-L} \tag{3.29}$$

where $\tilde{g}$ and g'_1 are free parameters determined experimentally.

The covariant derivative is given by,

$$D_\mu = \partial_\mu + ig_s T^\alpha G^\alpha_\mu + igT^a W^a_\mu + ig_1 Y B_\mu + i(\tilde{g}Y + g'_1 Y_{B-L})B'_\mu \tag{3.30}$$

Our model (the "pure " U_{B_L} model) is defined at the electroweak scale when the charge $\tilde{g} = 0$, so that $g_E Y^E = g'_1 Y_{B-L}$ there is no mixing between the the SM Z and Z' at tree level. in this case the covariant derivative is written as :

$$D_\mu = \partial_\mu + ig_s T^\alpha G^\alpha_\mu + igT^a W^a_\mu + ig_1 Y B_\mu + ig'_1 Y_{B-L} B'_\mu \tag{3.31}$$

In the sequential standard model SSM when the couplings of Z' and Z are the same, we have $g'_1 = g_1$ and $\tilde{g} = 0$ at the EW scale. In this case, the covariant derivative is written as:

$$D_\mu = \partial_\mu + ig_s T^\alpha G^\alpha_\mu + igT^a W^a_\mu + ig_1(Y B_\mu + Y_{B-L} B'_\mu) \tag{3.32}$$

$D_\mu q = qg'_1 Y_{B-L} B'_\mu q$ This term describe the interactions between the fermions of the standard model that have the conserved charge $q = B - L$ with the new gauge boson Z'.

Fermion sector

$\mathcal{L}_f$ is the lagrangian describing the interaction of SM fermion and the additional one is given by :

$$\mathcal{L}_f = \sum_{k=1}^{3}\left[i\bar{q}_{kL}\gamma_\mu D^\mu q_{kL} + i\bar{u}_{kR}\gamma_\mu D^\mu u_{kR} + i\bar{d}_{kR}\gamma_\mu D^\mu d_{kR} + i\bar{l}_{kL}\gamma_\mu D^\mu l_{kL} + i\bar{e}_{kR}\gamma_\mu D^\mu e_{kR} + i\bar{\nu}_{kR}\gamma_\mu D^\mu \nu_{kR}\right] \tag{3.33}$$

where q_{kL} and l_{kL} are the three generations of Left-handed quark and lepton doublets $(k = 1, 2, 3)$ and u_{kR} , d_{kR} and e_{kR} are the Right- handed components of up-type, down-type quarks and charged leptons, respectively.

The charges of fermions under $U(1)_{B-L}$ are introduced to cancel the gauge anomelies of Yukawa interactions term such as,

$$\begin{aligned} Q^u_{B-L} &= Q^q_{B-L} + Q^H_{B-L} = \frac{1}{3} \\ Q^d_{B-L} &= Q^q_{B-L} - Q^H_{B-L} = \frac{1}{3} \\ Q^e_{B-L} &= Q^l_{B-L} + Q^H_{B-L} = -1 \end{aligned} \tag{3.34}$$

The charges of these fermions under $U(1)_{B-L}$ shown in table (3.1). This lagrangian is the same as the standard model lagrangian with an additive term containing three new generation of neutrino are denoted by ν_{kR}, it is given by:

$$\mathcal{L}^{new}_f = \bar{\nu}_{kR}\gamma_\mu D^\mu \nu_{kR} \tag{3.35}$$

Consequently, $\mathcal{L}^{new}_f$ is the new term describing the interactions of the new right handed neutrinos participate to cancel the $B - L$ gauge anomelies of the theory [35] and to remain the theory gauge invariant.

Yukawa sector

$\mathcal{L}_Y$ is the Yukawa lagrangian which is given by:

$$\mathcal{L}_Y = -y^d_{jk}\bar{q}_{jL} d_{kR} H - y^u_{jk}\bar{q}_{jL} u_{kR}\tilde{H} - y^e_{jk}\bar{l}_{jL} e_{kR} H - y^\nu_{jk}\bar{l}_{jL}\nu_{kR}\tilde{H} - y^M_{jk}(\bar{\nu_R})^c_j \nu_{kR}\chi + h.c \tag{3.36}$$

where $\tilde{H} = i\sigma^2 H^*$

Higgs sector

$\mathcal{L}_S$ is the scalar lagrangian is of the form:

$$\mathcal{L}_s = (D^\mu H)^\dagger D_\mu H + (D^\mu \chi)^\dagger D_\mu \chi - V(H,\chi) \tag{3.37}$$

where χ is the new additional scalar (singlet field) and $V(H,\chi)$ is the scalar potential of Higgs H and χ, it is given by,

$$V(H,\chi) = m^2 H^\dagger H + \mu^2|\chi|^2 + \lambda_1 (H^\dagger H)^2 + \lambda_2 |\chi|^4 + \lambda_3 H^\dagger H, \qquad |\chi|^2 \tag{3.38}$$

$$= m^2 H^\dagger H + \mu^2|\chi|^2 + \left(H^\dagger H |\chi|^2\right) \begin{pmatrix} \lambda_1 & \frac{\lambda_3}{2} \\ \frac{\lambda_3}{2} & \lambda_2 \end{pmatrix} \begin{pmatrix} H^\dagger H \\ |\chi|^2 \end{pmatrix}, \qquad \mu > 0 \tag{3.39}$$

where the new charges $(B-L)$ of H and χ are respectively 0 and 2.

Symmetry Breaking and the mass of Z'

The $U(1)_{B-L}$ model includes a new singlet scalar boson χ in addition to the higgs boson so it is necessary to implement the spontaneous symmetry breaking of the $SU(2)_L \times U(1)_Y \times U(1)_{B-L}$. The new scalar χ contributes to break the $U(1)_{B-L}$ symmetry and to give mass to the new gauge boson Z'. The mechanism of $U(1)_{B-L}$ breaking is more complicated than the spontaneous electro-weak symmetry breaking (EWSB)[31, 32, 33, 34].

The last term in Eq (3.39) involve the large field values, so that the matrix must be positive :

$$\begin{pmatrix} \lambda_1 & \frac{\lambda_3}{2} \\ \frac{\lambda_3}{2} & \lambda_2 \end{pmatrix}$$

the minimisation of V must achieve the following conditions:

$$\begin{aligned} 4\lambda_1\lambda_2 - {\lambda_3}^2 &\geq 0 \\ \lambda_1\lambda_2 &\geq 0 \end{aligned} \tag{3.40}$$

The Higgs doublet H and the new scalar singlet χ can be parameterized as,

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} -iW^- \\ v + (h + iz) \end{pmatrix}$$

$$\chi = \frac{1}{\sqrt{2}} (x + (h' + iz'))$$

where H is the usual $SU_L(2)$ doublet which gives masses to SM fermions and responsible for the SM gauge symmetry breaking while χ is the new U_{B-L} scalar singlet responsible to generate the masses of the new gauge boson Z'. $W^\pm = W^1 \mp iW^2$, z and z' are the Goldstone bosons of $W\pm$, Z and Z', respectively.

After spontaneous symmetry breaking we have,

$$H_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix} \tag{3.41}$$

$$\chi_0 = \frac{x + h'}{\sqrt{2}} \tag{3.42}$$

where x ,v are real and non negative, it is the VEV for χ and H. We can express it as,

$$|H_0| = \frac{v^2}{2} = \frac{-\lambda_2 m^2 + \frac{\lambda_3}{2}\mu^2}{2\lambda_1\lambda_2 - \frac{{\lambda_3}^2}{2}} \tag{3.43}$$

$$|\chi_0| = \frac{x^2}{2} = \frac{-\lambda_1\mu^2 + \frac{\lambda_3}{2}m^2}{2\lambda_1\lambda_2 - \frac{\lambda_3{}^2}{2}} \tag{3.44}$$

The covariant derivate for the higgs doublet H with $Y = \frac{1}{2}$:

$$D_\mu H = (\partial_\mu + ig_s T^\alpha G^\alpha_\mu + igT^a W^a_\mu + ig_1 Y B_\mu - i\tilde{g}B'^\mu)H \tag{3.45}$$

where $T^\alpha = \frac{1}{2}\sigma_i$, σ_i is the Pauli matrices. The kinetic term in equation (3.37):

$$(D^\mu H)^\dagger D_\mu H = \frac{1}{2}\partial^\mu h \partial_\mu h + \frac{1}{8}(h+v)^2\left[g^2|W^{1\mu} - iW^{2\mu}|^2 + (gW^{3\mu} - g_1 B^\mu - \tilde{g}B'^\mu)^2\right] \tag{3.46}$$

For the Higgs doublet covariant derivate we consider the coupling $g_E Y^E = \tilde{g}Y$, where $g'_1 = 0$ and $Y = \frac{1}{2}$. For the new singlet scalar χ , $g_E Y^E = g'_1 Y_{B-L} Y$ (with Y^{B-L}_χ hypercharge of the new singlet χ). The covariant derivative of a scalar field χ can be written as,

$$D_\mu \chi = (\partial_\mu + ig'_1 Y_{B-L} B'_\mu)\chi \tag{3.47}$$

where $Y_{B-L} = 2$

$$D_\mu \chi = (\partial_\mu + 2ig'_1 B'_\mu)\chi \tag{3.48}$$

in this case, the kinetic term is given by:

$$(D^\mu \chi)^\dagger D_\mu \chi = \frac{1}{2}\partial^\mu h' \partial_\mu h' + \frac{1}{2}(h'+x)^2(g'_1 2B'^\mu)^2 \tag{3.49}$$

$$\frac{1}{8}v^2(gW^{3\mu} - g_1 B^\mu - \tilde{g}B'^\mu)^2 = \frac{1}{8}v^2\begin{pmatrix} B^\mu & W^{3\mu} & B'^\mu \end{pmatrix}\begin{pmatrix} g_1^2 & -g_1 g & g_1\tilde{g} \\ -g_1 g & g^2 & -g\tilde{g} \\ g_1\tilde{g} & -g\tilde{g} & \tilde{g}^2 \end{pmatrix}\begin{pmatrix} B_\mu \\ W_{3\mu} \\ B'^\mu \end{pmatrix}$$

We consider the orthogonal transformation, the rotation between the gauge and the mass bases is given by:

$$\begin{pmatrix} B^\mu \\ W^{3\mu} \\ B'^\mu \end{pmatrix} = \begin{pmatrix} \cos\vartheta_W & -\sin\vartheta_W\cos\vartheta' & \sin\vartheta_W\sin\vartheta' \\ \sin\vartheta_W & \cos\vartheta_W\cos\vartheta' & -\cos\vartheta_W\sin\vartheta' \\ 0 & \sin\vartheta' & \cos\vartheta' \end{pmatrix}\begin{pmatrix} A^\mu \\ Z^\mu \\ Z'^\mu \end{pmatrix}$$

where $-\frac{\pi}{4} < \vartheta < \frac{\pi}{4}$
after diagonalize the mass matrix, the mass of A, Z and the new gauge boson Z' are given by:

$$M_A = 0 \tag{3.50}$$

$$M_{Z,Z'} = \sqrt{g^2 + g_1^2}.\frac{v}{2}\left[\frac{1}{2}\left(\frac{\tilde{g}^2 + 16(\frac{x}{v})^2 g_1'^2}{g^2 + g_1^2} + 1\right) \mp \frac{\tilde{g}}{\sin 2\vartheta'\sqrt{g^2 + g_1^2}}\right]^{\frac{1}{2}} \tag{3.51}$$

where :

$$\tan 2\vartheta' = \frac{2\tilde{g}\sqrt{g^2 + g_1^2}}{\tilde{g}^2 + 16(\frac{x}{v})^2 g_1'^2 - g^2 - g_1^2} \tag{3.52}$$

and

$$\sin 2\vartheta' = \frac{2\tilde{g}\sqrt{g^2 + g_1^2}}{\sqrt{(\tilde{g}^2 + 16(\frac{x}{v})^2 g_1'^2 - g^2 - g_1^2)^2 + (2\tilde{g})^2(g^2 + g_1^2)}} \tag{3.53}$$

the Z and Z' mass can be written as,

$$M_Z = \frac{v}{2}\sqrt{g^2 + g_1^2} \tag{3.54}$$

$$M_{Z'}{}^{B-L} = 2g'_1 x. \tag{3.55}$$

3.8 Z' production and decay

The new heavy gauge boson "Z'" can be directly produced in the final state where one has to its decay products since it life time is very small since it very massive. At the LHC, one can produce it singly or in paire by the follwing reactions

$$pp \longrightarrow Z' + X \longrightarrow \text{SM particles} \tag{3.56}$$
$$pp \longrightarrow Z' + Z \longrightarrow \text{SM particles} \tag{3.57}$$

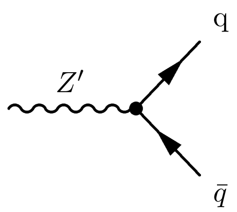
Also, it can be produced in indirectly as a virtul particle as in the following reaction for example

$$pp \longrightarrow Z' \longrightarrow t + \bar{t} \tag{3.58}$$

Z' Decay into SM fermions

The Z' gauge boson can decay following many modes to SM particles. We consider the case of the Z' decay into quark-anti-quark,

$$Z' \to q\bar{q} \tag{3.59}$$



Z' decays only to SM fermions at tree level, the decay width for $Z' \to q\bar{q}$ is given by :

$$\Gamma_{Z'\to q\bar{q}} = \frac{g^2_{Z'} M_{Z'}}{12\pi}\sqrt{1-\frac{4m_q^2}{M_{Z'}^2}}\Big[a^{t2}_{Z'}(1+\frac{2m_q^2}{M_{Z'}^2}) + b^{2t}_{Z'}(1-\frac{2m_q^2}{M_{Z'}^2})\Big] \tag{3.60}$$

where $M_{Z'}$ and m_q are the masses of Z' and the quarks, respectively.

There are other channel of Z' decay, the figure (3.6) shows some of Z' decays modes:

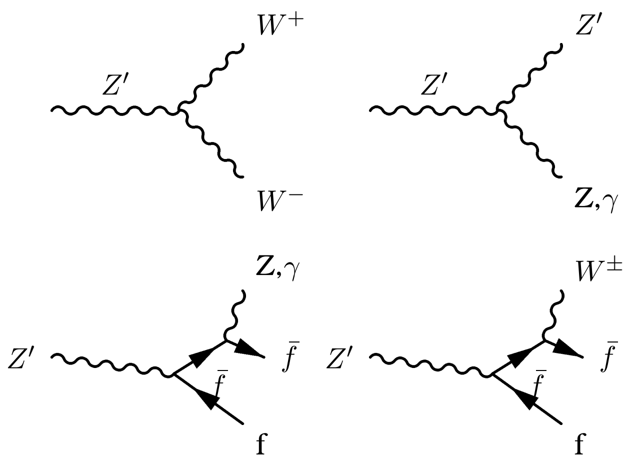
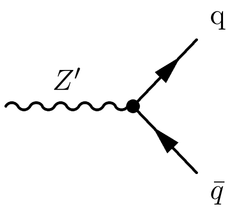


Figure 3.6:

3.9 Feynman rules

3.9.1 Feynman rules for the sequential model

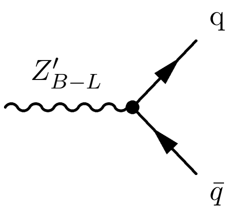


$$i(2\pi)^4 \frac{g_w}{4\cos\theta_w}\gamma^\mu (g_V^{q'} + g_A^{q'}\gamma_5)$$

$g_w = \sqrt{4\pi\alpha_W}$ is the electroweak coupling, $a^q_{Z'}$ and $b^q_{Z'}$ are the axial and the vector couplings, respectively.

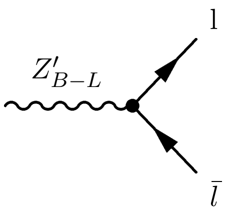
3.9.2 Feynman rules for the U_{B-L} Model

- $Z' - q - q$: In the U_{B-L} Model, the vector coupling $b^q_{Z'} = 0$, this model have only the axial coupling.



$$-\frac{1}{3}g'_1\gamma^\mu$$

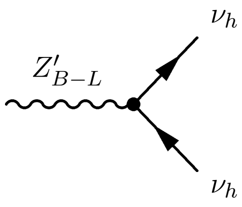
- $Z' - l - l$:



$$g'_1\gamma^\mu$$

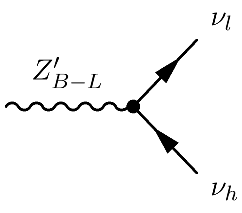
where $l = e, \mu, \tau$

- $Z' - \nu_h - \nu_h$: Z'_{B-L} couple to heavy neutrinos ν_v and light neutrino ν_l,



$$g'_1 \sin 2\alpha_\nu$$

- $Z' - \nu_l - \nu_h$:



$$g'_1 \cos 2\alpha_\nu$$

where $\sin 2\alpha_\nu = -\frac{2}{\sqrt{4+\frac{M_{\nu_h}}{M_{\nu_l}}}}$, $\cos 2\alpha_\nu = -\sqrt{\frac{M_{\nu_h}}{4m_{\nu_l}+M_{\nu_h}}}$

m_{ν_l} and M_{ν_h} are masses of light and heavy neutrinos.

CHAPTER 4

Leading order top-quark pair production at the LHC

In this chapter, we study the top quark pair production in a standard model extension including an extra gauge boson Z'. We present the analytical calculation of the LO partonic cross section for the reaction $pp \to t\bar{t}$ in the existence of Z', where we use the `hip` program to perform this calculation. To compute numerically the hadronic cross and the differential distributions, we use `MadGraph` and `MadAnalysis`.

4.1 Overview of hadronic physics

4.1.1 Large Hadron Collider

The Large Hadron Collider (LHC) is the center of particle physics experiments located at the European Council for Nuclear Research (CERN), in Geneva. It was designed to collide protons with higher energy up to the center-of-mass energy $\sqrt{s} = 14$ TeV with a design luminosity of 10^{34} cm^{-2} s^{-1}, in which, the proton beam could be accelerated to 7 TeV in opposite directions. The figure (4.1) shows the LHC and its detectors.

The primary object of the LHC is to produce new particles, and detect the phenomena that occur in particle accelerators. The collider contains four main particle detectors: ATLAS, CMS, ALICE and LHCb [18, 24]. The designs of these detectors are different, and each detector has its own characteristic. For example, CMS is dedicated to the study of heavy ion collisions, and the Large Hadron Collider (LHCb) is dedicated to the study of quark physics ... etc.

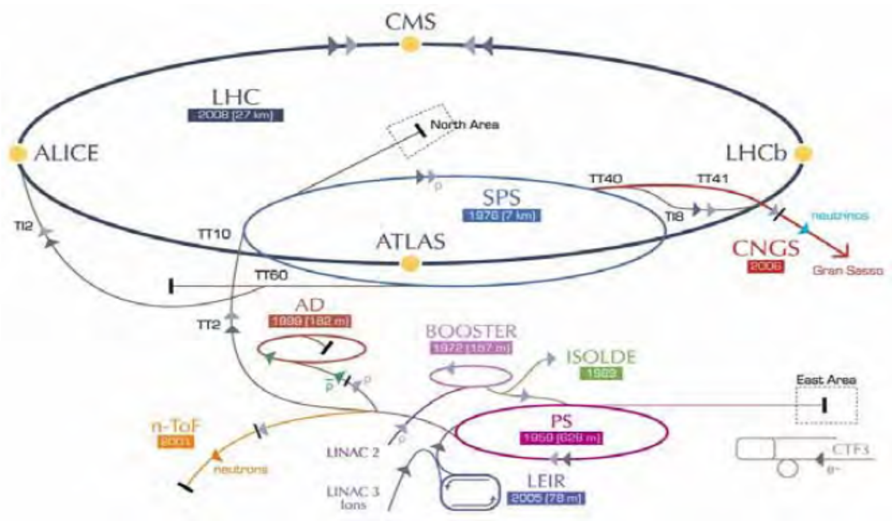


Figure 4.1: The CERN accelerator complex .

4.1.2 Hadronic Cross Section

The hadronic cross section measures the probability of particle production in collisions. We consider the following processes,

$$A + B \longrightarrow C + X \tag{4.1}$$

In QCD parton model, this process is described by some sub-processes "hard scattering" at high energy. We can calculate the cross section for a hadronic scattering of two particles A and B by calculation the cross section of the sub-processes involving the reaction of two partons a and b coming from the hadron A and B, respectively as shown in figure (4.2)[9, 37, 38]. In this case, the hadronic cross section $\sigma(A + B \rightarrow C + X)$ can be obtained by convolving the partonic cross section $\sigma(a + b \rightarrow c + d)$ with the Parton distribution function $f_a^A(x_A, Q^2)$. Then, the hadronic cross section can be written as,

$$\sigma^{had}_{AB\longrightarrow CX} = \sum_{a,b} \int dx_A dx_B f_a^A(x_A, Q^2) f_b^B(x_B, Q^2) \sigma^{part}_{ab\longrightarrow cd} \tag{4.2}$$

where Q^2 is the factorization scale, x_A and x_B are the fraction of moneta of particle a and b, $f_a^A(x_A, Q^2)$ and $f_b^B(x_B, Q^2)$ are the parton distribution functions which describe the probability for producing a and b partons from A and B hadrons (we can obtain them by fitting data) and $\sigma^{part}_{ab\longrightarrow cd}$ is the partonic cross section.

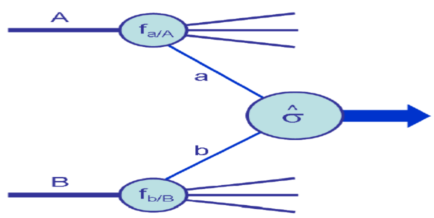


Figure 4.2: Structure of hard scattering process

4.1.3 Kinematic Variables

The kinematic variables are invariant quantities under Lorentz transformations. In the flowing, we suppose that the z-axis that represents the beam axis in accelerator physics[9, 37, 39, 40].

- **Transverse Momentum P_T:** The momentum of a particle can be expressed as:

$$P = P_T + P_L \tag{4.3}$$

where $P_T = (p_x, p_y, 0)$ is the orthogonal components and $P_L = (0, 0, p_z)$ is the longitudinal components. P_T is the transverse momentum that is present in the transverse plane to the beam direction (x, y), it is given by,

$$P_T = \sqrt{p_x^2 + p_y^2} = p \sin\theta \tag{4.4}$$

where θ is the polar angle (the angle between the particle direction and the beam direction).

- **Transverse Mass M_T:** The transverse mass M_T is defined by,

$$M_T = \sqrt{p_x^2 + p_y^2 + M_I^2} \tag{4.5}$$

where M_I is the invariant mass which is given by,

$$M_I^2 = (p_1 + p_2 + \cdots + p_n)^2 \tag{4.6}$$

We need to know the momentum of all the particles in the final states to calculate the invariant mass M_I. The transverse mass can be express in term of P_T as:

$$M_T = \sqrt{P_T^2 + M_I^2} \tag{4.7}$$

- **Rapidity y:** The rapidity is a dimensionless variable related to the relative velocity of a particle, used in accelerator physics. The rapidity of a particle is defined as:

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} \tag{4.8}$$

where

$$p_z = p \cos\theta \quad , \quad E = \sqrt{p^2 + m^2} \tag{4.9}$$

we can write the rapidity in term of M_T as:

$$y = \frac{1}{2} \ln \frac{E + p_z}{M_T c^2} \tag{4.10}$$

In the relativistic limit, the rapidity is equal to the **pseudorapidity**.

- **Pseudo-rapidity η:** The pseudo-rapidity is a variable can be measured without need to know the mass and momentum of the particle. It is related to the angle between the particle direction and the beam axis. The pseudo-rapidity is defined as :

$$\eta = \frac{1}{2} \ln \left[\frac{|p| + p_z}{|p| - p_z} \right] = -\ln \tan\left(\frac{\theta}{2}\right) \tag{4.11}$$

where $\cos\theta = \frac{p_z}{p}$, θ is the polar angle (the angle between the particle direction and the beam axis).

- **Transverse Energy E_T:** The transverse energy is defined as,

$$E_T = \sqrt{P_T^2 + M^2} \tag{4.12}$$

We notice that, in terms of the new variables, the components of the 4-momentum can be written as,

$$P = (M_T \cosh\eta, p_x, p_y, M_T \sinh\eta) \tag{4.13}$$

4.2 Tools for the cross section calculation

4.2.1 Hip Program

Hip program is a useful program which contains a set of packages written in the computer algebra language Mathematica, it helps us to calculate the amplitude and the cross section analytically [42]. Here are some basic commands of this code,

- **SetMass** [. . .]: to define the particle masses.
- **SetReal[...]**: to define the real variable.
- **setMandelstam[...]** : to write the scalar product in term of Mandelstam variables (s, t, u).
- **SpinorU[..]**: Dirac Spinor u.
- **SpinorUbar[..]** : Dirac Spinor $\bar{u}$.
- **SpinorV[..]**: Dirac Spinor v.
- **SpinorVbar[..]**: Dirac Spinor $\bar{v}$.
- **DiracGamma[..]** : Gamma matrices (γ_μ).

- **DiracGamma 5** : Gamma 5 (γ_5).
- **DiracTrace**[..]: to calculate the trace of Dirac matrices.
- **AbsSquared**[...]: to calculate the amplitude squared.
- **DotProduct** [..]: to calculate the scalar product.
- **contract**[...]: to contract over lorentz indices .
- **Prepareindex**[..]: to define the Lorentz indices.

4.2.2 MadGraph 5 Program

MadGraph 5 is a program written in Python language, which can automatically generate matrix elements and Feynman diagrams. We use it to perform the numerical calculation of the cross section for the standard model processes and beyond, since it can compute the LO and NLO cross section for any model [41]. We use MADGRAPH v2.7.3 version to calculate the cross section for top pair production $\sigma_{t\bar{t}}$. To generate a processes and calculate the cross section, we follow the following steps:

- open the terminal and type : **cd master** / $MG5\_aMC\_v2\_7\_3$
- we run the program by: $./bin/mg5\_aMC$
- import the model: **import model** $VPrime\_NLo$
- to generate the process: **generate p p** $>$ **t** $\tilde{t}$. We have three cases:
  - if we consider only the electroweak processes : **generate p p** $>$ **t** $\tilde{t}$ **QCD** $<= 0$
  - if we consider both electroweak and QCD processes: **generate p p** $>$ **t** $\tilde{t}$ **QED** $<= 10$ **QCD** $<= 10$
  - if we consider only the QCD processes: **generate p p** $>$ **t** $\tilde{t}$ **QED** $<= 0$
- to show the the Feynman diagrams: **display diagrams** we create files for each case :
- to create the repository containing the code: **output "file's name"**
- launch the numerical calculation of the cross section : **./bin/generate-events**
- type **0** in order to change some parameters such as the mass of Z' , the scale factor and the center of mass energy we use these command :
- **Scite Cards/run-card.dat**
- **Scite Cards/Param-card.dat**

4.3 Analytical calculation of the partonic cross section

In this section, we will calculate the partonic cross section analytically for both top pair production sub-processes $\sigma(gg \to t\bar{t})$ and $\sigma(q\bar{q} \to t\bar{t})$ in the existence of an additional electroweak sub-process mediated by the new Z' boson. We use hip program to calculate the amplitude squared.

4.3.1 Sub-process $q\bar{q} \to t\bar{t}$

Electroweak contribution

Consider the reaction :

$$q(p_1) + \bar{q}(p_2) \to t(p_3) + \bar{t}(p_4)$$

For this reaction, the are three possible Feynman diagrams mediated by γ, Z boson and the new gauge boson Z'. The diagrams are shown in the figure (4.3),

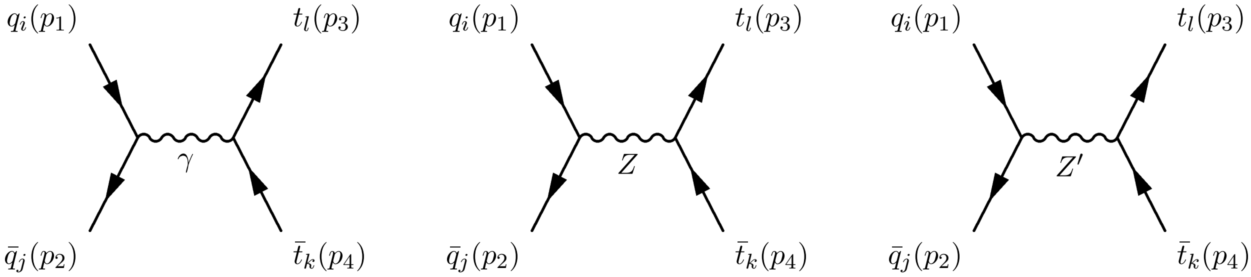


Figure 4.3: Electroweak Feynman diagrams for $q\bar{q} \to t\bar{t}$

Calculation of the Amplitudes:

The amplitude and its complex conjugate for the first diagram (by photon exchange) are given by,

$$\mathcal{M}_1 = \frac{ie^2}{(p_1+p_2)^2} Q_q Q_t \delta_{ji}\delta_{lk}\bar{v}_j(p_2)\gamma^{\mu}u_i(p_1)\bar{u}_l(p_3)\gamma_{\mu}v_k(p_4). \tag{4.14}$$

$$\bar{\mathcal{M}_1} = -\frac{ie^2}{(p_1+p_2)^2} Q_q Q_t \delta_{i'j'}\delta_{k'l'}\bar{u}_{i'}(p_1)\gamma^{\mu'}v_{j'}(p_2)\bar{v}_{k'}(p_4)\gamma_{\mu'}u_{l'}(p_3) \tag{4.15}$$

The amplitude summed and averaged over spins and colors is,

$$\begin{aligned}\bar{\sum}|\mathcal{M}_1|^2 &= \frac{1}{4N^2}\sum_{spin,color}|\mathcal{M}_1|^2 \\ &= \frac{e^4 Q_q^2 Q_t^2}{4N^2(p_1+p_2)^4}\sum_{spin,color}\delta_{ji}\delta_{lk}\delta_{i'j'}\delta_{k'l'}\left[\bar{v}_j(p_2)\gamma^{\mu}u_i(p_1)\right. \\ &\left.\bar{u}_l(p_3)\gamma_{\mu}v_k(p_4)\right] \times \left[\bar{u}_{i'}(p_1)\gamma^{\mu'}v_{j'}(p_2)\bar{v}_{k'}(p_4)\gamma_{\mu'}u_{l'}(p_3)\right]\end{aligned} \tag{4.16}$$

where i, j, k, l are color indices and N is the number of colors of quarks which equals to $N = 3$. We sum over spin and color of the initial and final states by making use of the relations,

$$\sum_{spin} u_i(p)\bar{u}_j(p) = \delta_{ij}(\not{p}+m) \tag{4.17}$$

$$\sum_{spin} v_i(p)\bar{v}_j(p) = \delta_{ij}(\not{p}-m) \qquad \sum_{col}\delta_{ji}\delta_{ij'}\delta_{ii'}\delta_{jj'} = \delta_{jj} = N$$

We notice that the mass of the initial state particles are neglected ($m_q = 0$). The calculation then reduces to a product of traces,

$$\bar{\sum}|\mathcal{M}_1|^2 = \frac{e^4 Q_q^2 Q_t^2}{4(p_1+p_2)^4} Tr[\not{p}_2\gamma^{\mu}\ \not{p}_1\gamma^{\mu'}] \times Tr[(\not{p}_4 - m_t)\gamma_{\mu'}(\not{p}_3+m_t)\gamma_{\mu}] \tag{4.18}$$

We remind that,

$$\begin{aligned} Tr[\gamma_{\alpha}\gamma_{\beta}] &= 4g_{\alpha}g_{\beta} \\ Tr[\gamma_{\lambda}\gamma_{\alpha}\gamma_{\rho}\gamma_{\beta}] &= 4[g_{\lambda\alpha}g_{\rho\beta} - g_{\lambda\rho}g_{\alpha\beta} + g_{\lambda\beta}g_{\alpha\rho}] \\ Tr[\gamma_5\gamma_{\alpha}\gamma_{\rho}\gamma_{\beta}\gamma_{\mu}] &= 4i\varepsilon_{\alpha\rho\beta\mu} \\ Tr[\gamma_{\alpha 1}\gamma_{\alpha 2}\gamma_{\alpha 3}...\gamma_{\alpha n}] &= 0, \quad For\ \ each\ \ odd\ \ number\ \ n \end{aligned} \tag{4.19}$$

after simplifying the traces, we get:

$$Tr[\not{p}_2\gamma^{\mu}\ \not{p}_1\gamma^{\mu\prime}] \times Tr[(\not{p}_4 - m_t)\gamma_{\mu\prime}(\not{p}_3+m_t)\gamma_{\mu}] = 32[p_2.p_3 p_1.p_4 + p_2.p_4 p_1.p_3 + m_t^2 p_2.p_1] \tag{4.20}$$

We define the Mandelstam variables as,

$$\begin{aligned} s &= (p_1+p_2)^2 = (p_3+p_4)^2 \\ u &= (p_2-p_3)^2 = (p_4-p_1)^2 \\ t &= (p_1-p_3)^2 = (p_4-p_2)^2 \end{aligned} \tag{4.21}$$

The scalar products can be expressed in terms of Mandelstam variables as:

$$\begin{aligned} p_1.p_2 = \frac{s}{2} \quad &, \quad p_3.p_4 = \frac{s-2m_t^2}{2} \\ p_2.p_3 = p_1.p_4 = \frac{m_t^2-u}{2} \quad &, \quad p_2.p_4 = p_1.p_3 = \frac{m_t^2-t}{2} \end{aligned} \tag{4.22}$$

where

$$s+t+u = \sum_{i=1,4} m_i = 2m_t^2 \tag{4.23}$$

We obtain

$$\bar{\sum}|\mathcal{M}_1|^2 = \frac{2e^4Q_q^2Q_t^2}{s^2}\left[u^2+t^2+2m_t^4-2m_t^2(u+t-s)\right] \tag{4.24}$$

For the second diagram , the amplitude and its conjugate are given by:

$$\begin{aligned} \mathcal{M}_2 = &\frac{ie^2Q_qQ_t\delta_{ji}\delta_{lk}}{4\sin^2\theta_w\cos^2\theta_w[(p_1+p_2)^2-m_z^2]}\left[\bar{v}_j(p_2)\gamma_\mu\big(a_L^q(1-\gamma_5)+a_R^q(1+\gamma_5)\big)u_i(p_1)\right] \\ &\times\left[\bar{u}_l(p_3)\gamma_\nu(a_L^t(1-\gamma_5)+a_R^t(1+\gamma_5))v_k(p_4)\right]\left[g^{\mu\nu}-\frac{(p_1+p_2)^\mu(p_1+p_2)^\nu}{m_z^2}\right]. \end{aligned} \tag{4.25}$$

$$\begin{aligned} \bar{\mathcal{M}}_2 = &\frac{-ie^2Q_qQ_t\delta_{i'j'}\delta_{k'l'}}{4\sin^2\theta_w\cos^2\theta_w[(p_1+p_2)^2-m_z^2]}\left[\bar{u}_{i'}(p_1)\gamma_{\mu'}\big(a_L^q(1-\gamma_5)+a_R^q(1+\gamma_5)\big)v_{j'}(p_2)\right] \\ &\times\left[\bar{v}_{k'}(p_4)\gamma_{\nu'}\big(a_L^t(1-\gamma_5)+a_R^t(1+\gamma_5)\big)u_{l'}(p_3)\right]\left[g^{\mu'\nu'}-\frac{(p_1+p_2)^{\mu'}(p_1+p_2)^{\nu'}}{m_z^2}\right]. \end{aligned} \tag{4.26}$$

Where θ_W is the weak mixing angle. The amplitude squared is given by,

$$\begin{aligned} \bar{\sum}|\mathcal{M}_2|^2 = &\frac{e^4Q_q^2Q_t^2}{16\times 4N^2\sin^4\theta_w\cos^4\theta_w\left[(p_1+p_2)^2-m_z^2\right]^2}\sum_{spin,color}\delta_{ji}\delta_{lk}\delta_{i'j'}\delta_{k'l'} \\ &\left[\bar{v}_j(p_2)\gamma_\mu(a_L^q(1-\gamma_5)+a_R^q(1+\gamma_5))u_i(p_1)\right]\left[\bar{u}_l(p_3)\gamma_\nu(a_L^t(1-\gamma_5)+a_R^t(1+\gamma_5))v_k(p_4)\right] \\ &\left[\bar{u}_{i'}(p_1)\gamma_{\mu'}(a_L^q(1-\gamma_5)+a_R^q(1+\gamma_5))v_{j'}(p_2)\right]\left[\bar{v}_{k'}(p_4)\gamma_{\nu'}(a_L^t(1-\gamma_5)+a_R^t(1+\gamma_5))u_{l'}(p_3)\right] \\ &\left[g^{\mu\nu}-\frac{(p_1+p_2)^\mu(p_1+p_2)^\nu}{m_z^2}\right]\left[g^{\mu'\nu'}-\frac{(p_1+p_2)^{\mu'}(p_1+p_2)^{\nu'}}{m_z^2}\right] \end{aligned} \tag{4.27}$$

we use the relations :

$$\begin{aligned} (1-\gamma_5)^2 &= 2(1-\gamma_5) \\ (1+\gamma_5)^2 &= 2(1+\gamma_5) \\ (1-\gamma_5)(1+\gamma_5) &= 0 \end{aligned} \tag{4.28}$$

$$\not{p}(1-\gamma_5) = (1+\gamma_5)\not{p} \tag{4.29}$$

we obtain

$$\begin{aligned} \bar{\sum}|\mathcal{M}_2|^2 = &\frac{e^4Q_q^2Q_t^2}{16\sin^4\theta_w\cos^4\theta_w[(p_1+p_2)^2-m_z^2]^2}\left[({a_L^q}^2+{a_R^q}^2)Tr[\not{p}_2\gamma_\mu\not{p}_1\gamma_{\mu'}]+({a_R^q}^2-{a_L^q}^2)\right. \\ &\left.\times Tr[\not{p}_2\gamma_\mu\gamma_5\not{p}_1\gamma_{\mu'}]\right] \\ &\left[({a_L^t}^2+{a_R^t}^2)Tr[\not{p}_3\gamma_\nu\not{p}_4\gamma_{\nu'}]+({a_L^t}^2-{a_R^t}^2)Tr[\not{p}_3\gamma_\nu\gamma_5\not{p}_4\gamma_{\nu'}]-2m_t^2a_L^ta_R^tTr[\gamma_\nu\gamma_{\nu'}]\right] \\ &\left[g^{\mu\nu}-\frac{(p_1+p_2)^\mu(p_1+p_2)^\nu}{m_z^2}\right]\left[g^{\mu'\nu'}-\frac{(p_1+p_2)^{\mu'}(p_1+p_2)^{\nu'}}{m_z^2}\right] \end{aligned} \tag{4.30}$$

We contract over Lorentz indices by using the relations in (4.20), after simplifying, we get:

$$\bar{\sum}|\mathcal{M}_2|^2 = \frac{e^4Q_q^2Q_t^2}{\sin^4\theta_w\cos^4\theta_w[s-m_z^2]^2}\left[{a_L^q}^2(2a_L^ta_R^tm_t^2s+{a_R^t}^2(m_t^2-t)^2+{a_L^t}^2(m_t^2-u)^2)\right.$$
$$\left.+{a_R^q}^2(2a_L^ta_R^tm_t^2s+{a_L^t}^2(m_t^2-t)^2+{a_R^t}^2(m_t^2-u)^2)\right] \quad (4.31)$$

For the third diagram the amplitude and it's conjugate are given by:

$$\mathcal{M}_3 = \frac{ie^2Q_qQ_t\delta_{ji}\delta_{lk}}{4\sin^2\theta_w\cos^2\theta_w[(p_1+p_2)^2-m_{z'}^2]}\left[\bar{v}_j(p_2)\gamma_\mu(a_L^q{'}(1-\gamma_5)+a_R^q{'}(1+\gamma_5))u_i(p_1)\right]$$
$$\times\left[\bar{u}_l(p_3)\gamma_\nu(a_L^t{'}(1-\gamma_5)+a_R^t{'}(1+\gamma_5))v_k(p_4)\right]\left[g^{\mu\nu}-\frac{(p_1+p_2)^\mu(p_1+p_2)^\nu}{m_{z'}^2}\right]. \quad (4.32)$$

$$\bar{\mathcal{M}}_3 = \frac{-ie^2Q_qQ_t\delta_{i'j'}\delta_{k'l'}}{4\sin^2\theta_w\cos^2\theta_w[(p_1+p_2)^2-m_{z'}^2]}\left[\bar{u}_{i'}(p_1)\gamma_{\mu'}(a_L^q{'}(1-\gamma_5)+a_R^q{'}(1+\gamma_5))v_j'(p_2)\right]$$
$$\times\left[\bar{v}_k'(p_4)\gamma_{\nu'}(a_L'^t(1-\gamma_5)+a_R'^t(1+\gamma_5))u_{l'}(p_3)\right]\left[g^{\mu'\nu'}-\frac{(p_1+p_2)^{\mu'}(p_1+p_2)^{\nu'}}{m_{z'}^2}\right]. \quad (4.33)$$

where $a_{L,R}'^q$ and $a_{L,R}'^t$ are the left and right coupling of quark with Z'.

For the third diagram, the amplitude is the same as the second diagram exchanged by Z boson, where Z' has a different coupling to quark and different mass. In this case , the squared amplitude is given by,

$$\bar{\sum}|\mathcal{M}_3|^2 = \frac{e^4Q_q^2Q_t^2}{\sin^4\theta_w\cos^4\theta_w[s-m_{z'}^2]^2}\left[a_L^q{'}^2(2a_L^t{'}a_R^t{'}m_t^2s+a_R^t{'}^2(m_t^2-t)^2+a_L^t{'}^2(m_t^2-u)^2)\right.$$
$$\left.+a_R^q{'}^2(2a_L^t{'}a_R^t{'}m_t^2s+a_L^t{'}^2(m_t^2-t)^2+a_R^t{'}^2(m_t^2-u)^2)\right] \quad (4.34)$$

Now we calculate the interference,

$$\bar{\sum}|\mathcal{M}_1\bar{\mathcal{M}}_2| = \frac{1}{4N^2}\sum_{spin,color}|\mathcal{M}_1\bar{\mathcal{M}}_2|$$
$$= \sum_{spin,color}\frac{e^4Q_q^2Q_t^2\delta_{ji}\delta_{lk}\delta_{sm}\delta_{nr}}{16N^2[(p_1+p_2)^2-m_z^2][p_1+p_2]^2\sin^2\theta_w\cos^2\theta_w}\bar{v}_j(p_2)\gamma^\mu u_i(p_1)\bar{u}_l(p_3)\gamma_\mu v_k(p_4)\bar{u}_s(p_1)$$
$$(a_L^q(1+\gamma_5)+a_R^q(1-\gamma_5))\gamma^\alpha v_m(p_3)\bar{v}_n(p_4)(a_L^t(1+\gamma_5)+a_R^t(1-\gamma_5))\gamma_\beta u_r(p_3)\big[g^{\alpha\beta}$$
$$-\frac{(p_1+p_2)^\alpha(p_1+p_2)^\beta}{m_z^2}\big] \quad (4.35)$$

we sum over spin and color then we obtain

$$\bar{\sum}|\mathcal{M}_1\bar{\mathcal{M}}_2| = \frac{e^4Q_q^2Q_t^2}{16[(p_1+p_2)^2-m_z^2](p_1+p_2)^2\sin\theta_w\cos\theta_w}Tr[\not{p}_2\gamma^\mu\ \not{p}_1(a_L^q(1+\gamma_5)+a_R^q(1-\gamma_5)\gamma_\alpha)]$$
$$Tr[(\not{p}_3+m_t)]\gamma_\mu(\not{p}_4-m_t)(a_L^t(1+\gamma_5)+a_R^t(1-\gamma_5))\gamma_\beta]\times[g^{\alpha\beta}-\frac{(p_1+p_2)^\alpha(p_1+p_2)^\beta}{m_z^2}]$$
$$(4.36)$$

after simplifying , we get :

$$2Re[\bar{\sum}\mathcal{M}_1\bar{\mathcal{M}}_2] = \frac{2e^4Q_q^2Q_t^2}{[s-m_z^2]s\sin^2\theta_w\cos^2\theta_w}\big[a_L^q\big[[(a_R^t(m_t^4+m_t^2(s-2t)+t^2))$$
$$+a_L^t(m_t^4+m_t^2(s-2u)+u^2)]+(a_R^q(a_L^t(m_t^4+m_t^2(s-2t)+t^2)+a_R^t(m_t^4+m_t^2(s-2u)+u^2)))\big]\big]$$
$$(4.37)$$

$$2Re[\bar{\sum}\mathcal{M}_1\bar{\mathcal{M}}_3] = \frac{2e^4Q_q^2Q_t^2}{[(p_1+p_2)^2-m_{z'}^2][p_1+p_2]^2\sin^2\theta_w\cos^2\theta_w}\big[a_{L'}^q\big[[(a_{R'}^t(m_t^4+m_t^2(s-2t)+t^2)) + a_{L'}^t(m_t^4+m_t^2(s-2u)+u^2)] + (a_{R'}^q(a_{L'}^t(m_t^4+m_t^2(s-2t)+t^2) + a_{R'}^t(m_t^4+m_t^2(s-2u)+u^2)))\big]\big] \tag{4.38}$$

the interference between 2 and 3 are given by:

$$\bar{\sum}|\mathcal{M}_2\bar{\mathcal{M}}_3| = \frac{1}{4N^2}\sum_{spin,color}|\mathcal{M}_2\bar{\mathcal{M}}_3|$$
$$= \sum_{spin,color}\frac{e^4Q_q^2Q_t^2\delta_{ji}\delta_{lk}\delta_{nm}\delta_{rs}}{16\times 4N^2[(p_1+p_2)^2-m_z^2][(p_1+p_2)^2-m_{z'}^2]\sin^4\theta_w\cos^4\theta_w}\big[\bar{v}_j(p_2)\gamma_\mu(a_L^q(1-\gamma_5)+ a_R^q(1+\gamma_5))u_i(p_1)\bar{u}_l(p_3)\gamma_\nu(a_L^t(1-\gamma_5)+a_R^t(1+\gamma_5))v_k(p_4)\big]\big[\bar{u}_n(p_1)(a_{L'}^q(1-\gamma_5)+a_{R'}^q(1+\gamma_5))\gamma_\alpha v_m(p_2)\, \bar{v}_r(p_4)(a_{L'}^t(1+\gamma_5)+a_{R'}^t(1-\gamma_5))\gamma_\beta u_s(p_3)\big]\big[g^{\alpha\beta}-\frac{(p_1+p_2)^\alpha(p_1+p_2)^\beta}{m_z^2}\big]\big[g^{\mu\nu}-\frac{(p_1+p_2)^\mu(p_1+p_2)^\nu}{m_{z'}^2}\big] \tag{4.39}$$

summing over spins and colors:

$$\bar{\sum}|\mathcal{M}_2\bar{\mathcal{M}}_3| = \frac{e^4Q_q^2Q_t^2\delta_{ji}\delta_{lk}\delta_{nm}\delta_{rs}}{16\times 4N^2[(p_1+p_2)^2-m_z^2][(p_1+p_2)^2-m_{z'}^2]\sin^4\theta_w\cos^4\theta_w}\, Tr[\not{p}_2\gamma_\mu(a_L^q(1-\gamma_5)+a_R^q(1+\gamma_5))\,\not{p}_1(a_{L'}^q(1+\gamma_5)+a_{R'}^q(1-\gamma_5))\gamma_\alpha]\, Tr[(\not{p}_4-m_t)(a_{L'}^t(1+\gamma_5)+a_{R'}^t(1-\gamma_5))\gamma_\beta(\not{p}_3+m_t)\gamma_\nu(a_L^t(1-\gamma_5)+a_R^t(1+\gamma_5))] \tag{4.40}$$

after simplifying, we get:

$$2Re[\bar{\sum}\mathcal{M}_2\bar{\mathcal{M}}_3] = -\frac{2e^4Q_q^2Q_t^2}{\sin^4\theta_w\cos^4\theta_w(m_z^2-s)(m_{z'}^2-s)}\big[a_L^qa_{L'}^q(a_R^t(a_{L'}^tm_t^2s+a_{R'}^t(m_t^2-t)^2) + a_L^t(a_{R'}^tm_t^2s+a_{L'}^t(m_t^2-u)^2)) + a_R^qa_{R'}^q(a_L^t(a_{R'}^tm_t^2s+a_{L'}^t(m_t^2-t)^2)+a_R^t(a_{L'}^tm_t^2s+a_{R'}^t(m_t^2-u)^2))\big] \tag{4.41}$$

the total amplitude summed over spins and colors is written as :

$$\bar{\sum}|\mathcal{M}_{tot}|^2 = \bar{\sum}|\mathcal{M}_1|^2+\bar{\sum}|\mathcal{M}_2|^2+\bar{\sum}|\mathcal{M}_3|^2+2Re[\bar{\sum}\mathcal{M}_1\bar{\mathcal{M}}_2] + 2Re[\bar{\sum}\mathcal{M}_1\bar{\mathcal{M}}_3]+2Re[\bar{\sum}\mathcal{M}_2\bar{\mathcal{M}}_3] \tag{4.42}$$

the total amplitude is given by :

$$\bar{\sum}|\mathcal{M}_{tot}|^2 = \frac{2e^4Q_q^2Q_t^2}{\sin^4\theta_w\cos^4\theta_w(s-m_z^2)^2(s-m_{z'}^2)^2s^2}\Big[\sin^4\theta_w\cos^4\theta_w(s-m_z^2)^2(s-m_{z'}^2)^2\big[u^2+t^2 + 2m_t^4-2m_t^2(u+t-s)\big]+s^2(s-m_{z'}^2)^2\big[a_L^{q\,2}(2a_L^ta_R^tm_t^2s+a_R^{t\,2}(m_t^2-t)^2+a_L^{t\,2}(m_t^2-u)^2) + a_R^{q\,2}(2a_L^ta_R^tm_t^2s+a_L^{t\,2}(m_t^2-t)^2+a_R^{t\,2}(m_t^2-u)^2)\big]+s^2(s-m_z^2)^2\big[a_{L'}^{q\,2}(2a_{L'}^ta_{R'}^tm_t^2s+a_{R'}^{t\,2}(m_t^2-t)^2 + a_{L'}^{t\,2}(m_t^2-u)^2)+a_{R'}^{q\,2}(2a_{L'}^ta_{R'}^tm_t^2s+a_{L'}^{t\,2}(m_t^2-t)^2+a_{R'}^{t\,2}(m_t^2-u)^2)\big] + s\sin^2\theta_w\cos^2\theta_w(s-m_z^2)(s-m_{z'}^2)^2 \big[a_L^q\big[[(a_R^t(m_t^4+m_t^2(s-2t)+t^2))+a_L^t(m_t^4+m_t^2(s-2u)+u^2)]+(a_R^q(a_L^t(m_t^4+m_t^2(s-2t)+t^2) + a_R^t(m_t^4+m_t^2(s-2u)+u^2)))\big]\big]+s\sin^2\theta_w\cos^2\theta_w(s-m_z^2)^2(s-m_{z'}^2)\times \big[a_{L'}^q\big[[(a_{R'}^t(m_t^4+m_t^2(s-2t)+t^2))+a_{L'}^t(m_t^4+m_t^2(s-2u)+u^2)]+(a_{R'}^q(a_{L'}^t(m_t^4+m_t^2(s-2t)+t^2) + a_{R'}^t(m_t^4+m_t^2(s-2u)+u^2)))\big]\big]-(s-m_z^2)(s-m_{z'}^2)\big[a_L^qa_{L'}^q(a_R^t(a_{L'}^tm_t^2s+a_{R'}^t(m_t^2-t)^2) + a_L^t(a_{R'}^tm_t^2s+a_{L'}^t(m_t^2-u)^2))+a_R^qa_{R'}^q(a_L^t(a_{R'}^tm_t^2s+a_{L'}^t(m_t^2-t)^2)+a_R^t(a_{L'}^tm_t^2s+a_{R'}^t(m_t^2-u)^2))\big]\Big] \tag{4.43}$$

Calculation of the partonic cross section

The partonic cross section is given by:

$$\sigma = \frac{1}{2s(2\pi)^2}\int\frac{d^3p_3}{2E_3}\int\frac{d^3p_4}{2E_4}\delta^4(p_1+p_2-p_3-p_4)\int\bar{\sum}|\mathcal{M}_{tot}|^2 \quad (4.44)$$

where

$$\int\frac{d^3p_3}{2E_3} = \int d^4p_3\delta^+(p_3^2-m_t^2) \quad (4.45)$$

we substitute this relation in (4.44), we get:

$$\sigma = \frac{1}{2s(2\pi)^2}\int d^4p_3\delta^4(p_3-(p_1+p_2-p_4))\delta^+(p_3^2-m_t^2)\int\frac{d^3p_4}{2E_4}\bar{\sum}|\mathcal{M}_{tot}|^2 \quad (4.46)$$

The integration over p_3 is evident, thanks to the delta function. Then, we use the spherical coordinate system as:

$$d^3p_4 = |\overrightarrow{p_4}|^2 d|\overrightarrow{p_4}|d\Omega \quad (4.47)$$

we have also

$$E_4 = \sqrt{|\overrightarrow{p_4}|^2+m_t^2} \quad (4.48)$$

so we obtain

$$\sigma = \frac{1}{4s(2\pi)^2}\int\frac{|\overrightarrow{p_4}|^2 d|\overrightarrow{p_4}|}{\sqrt{|\overrightarrow{p_4}|^2+m_t^2}}d\Omega\delta^+((p_1+p_2-p_4)^2-m_t^2)\bar{\sum}|\mathcal{M}_{tot}|^2 \quad (4.49)$$

we can write the delta function in term of Mandelstam variables as :

$$\delta^+((p_1+p_2-p_4)^2-m_t^2) = \delta^+(s-2\sqrt{s}\sqrt{|\overrightarrow{p_4}|^2+m_t^2}) \quad (4.50)$$

to simplify $\delta^+(s-2\sqrt{s}\sqrt{|\overrightarrow{p_4}|^2+m_t^2})$, we use the relation

$$\delta[g(x)] = \sum_i\frac{\delta(x-x_i)}{|g\prime(x_i)|} \quad (4.51)$$

we set $x = |\overrightarrow{p_4}|$, and x_i is the solution of $g(|\overrightarrow{p_4}|_+) = 0$ (we take the positive solution) :

$$x_i = |\overrightarrow{p_4}|_+ = +\sqrt{\frac{s}{4}-m_t^2} \qquad and \qquad |g'(|\overrightarrow{p_4}_+|)|{=}2\sqrt{s}\sqrt{1-\frac{4m_t^2}{s}} \quad (4.52)$$

then

$$\delta^+(s-2\sqrt{s}\sqrt{|\overrightarrow{p_4}|^2+m_t^2}) = \frac{\delta^+(|\overrightarrow{p_4}|-\frac{\sqrt{s}}{2}\sqrt{1-\frac{4m_t^2}{s}})}{2\sqrt{s}\sqrt{1-\frac{4m_t^2}{s}}} \quad (4.53)$$

the cross section becomes :

$$\sigma = \frac{1}{8s\sqrt{s}(2\pi)^2\sqrt{1-\frac{4m_t^2}{s}}}\int\frac{|\overrightarrow{p_4}|^2 d|\overrightarrow{p_4}|}{\sqrt{|\overrightarrow{p_4}|^2+m_t^2}\sqrt{1-\frac{4m_t^2}{s}}}d\Omega\delta^+(|\overrightarrow{p_4}|-\frac{\sqrt{s}}{2})\bar{\sum}|\mathcal{M}_{tot}|^2 \quad (4.54)$$

where $d\Omega = \sin(\theta)d\theta d\phi$ is the solid angle , we have :

$$|\overrightarrow{p_4}|^2 = \frac{s}{4}(1-\frac{4m_t^2}{s}) \quad (4.55)$$

substitute in Eq. (4.6) and we integrate on ϕ, we obtain :

$$\sigma = \frac{\sqrt{1-\frac{4m_t^2}{s}}}{16s(2\pi)} \int_0^\pi \sin\theta d\theta \bar{\sum} |\mathcal{M}_{tot}|^2 \tag{4.56}$$

We change the integration variables

$$x = \cos\theta \quad , \quad \theta = 0 \Rightarrow x = 1 \tag{4.57}$$

$$dx = d\cos\theta \quad , \quad \theta = \pi \Rightarrow x = -1 \tag{4.58}$$

then , the total cross section becomes as :

$$\sigma = -\frac{\sqrt{1-\frac{4m_t^2}{s}}}{16s(2\pi)} \int_{-1}^1 dx \bar{\sum} |\mathcal{M}_{tot}|^2 \tag{4.59}$$

Now, we will calculate the total cross section for this reaction. we work in the center of mass frame and neglect the mass of quarks. In the CM frame, we have:

$$p_1 = \frac{\sqrt{s}}{2}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}, \quad p_2 = \frac{\sqrt{s}}{2}\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}, \quad p_3 = \frac{\sqrt{s}}{2}\begin{pmatrix}1\\\rho\sin\theta\\0\\\rho\cos\theta\end{pmatrix}, \quad p_4 = \frac{\sqrt{s}}{2}\begin{pmatrix}1\\-\rho\sin\theta\\0\\-\rho\cos\theta\end{pmatrix} \tag{4.60}$$

where θ is the angle between the incoming and the outgoing particles and

$$\rho = \sqrt{1-\frac{4m_t^2}{s}} \tag{4.61}$$

we have

$$t = -\frac{s}{4}(1+\rho^2-2\rho\cos\theta) \quad , \quad u = -\frac{s}{4}(1+\rho^2+2\rho\cos\theta) \tag{4.62}$$

We substitute this relation in the total amplitude and then we integrate over x. After simplification, we obtain :

$$\begin{aligned} \sigma_{EW} = &-\frac{e^4Q_q^2Q_t^2}{48\pi s^2\cos^4\theta_w\sin^4\theta_w(m_z^2-s)^2(m_z'^2-s)^2}\sqrt{1-\frac{4m_t^2}{s}}\Big[(a_L^{q^2}+a_R^{q^2})(m_z'^2-s)^2s^2(6a_L^ta_R^tm_t^2 \\ &+a_L^{t2}(-m_t^2+s)+a_R^{t2}(-m_t^2+s))+(a_L^{\prime q^2}+a_R^{\prime q^2})(m_z^2-s)^2s^2(6a_L^{\prime t}a_R^{\prime t}m_t^2+a_L^{\prime t^2}(-m_t^2+s)+ \\ &a_R^{\prime t^2}(-m_t^2+s))+2(a_L^qa_L^{\prime q}+a_R^qa_R^{\prime q})(m_z^2-s)(m_z'^2-s)s^2(a_L^t(3a_R^{\prime t}m_t^2+a_L^{\prime t}(-m_t^2+s))+a_R^t(3a_L^{\prime t}m_t^2 \\ &+a_R^{\prime t}(-m_t^2+s)))-2(a_L^q+a_R^q)(a_L^t+a_R^t)\cos^2\theta_w(mz^2-s)(m_z'^2-s)^2s(2m_t^2+s)\sin^2\theta_w+ \\ &2(a_L^{\prime q}+a_R^{\prime q})(a_L^{\prime t}+a_R^{\prime t})\cos^2\theta_w(m_z^2-s)^2s(2m_t^2+s)(-m_z'^2+s)\sin^2\theta_w+ \\ &8\cos^4\theta_wm_t^2(m_z^2-s)^2(m_z'^2-s)^2\sin^4\theta_w+4\cos^4\theta_w(m_z^2-s)^2(m_z'^2-s)^2s\sin^4\theta_w\Big] \end{aligned} \tag{4.63}$$

QCD contribution

The QCD contribution to the sub-process $qq \to t\bar{t}$ is given by the Feynman diagram mediated by the gluon, see figure (4.4).

$$q(p_1) + q(p_2) \to t(p_3) + \bar{t}(p_4)$$

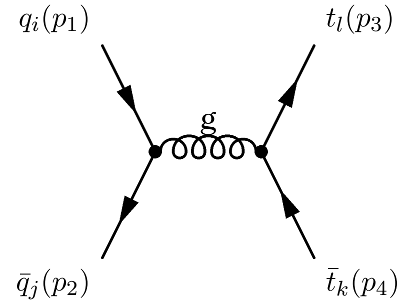


Figure 4.4: QCD sub-process

Calculation of the amplitude of $q\bar{q} \to t\bar{t}$:

For this diagram, the amplitude and it's complex conjugate are given by,

$$\mathcal{M}_4 = \frac{ig_s^2}{(p_1+p_2)^2}(T^a)_{ji}(T^a)_{lk}\bar{v}_j(p_2)\gamma^\mu u_i(p_1)\bar{u}_l(p_3)\gamma_\mu v_k(p_4). \tag{4.64}$$

$$\bar{\mathcal{M}}_4 = -\frac{ig_s^2}{(p_1+p_2)^2}(T^{a'})_{i'j'}(T^{a'})_{k'l'}\bar{u}_{i'}(p_1)\gamma^{\mu'}v_{j'}(p_2)\bar{v}_{k'}(p_4)\gamma_{\mu'}u_{l'}(p_3). \tag{4.65}$$

The amplitude squared summed over spin and color is,

$$\begin{aligned}\bar{\sum}|\mathcal{M}_4|^2 &= \frac{1}{4N^2}\sum_{spin,color}|\mathcal{M}_4|^2 \\ &= \frac{g_s^4}{4N^2(p_1+p_2)^4}\sum_{spin,color}(T^a)_{ji}(T^a)_{lk}(T^{a'})_{i'j'}(T^{a'})_{k'l'}[\bar{v}_j(p_2)\gamma^\mu u_i(p_1)\bar{u}_l(p_3)\gamma_\mu v_k(p_4)] \\ &\times [\bar{u}_{i'}(p_1)\gamma^{\mu'}v_{j'}(p_2)\bar{v}_{k'}(p_4)\gamma_{\mu'}u_{l'}(p_3)]\end{aligned} \tag{4.66}$$

$$\begin{aligned}&= \frac{g_s^4}{4N^2(p_1+p_2)^4}\sum_{color}(T^a)_{ji}(T^a)_{lk}(T^{a'})_{i'j'}(T^{a'})_{k'l'}\delta_{j'j}\delta_{kk'}\delta_{ii'}\delta_{ll'} \\ &Tr[\not{p}_1\gamma^{\mu'}\not{p}_2\gamma^\mu] \times Tr[(\not{p}_3+m_t)\gamma_\mu(\not{p}_4-m_3)\gamma_{\mu'}]\end{aligned} \tag{4.67}$$

where the color factor is given by,

$$\begin{aligned}\sum_{col}(T^a)_{ji}(T^a)_{lk}(T^{a'})_{i'j'}(T^{a'})_{k'l'}\delta_{j'j}\delta_{kk'}\delta_{ii'}\delta_{ll'} &= Tr[T^aT^{a'}] \times Tr[T^aT^{a'}] \\ &= \frac{\delta_{aa}}{4} \\ &= \frac{N^2-1}{4}\end{aligned} \tag{4.68}$$

We contract over the Lorentz indices and we express the scalar products of the 4-momenta in terms of the MandelStam variables, we get,

$$\bar{\sum}|\mathcal{M}_4|^2 = \frac{16g_s^4}{9s^2}[u^2+t^2+2m_t^4-2m_t^2(t+u-s)] \tag{4.69}$$

Calculation of the cross section:

Now, we will calculate the cross section. From the general formula presented obove, we have

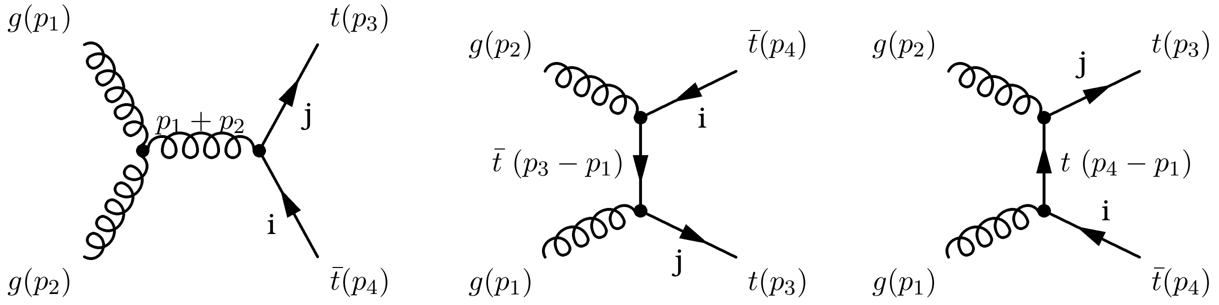
$$\sigma_S = -\frac{\sqrt{1-\frac{4m_t^2}{s}}}{16s(2\pi)}\int_{-1}^{1}dx\bar{\sum}|\mathcal{M}_4|^2 \tag{4.70}$$

In the same way, we consider particles in the center of mass frame and performing the integration over all the variables we get,

$$\sigma_S = -\frac{g_s^4}{6(2\pi)s^2}\sqrt{1-\frac{4m_t^2}{s}}(s+2m_t^2) \tag{4.71}$$

Sub-process $gg \to tt$**:**

Now, we will calculate the cross section for the $gg \to t\bar{t}$ reaction. The three possible Feynman diagrams for this reaction are shown in the figure (4.3.1),



Calculation of the amplitudes amplitude:

For the first diagram the amplitude and it's conjugate are given by,

$$\begin{aligned}
\mathcal{M}_1 &= \frac{g_s^2}{(p_1+p_2)^2}(T^a)_{ij} f^{dca}\big[g^{\gamma\rho}(p_2-p_1)^\nu + g^{\rho\nu}(-p_2)^\gamma + g^{\nu\gamma}(p_1)^\rho\big]\bar{u}_i(p_3)\gamma_\nu v_j(p_4)\varepsilon_\rho^c(p_1)\varepsilon_\gamma^d(p_2) \\
&= \frac{g_s^2}{(p_1+p_2)^2}(T^a)_{ij} f^{dca}\bar{u}_i(p_3)\big[g^{\gamma\rho}(\not{p}_2 - \not{p}_1) + \gamma^\rho(-p_2)^\gamma + \gamma^\gamma(p_1)^\rho\big]v_j(p_4)\varepsilon_\rho^c(p_1)\varepsilon_\gamma^d(p_2) \\
\bar{\mathcal{M}}_1 &= \frac{g_s^2}{(p_1+p_2)^2}(T^{a'})_{j'i'} f^{d'c'a'}\bar{v}_{j'}(p_4)\big[g^{\gamma'\rho'}(\not{p}_2 - \not{p}_1) + \gamma^{\rho'}(-p_2)^{\gamma'} + \gamma^{\gamma'}(p_1)^{\rho'}\big]u_{i'}(p_3)\varepsilon_{\rho'}^{*c'}(p_1)\varepsilon_{\gamma'}^{*d'}(p_2)
\end{aligned} \tag{4.72}$$

where

$$\sum_\lambda^2 \varepsilon_\mu^\lambda(q)\varepsilon_\nu^{*\lambda}(q) = -g_{\mu\nu} \tag{4.73}$$

The amplitude squared is,

$$\begin{aligned}
\bar{\sum}|\mathcal{M}_1|^2 &= \frac{1}{4(N^2-1)^2}\sum_{spin,color}|\mathcal{M}_1|^2 \\
&= \frac{g_s^4}{4(N^2-1)^2(p_1+p_2)^4}\sum_{spin,color}(T^a)_{ij}(T^{a'})_{j'i'} f^{dca} f^{d'c'a'} g_{\rho\rho'} g_{\gamma\gamma'}\delta^{cc'}\delta^{dd'}\bar{u}_i(p_3)\big[g^{\gamma\rho}(\not{p}_2 - \not{p}_1) \\
&+ \gamma^\rho(-p_2)^\gamma + \gamma^\gamma(p_1)^\rho\big]v_j(p_4)\bar{v}_{j'}(p_4)\big[g^{\gamma'\rho'}(\not{p}_2 - \not{p}_1) + \gamma^{\rho'}(-p_2)^{\gamma'} + \gamma^{\gamma'}(p_1)^{\rho'}\big]u_{i'}(p_3)
\end{aligned} \tag{4.74}$$

we get

$$\begin{aligned}
\bar{\sum}|\mathcal{M}_1|^2 = \frac{g_s^4}{4(N^2-1)^2(p_1+p_2)^4}\sum_{color}(T^a)_{ij}(T^{a'})_{j'i'} f^{dca} f^{dca'}\delta_{ii'}\delta_{jj'} \\
\times Tr\Big[(\not{p}_3 + m_t)\big[g^{\gamma\rho}(\not{p}_2 - \not{p}_1) + \gamma^\rho(-p_2)^\gamma \\
+ \gamma^\gamma(p_1)^\rho\big](\not{p}_4 - m_t)\big[g_{\gamma\rho}(\not{p}_2 - \not{p}_1) + \gamma_\rho(-p_2)_\gamma + \gamma_\gamma(p_1)_\rho\big]\Big]
\end{aligned} \tag{4.75}$$

we have:

$$f_{abc} f_{a'bc} = N\delta_{aa'} \tag{4.76}$$

the color factor,

$$\sum_{color}(T^a)_{ij}(T^{a'})_{j'i'} f^{dca} f^{dca'}\delta_{ii'}\delta_{jj'} = \frac{N(N^2-1)}{2} \tag{4.77}$$

After simplification and replacing by the MandelStam variables, we get

$$\bar{\sum}|\mathcal{M}_1|^2 = \frac{3g_s^4}{32s^2}[-2m_t^4 + 3(s^2 - t^2 - u^2) + 4tu + 2m_t^2(t+u)] \tag{4.78}$$

For the second diagram the amplitude and its conjugate are given by :

$$\begin{aligned}
\mathcal{M}_2 &= \frac{-ig_s^2}{(p_3-p_1)^2 - m_t^2}(T^a)_{ki}(T^b)_{jk}\bar{u}_j(p_3)\gamma^\nu(\not{p}_3 - \not{p}_1 + m_t)\gamma^\mu v_i(p_4)\varepsilon^a_\mu(p_2)\varepsilon^b_\nu(p_1) \qquad (4.79)\\
\bar{\mathcal{M}}_2 &= \frac{ig_s^2}{(p_3-p_1)^2 - m_t^2}(T^{a'})_{i'k'}(T^{b'})_{k'j'}\bar{v}_{i'}(p_4)\gamma^{\mu'}(\not{p}_3 - \not{p}_1 + m_t)\gamma^{\nu'} u_{j'}(p_3)\varepsilon^{*a'}_{\mu'}(p_2)\varepsilon^{*b'}_{\nu'}(p_1)
\end{aligned}$$

The amplitude squared summing over spin and color, and we use (1.70), we get :

$$\begin{aligned}
\bar{\sum}|\mathcal{M}_2|^2 &= \frac{1}{4(N^2-1)^2}\sum_{spin,color}|\mathcal{M}_2|^2\\
&= \frac{g_s^4}{4(N^2-1)^2((p_3-p_1)^2-m_t^2)^2}\sum_{spin,color}(T^a)_{ki}(T^b)_{jk}(T^{a'})_{i'k'}(T^{b'})_{k'j'}\delta^{aa'}\delta^{bb'}g_{\mu\mu'}g_{\nu\nu'}\\
&\left[\bar{u}_j(p_3)\gamma^\nu(\not{p}_3 - \not{p}_1 + m_t)\gamma^\mu v_i(p_4)\right]\left[\bar{v}_{i'}(p_4)\gamma^{\mu'}(\not{p}_3 - \not{p}_1 + m_t)\gamma^{\nu'}u_{j'}(p_3)\right]\\
&= \frac{g_s^4}{4(N^2-1)^2((p_3-p_1)^2-m_t^2)^2}\sum_{spin,color}(T^a)_{ki}(T^b)_{jk}(T^a)_{i'k'}(T^b)_{k'j'}\left[\bar{u}_j(p_3)\gamma^\nu(\not{p}_3 - \not{p}_1 + m_t)\right.\\
&\left.\gamma^\mu v_i(p_4)\right]\left[\bar{v}_{i'}(p_4)\gamma_\mu(\not{p}_3 - \not{p}_1 + m_t)\gamma_\nu u_{j'}(p_3)\right] \qquad (4.80)
\end{aligned}$$

$$= \frac{g_s^4}{16(N^2-1)[(p_3-p_1)^2-m_t^2]^2}Tr[(\not{p}_3 + m_t)\gamma^\nu(\not{p}_3 - \not{p}_1 + m_t)\gamma^\mu(\not{p}_4 - m_t)\gamma_\mu(\not{p}_3 - \not{p}_1 + m_t)\gamma_\nu] \tag{4.81}$$

After simplifying, we get:

$$\bar{\sum}|\mathcal{M}_2|^2 = \frac{g_s^4}{16(t-m_t^2)^2}(3m_t^4 + tu - m_t^2(2s + 5t + 3u)) \tag{4.82}$$

for the third diagram the amplitude and its conjugate are given by :

$$\begin{aligned}
\mathcal{M}_3 &= \frac{-ig_s^2}{(p_4-p_1)^2 - m_t^2}(T^a)_{jk}(T^b)_{ki}\bar{u}_j(p_3)\gamma^\mu(\not{p}_4 - \not{p}_1 + m_t)\gamma^\nu v_i(p_4)\varepsilon^a_\mu(p_2)\varepsilon^b_\nu(p_1) \qquad (4.83)\\
\bar{\mathcal{M}}_3 &= \frac{ig_s^2}{(p_4-p_1)^2 - m_t^2}(T^{a'})_{k'j'}(T^{b'})_{i'k'}\bar{v}_{i'}(p_4)\gamma^{\nu'}(\not{p}_4 - \not{p}_1 + m_t)\gamma^{\mu'} u_{j'}(p_3)\varepsilon^{*a'}_{\mu'}(p_2)\varepsilon^{*b'}_{\nu'}(p_1)
\end{aligned}$$

The amplitude squared is :

$$\begin{aligned}
\bar{\sum}|\mathcal{M}_3|^2 &= \frac{g_s^4}{4(N^2-1)^2((p_4-p_1)^2-m_t^2)^2}\sum_{spin,color}(T^a)_{jk}(T^b)_{ki}(T^{a'})_{k'j'}(T^{b'})_{i'k'}\delta^{aa'}\delta^{bb'}g_{\mu\mu'}g_{\nu\nu'}\\
&\left[\bar{u}_j(p_3)\gamma^\mu(\not{p}_4 - \not{p}_1 + m_t)\gamma^\nu v_i(p_4)\right]\left[\bar{v}_{i'}(p_4)\gamma^{\nu'}(\not{p}_4 - \not{p}_1 + m_t)\gamma^{\mu'}u_{j'}(p_3)\right]\\
&= \frac{g_s^4}{16(N^2-1)((p_4-p_1)^2-m_t^2)^2}\\
&\times Tr[(\not{p}_3 + m_t)\gamma^\mu(\not{p}_4 - \not{p}_1 + m_t)\gamma^\nu(\not{p}_4 - m_t)\gamma_\nu(\not{p}_4 - \not{p}_1 + m_t)\gamma_\mu] \qquad (4.84)
\end{aligned}$$

$$\bar{\sum}|\mathcal{M}_3|^2 = \frac{g_s^4}{16(u-m_t^2)^2}(-29m_t^4 + tu + m_t^2(6s + 5t - 13u)) \tag{4.85}$$

$$\bar{\sum}|\mathcal{M}_1\bar{\mathcal{M}}_2| = \frac{1}{4(N^2-1)^2}\sum_{spin,color}|\mathcal{M}_1\bar{\mathcal{M}}_2|$$
$$= \frac{ig_s^4}{4(N^2-1)^2(p_1+p_2)^2[(p_3-p_1)^2-m_t^2]}\sum_{spin,color}(T^a)_{ij}f^{dca}(T^{a'})_{i'k'}(T^{b'})_{k'j'}\delta^{cb'}\delta^{da'}g_{\rho\nu'}g_{\gamma\mu'}$$
$$\big[\bar{u}_i(p_3)\big[g^{\gamma\rho}(\not{p}_2-\not{p}_1)+\gamma^\rho(-p_2)^\gamma+\gamma^\gamma(p_1)^\rho\big]v_j(p_4)\big]\big[\bar{v}_{i'}(p_4)\gamma^{\mu'}(\not{p}_3-\not{p}_1+m_t)\gamma^{\nu'}u_{j'}(p_3)\big]$$
$$= \frac{ig_s^4}{4(N^2-1)^2(p_1+p_2)^2[(p_3-p_1)^2-m_t^2]}\sum_{color}(T^a)_{ij}f^{dca}(T^d)_{jk'}(T^c)_{k'i}$$
$$Tr\big[(\not{p}_3+m_t)[g^{\gamma\rho}(\not{p}_2-\not{p}_1)+\gamma^\rho(-p_2)^\gamma+\gamma^\gamma(p_1)^\rho](\not{p}_4-m_t)[\gamma_\gamma(\not{p}_3-\not{p}_1+m_t)\gamma_\rho]\big] \quad (4.86)$$

the color factor,

$$\sum_{color}(T^a)_{ij}f^{dca}(T^d)_{jk'}(T^c)_{k'i} = Tr[T^aT^dT^c]f^{dca} = \frac{1}{4}(d^{adc}+if^{adc})f^{dca} = \frac{iN}{4} \quad (4.87)$$

where

$$d^{adc}f^{dca}=0 \quad , \quad f^{adc}f^{dca}=N \quad (4.88)$$

$$2Re\Big[\bar{\sum}|\mathcal{M}_1\bar{\mathcal{M}}_2|\Big] = \frac{3g_s^4}{128s(t-m_t^2)}[2m_t^4+2m_t^2(s-t)+(s-u)(s-t+u)] \quad (4.89)$$

$$\bar{\sum}|\mathcal{M}_1\bar{\mathcal{M}}_3| = \frac{1}{4(N-1)^2}\sum_{spin,color}|\mathcal{M}_1\bar{\mathcal{M}}_3|$$
$$= \frac{ig_s^4}{4(N^2-1)^2(p_1+p_2)^2[(p_4-p_1)^2-m_t^2]}\sum_{spin,color}(T^a)_{ij}f^{dca}(T^{a'})_{k'j'}(T^{b'})_{i'k'}\delta^{cb'}\delta^{da'}g_{\rho\nu'}g_{\gamma\mu'}$$
$$\big[\bar{u}_i(p_3)\big[g^{\gamma\rho}(\not{p}_2-\not{p}_1)+\gamma^\rho(-p_2)^\gamma+\gamma^\gamma(p_1)^\rho\big]v_j(p_4)\big]\big[\bar{v}_{i'}(p_4)\gamma^{\nu'}(\not{p}_4-\not{p}_1+m_t)\gamma^{\mu'}u_{j'}(p_3)\big] \quad (4.90)$$

$$= \frac{ig_s^4}{4(N-1)^2(p_1+p_2)^2[(p_4-p_1)^2-m_t^2]}\sum_{color}(T^a)_{ij}f^{dca}(T^d)_{k'i}(T^c)_{jk'}$$
$$Tr\big[(\not{p}_3+m_t)\big[g^{\gamma\rho}(\not{p}_2-\not{p}_1)+\gamma^\rho(-p_2)^\gamma+\gamma^\gamma(p_1)^\rho\big](\not{p}_4-m_t)\gamma_\rho(\not{p}_4-\not{p}_1+m_t)\gamma_\gamma\big] \quad (4.91)$$

$$2Re\Big[\bar{\sum}|\mathcal{M}_1\bar{\mathcal{M}}_3|\Big] = \frac{3g_s^4}{128s(u-m_t^2)}[10m_t^4-(s-t)(s+t-u)-2m_t^2(s+6t-u)] \quad (4.92)$$

$$\bar{\sum}|\mathcal{M}_2\bar{\mathcal{M}}_3| = \frac{1}{4(N-1)^2}\sum_{color}|\mathcal{M}_2\bar{\mathcal{M}}_3|$$
$$= \frac{g_s^4}{4(N-1)^2[(p_3-p_1)^2-m_t^2][(p_4-p_1)^2-m_t^2]}\sum_{spin,color}(T^a)_{ki}(T^b)_{jk}(T^{a'})_{k'j'}(T^{b'})_{i'k'}\delta^{aa'}\delta^{bb'}$$
$$g_{\mu\mu'}g_{\nu\nu'}[\bar{v}_{i'}(p_4)\gamma^{\nu'}(\not{p}_4-\not{p}_1+m_t)\gamma^{\mu'}u_{j'}(p_4)][\bar{u}_j(p_3)\gamma^{\nu}(\not{p}_3-\not{p}_1+m_t)\gamma^{\mu}v_i(p_4)] \quad (4.93)$$

$$= \frac{g_s^4}{4(N-1)^2[(p_3-p_1)^2-m_t^2][(p_4-p_1)^2-m_t^2]} \sum_{color} (T^a)_{ki}(T^b)_{jk}(T^a)_{k'j}(T^b)_{ik'}$$
$$Tr\left[(\not{p}_4 - m_t)\gamma_\nu(\not{p}_4 - \not{p}_1 + m_t)\gamma_\mu(\not{p}_3 + m_t)\gamma^\nu(\not{p}_3 - \not{p}_1 + m_t)\gamma^\mu\right] \quad (4.94)$$

$$2Re\left[\bar{\sum}|\mathcal{M}_2\bar{\mathcal{M}}_3|\right] = \frac{g_s^4}{8(u-m_t^2)(t-m_t^2)}[-2m_t^4 - s(s+t+u) + m_t^2(6s-t+3u)] \quad (4.95)$$

the total amplitude is given by:

$$\bar{\sum}|\ \mathcal{M}_{tot}|^2 = \frac{g_s^4}{128s^2(-m_t^2+u)^2(-m_t^2+t)^2}\Big[8s^2(-m_t^2+t)^2(-29m_t^4+m_t^2(6s+5t-13u)+tu)$$
$$+ 8s^2(-m_t^2+u)^2[3m_t^4+tu-m_t^2(2s+5t+3u)] + 12(-m_t^2+t)^2(-m_t^2+u)^2[-2m_t^4+4tu$$
$$+ 2m_t^2(t+u) + 3(s^2-t^2-u^2)] +$$
$$3s(-m_t^2+u)^2(-m_t^2+t)[2m_t^4+2m_t^2(s-t)+(s-u)(s-t+u)]$$
$$+ 3s(-m_t^2+u)(-m_t^2+t)^2[10m_t^4-(s-t)(s+t-u)-2m_t^2(s+6t-u)] + 16s^2(-m_t^2+u)$$
$$\times(-m_t^2+t)[-2m_t^4 - s(s+t+u) + m_t^2(6s-t+3u)]\Big] \quad (4.96)$$

4.4 Hadronic Cross Section Calculations

In this section, we will use the MADGRAPH version 2.7.3 program to perform numerical calculations on the hadronic cross section of the top-quark pair production at leading order (LO).

In the standard model, we can write the leading order partonic cross section as:

$$\sigma_{tot}^{LO} = \sigma_{ew}^{LO}(\alpha_W^2) + \sigma_s^{LO}(\alpha_s^2) \quad (4.97)$$

where α_S and α_W are the couplings of the electroweak and strong forces, σ_{ew}^{LO} is the partonic cross section for the electroweak subprocesses mediated by γ and Z gauge bosons in $q\bar{q} \to t\bar{t}$ reactions, while σ_S^{LO} is the QCD subprocesses mediated by gluon boson in $q\bar{q} \to t\bar{t}$ and $g\bar{g} \to t\bar{t}$ reactions.

The existence of the new Z' boson contributes to an additional electroweak sub-process by exchanging Z', as shown in the figure (4.3). In this case, the σ_{ew}^{LO} cross section in (4.97) will be affected and change, while σ_s^{LO} is not affected and it is remains unchanged.

In order to know the change caused by the extensions of the SM that predict the existence of a boson such Z'. We will calculate the variation of hadronic cross section in term of Z' mass and for the different center of mass energies $\sqrt{S}$ = 14 TeV , 13 TeV, 8 TeV and 7 TeV for both the electroweak and QCD processes.

4.4.1 Purely electroweak processes

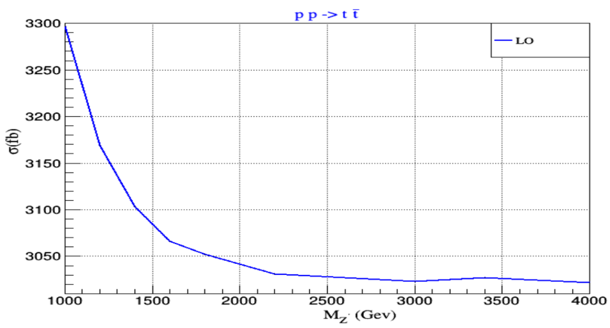
We will calculate the hadronic cross section for the electroweak processes. We have changed the values of $M_{Z'}$ for each value of center mass energy 14 TeV, 13 TeV, 8 TeV and 7 TeV. The steps of this calculation are shown above.

The table (4.1) summarizes the different values of the hadronic cross section for different Z' masses and center of mass energies,

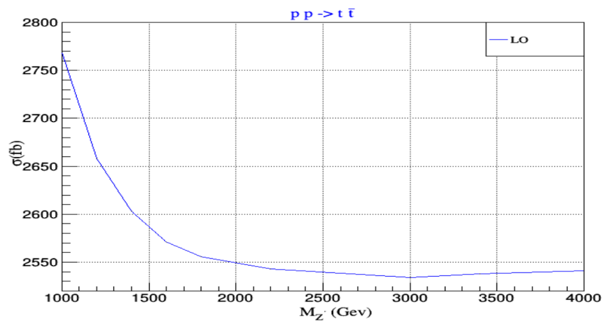
$\sqrt{s}$	14 Tev		13 Tev		8 Tev		7 Tev	
	$M_{Z'}$ Gev	σ (fb)	$M_{Z'}$ (Gev)	σ (fb)	$M_{Z'}$ (Gev)	σ (fb)	$M_{Z'}$(Gev)	σ (fb)
	1000	3297	1000	2769	1000	806	1000	555
	1200	3169	1200	2658	1200	767	1200	529.3
	1400	3103	1400	2603	1400	751	1400	518
	1600	3066	1600	2571	1600	744.2	1600	512.9
	1800	3052	1800	2556	1800	744.1	1800	512.9
	2200	3031	2200	2543	2200	741.2	2200	512.6
	3000	3023	3000	2534	3000	741.9	3000	512.6
	3400	3027	3400	2538	3400	742.1	3400	515.5
	4000	3022	4000	2541	4000	743.7	4000	516.4

Table 4.1: Variation of LO cross section in terms of the mass

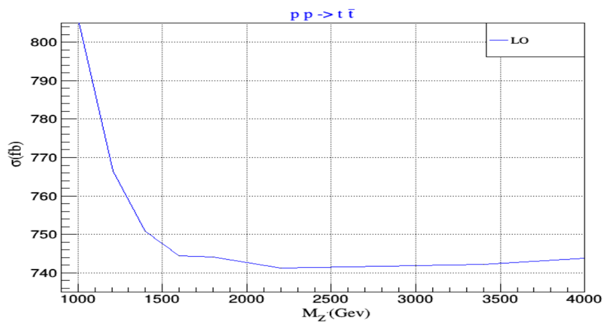
The figure (4.5) represents the variation of the hadronic cross section in term of Z' mass for different values of the energy $\sqrt{s}$. We observe that the values of cross section increase when the center of mass energy increases because the production of final state particles (top quark pair) increases at higher energy values. The production of top quark pair is a result to the decay of Z'. For this reason, We have seen that the cross section is inversely proportional to the Z' mass. We observe that when the mass of Z' increases, the cross section decreases, but it becomes almost constant for large values of M_Z. This means that the cross section becomes independent of the mass of Z' for large values.



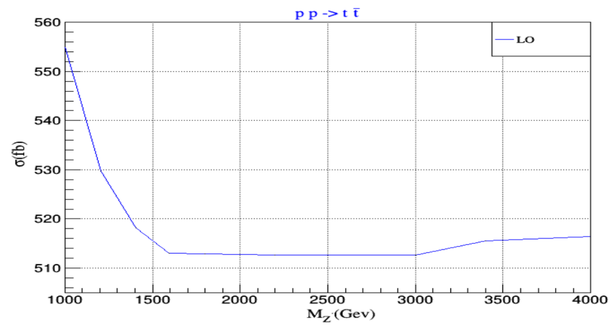
for $\sqrt{s}$= 14 Tev



for $\sqrt{s}$= 13 Tev



for $\sqrt{s}$= 8 Tev



for $\sqrt{s}$= 7 Tev

Figure 4.5: Variation of the hadronic cross section σ_{EW} in term of $M_{Z'}$

4.4.2 Purely QCD processes

We calculate the hadronic cross section for the process mediated by the the strong interaction (gluon) σ_s. We follow the same previous steps in MadGraph. The result are shown in the table (4.2). We observe that the cross section is the same for each change of Z' mass. Since Z' mediates only the electroweak interaction. So the existence of Z' doesn't affect the cross section for the QCD process. The table (4.2) represents the variation of hadronic cross section for each value of the center of mass energy $\sqrt{s}$. The value of cross section is large compared to the electroweak process because The strength of the strong interaction is larger $\alpha_S >> \alpha_W$.

$\sqrt{s}$ (Tev)	14 Tev	13 Tev	8 Tev	7 Tev
σ (fb)	593100	502400	154200	108800

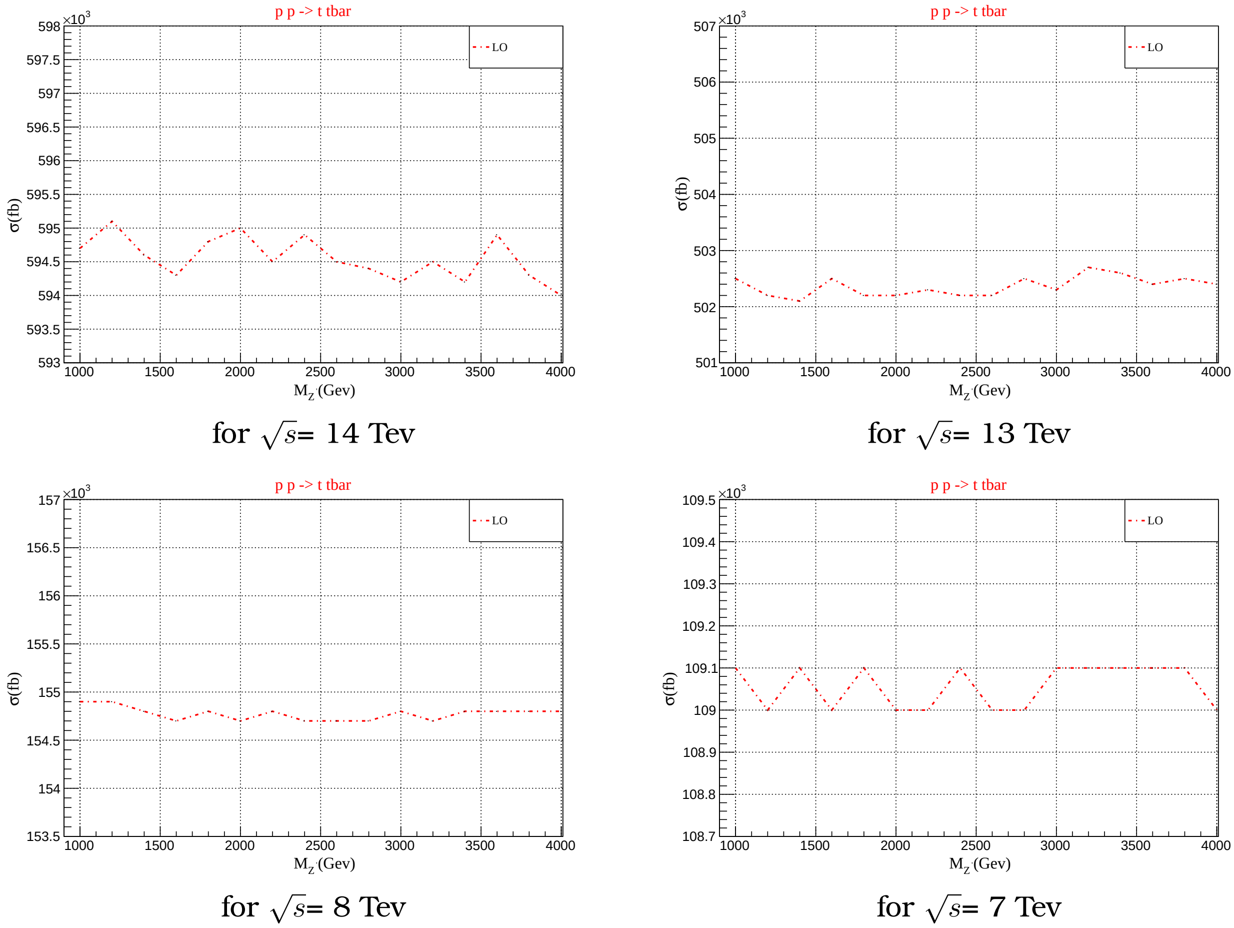
Table 4.2: ..

4.4.3 Electroweak and QCD processes

We calculate the total cross section for both electroweak and QCD processes. The result are shown in the table (4.3). The variation of the cross section in term of Z' mass are shown in the figure (4.6). We observe that the cross section is not affected too much by the change of $M_{Z'}$, because the strong interaction coupling to the produced quarks is much stronger than the weak interaction coupling, and therefore the strong interaction is the dominant process in this case.

$\sqrt{s}$	14 Tev		13 Tev		8 Tev		7 Tev	
	$M_{Z'}$ Gev	σ (fb)	$M_{Z'}$ (Gev)	σ (fb)	$M_{Z'}$ (Gev)	σ (fb)	$M_{Z'}$(Gev)	σ (fb)
	1000	594700	1000	502500	1000	154900	1000	109100
	1200	595100	1200	502200	1200	154900	1200	109000
	1400	594600	1400	502100	1400	154800	1400	109100
	1600	594300	1600	502500	1600	154700	1600	109000
	1800	594800	1800	502200	1800	154800	1800	109100
	2200	594500	2200	502300	2200	154800	2200	109000
	3000	594200	3000	502300	3000	154800	3000	109100
	3400	594200	3400	502600	3400	154800	3400	109100
	4000	594000	4000	502400	4000	154800	4000	109000

Table 4.3: Variation of the cross section in term of m'_Z



for $\sqrt{s}$= 14 Tev

for $\sqrt{s}$= 13 Tev

for $\sqrt{s}$= 8 Tev

for $\sqrt{s}$= 7 Tev

Figure 4.6: Variation of the cross section in term of m'_Z

4.4.4 Differential distributions

In this section, We will calculate the differential cross section for top pair production that represents the variation of cross section in term of the kinimatic variable P_T, η, M_I ... etc.

For this calculation, We use MADGRAPH v2.7.3, MADANALYSIS and Root to produce the differential distribution.

The figure (4.7) represents the variation of the distribution cross section in term of top quark transverse momentum $P_T(t)$ with the center of mass energy $\sqrt{s} = 7$ TeV and for different value of Z' masses $M_{Z'} = 1000$, 1400 and 2200 GeV.

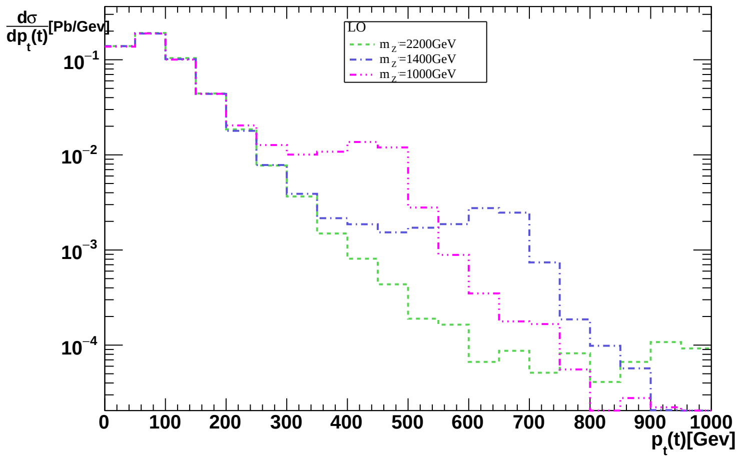


Figure 4.7: Transverse momentum distribution of the top quark production

The Z' gauge boson is expected to be produced at rest when two protons collide, and then decay into a top quark pair. For this reason, both top quarks produced in the final state must have higher transverse momentum coming from Z'. In Figure (4.7), we observe that when the transverse momentum of top quark increases, the cross section decreases and for each mass of Z', $P_T(t)$ achieves a maximum value then decrease again. For example, we observe that the cross section achieves the maximum value of Z' mass at $P_T(t) = 500Gev$ for $M_{Z'} = 1000GeV$ and also reach its maximum at $P_T(t) = 700Gev$ for $M_{Z'} = 1400GeV$. So for each mass of Z', the transverse momentum of top quark $P_T(t)$ achieves the maximum value near the value $P_T(t) = \frac{M_{Z'}}{2}$. Therefore, the transverse momentum $P_T(t)$ is considered as a measure of $M_{Z'}$.

The Rapidity and Pseudo-Rapidity distributions of the top-quark pair are shown in figure (4.8) and (4.9). We observe that the distribution cross section achieves its maximum value around $\eta = 0$.

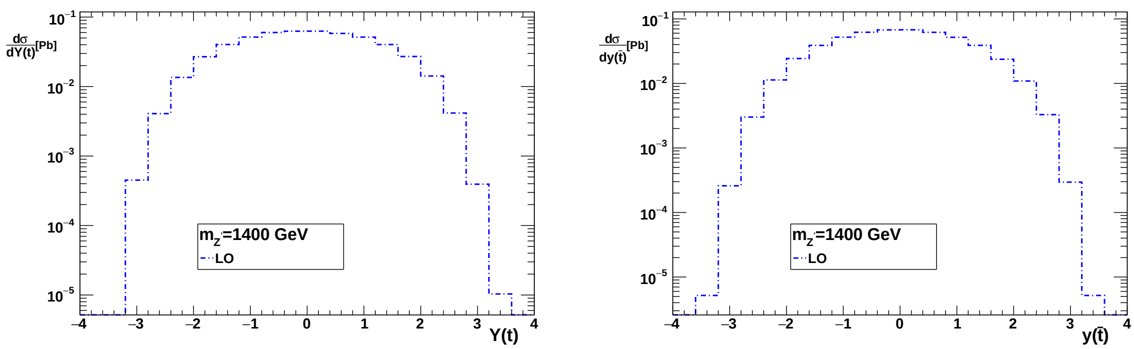


Figure 4.8: Rapidity distributions of top-quark pair production

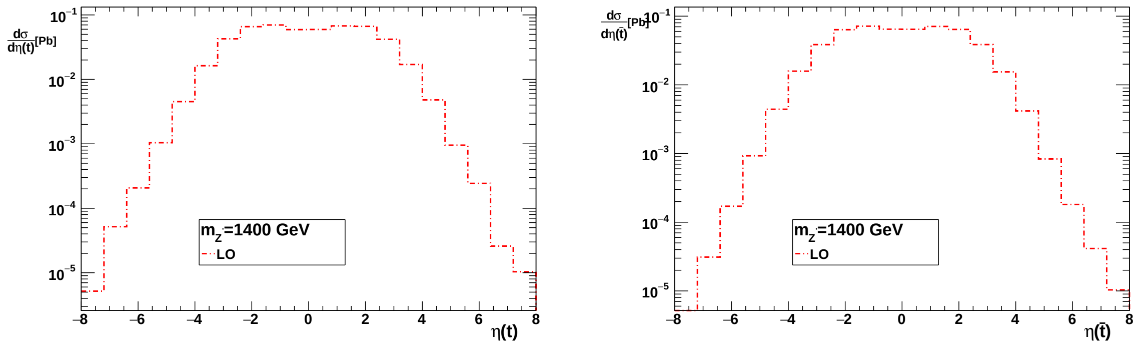


Figure 4.9: pseudo-Rapidity distributions of top-quark pair production

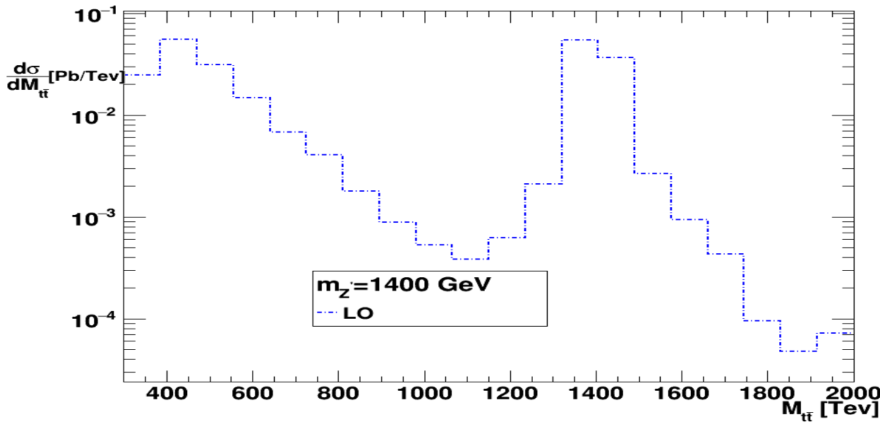


Figure 4.10: Invariant mass distributions of top-quark pair production

4.5 The Z' Decay Width

In this section, we will calculate the Z' decay width for quarks. Lets consider the process,

$$Z' \to q\bar{q} \tag{4.98}$$

The Z' gauge boson can decays into SM quark as shown in the figure (4.11)

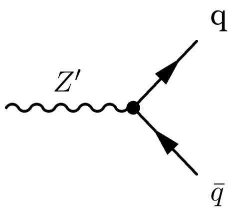


Figure 4.11: Z' decay to quarks

For this process, the amplitude and it's conjugate is given by:

$$\begin{aligned}
\mathcal{M}_{Z'} &= i(2\pi)^4 \frac{g_w}{4\cos\theta_W} \bar{u}(p_1)\gamma^\mu (a^t_{Z'} + b^t_{Z'}\gamma_5) v(p_2) \varepsilon_\mu(q) \\
\bar{\mathcal{M}}_{Z'} &= -i(2\pi)^4 \frac{g_w}{4\cos\theta_W} \bar{v}(p_2)(a^t_{Z'} - b^t_{Z'}\gamma_5)\gamma^\alpha u(p_1)\varepsilon^*_\alpha(q)
\end{aligned} \tag{4.99}$$

The amplitude squared,

$$\bar{\sum}|\mathcal{M}_{Z'}|^2 = \frac{(2\pi)^8 g_w^2}{3\times 16\cos^2\theta_W}\sum_{spin,polar}\big[\bar{u}(p_1)\gamma^\mu(a^t_{Z'}+b^t_{Z'}\gamma_5)v(p_2)\bar{v}(p_2)(a^t_{Z'}-b^t_{Z'}\gamma_5)\gamma^\alpha u(p_1)\big]\varepsilon_\mu(q)\varepsilon^*_\alpha(q) \tag{4.100}$$

We use the relations,

$$\sum_{spin} u_i(p)\bar{u}_j(p) = \delta_{ij}(\not{p}+m) \quad , \quad \sum_{spin} v_i(p)\bar{v}_j(p) = \delta_{ij}(\not{p}-m)$$
$$\sum_\lambda^2 \varepsilon^\lambda_\mu(q)\varepsilon^{*\lambda}_\nu(q) = -g_{\mu\nu}+\frac{q_\mu q_\nu}{M^2_{Z'}} \tag{4.101}$$

The amplitude squared becames :

$$\begin{aligned}\bar{\sum}|\mathcal{M}_{Z'}|^2 &= \frac{(2\pi)^8 g_w^2}{3\times 16\cos^2\theta_W}Tr\big[(\not{p}_1+m_t)\gamma^\mu(a^t_{Z'}+b^t_{Z'}\gamma_5)(\not{p}_2-m_t)(a^t_{Z'}-b^t_{Z'}\gamma_5)\gamma^\alpha\big]\big[-g_{\mu\alpha}+\frac{q_\mu q_\alpha}{M^2_{Z'}}\big]\\ &= \frac{(2\pi)^8 g_w^2}{3\times 16\cos^2\theta_W}\big[(a^{t2}_{Z'}+b^{2t}_{Z'})Tr[\not{p}_1\gamma^\mu\ \not{p}_2\gamma^\alpha]-2a^t_{Z'}b^t_{Z'}Tr[\not{p}_1\gamma^\mu\ \not{p}_2\gamma_5\gamma^\alpha]\\ &-m_t^2(a^{t2}_{Z'}-b^{2t}_{Z'})Tr[\gamma^\mu\gamma^\alpha]\big]\\ &\times\big[-g_{\mu\alpha}+\frac{q_\mu q_\alpha}{M^2_{Z'}}\big]\end{aligned} \tag{4.102}$$

We have

$$Tr[\not{p}_1\gamma^\mu\ \not{p}_2\gamma_5\gamma^\alpha]\big[-g_{\mu\alpha}+\frac{q_\mu q_\alpha}{M^2_{Z'}}\big]=0$$
$$Tr[\not{p}_1\gamma^\alpha\ \not{p}_2\gamma_\alpha]=-8p_1.p_2,\quad Tr[\not{p}_1\ \not{q}\ \not{p}_2\ \not{q}]=8m_t^2(m_t^2+p_1.p_2),\quad Tr[\not{q}\ \not{q}]=8(m_t^2+p_1.p_2) \tag{4.103}$$

After simplification, we obtain:

$$\bar{\sum}|\mathcal{M}_{Z'}|^2 = \frac{8(2m_q^2(a^{t2}_{Z'}M^2_{Z'}+b^{2t}_{Z'}(m_q^2-M^2_{Z'}))+(a^{t2}_{Z'}M^2_{Z'}+b^{2t}_{Z'}(2m_q^2+M^2_{Z'}))p1.p2)}{M^2_{Z'}} \tag{4.104}$$

with

$$p_1.p_2=(M_{Z'}-2m_q^2)/2 \tag{4.105}$$

we get :

$$\bar{\sum}|\mathcal{M}_{Z'}|^2 = 4(b^{2t}_{Z'}(-4m_q^2+M^2_{Z'})+a^{t2}_{Z'}(2m_q^2+M^2_{Z'})) \tag{4.106}$$

We have

$$\Gamma_{Z\prime}=\frac{1}{2M_{Z'}}\int\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(q-p_1-p_2)\bar{\sum}|M|^2 \tag{4.107}$$

We use,

$$\int\frac{d^3p_1}{2E_1}=\int d^4p_1\delta^+(p_1^2-m_q^2) \tag{4.108}$$

$$\Gamma_{Z\prime}=\frac{1}{2(2\pi)^2M_{Z'}}\int\frac{d^3p_2}{2E_2}d^4p_1\delta^+(p_1^2-m_q^2)\delta^4(q-p_1-p_2)\bar{\sum}|M|^2 \tag{4.109}$$

with $p_1 = q - p_2$. We get,

$$\Gamma_{Z\prime} = \frac{1}{2(2\pi)^2 M_{Z'}} \int \frac{d^3 p_2}{2E_2} \delta^+((q-p_2)^2 - m_q^2) \bar{\sum} |M|^2 \tag{4.110}$$

In the Z' frame, we have:

$$q = \begin{pmatrix} M_{Z'} \\ 0 \end{pmatrix}, \quad p_1 = \begin{pmatrix} E_1 \\ \overrightarrow{p_1} \end{pmatrix}, \quad p_2 = \begin{pmatrix} E_2 \\ \overrightarrow{p_2} \end{pmatrix}$$

with

$$q^2 = M_{Z'}^2 \quad , \quad p_1^2 = p_2^2 = m_q^2, \quad p_1 + p_2 = 0 \tag{4.111}$$

and

$$\delta^+[(q-p_2)^2 - m_t^2] = \delta^+(M_{Z'}^2 - 2M_{Z'} E_2) \tag{4.112}$$

where

$$E_2 = \sqrt{|\overrightarrow{p_2}|^2 + m_q^2} \tag{4.113}$$

we obtain :

$$\delta^+[(q-p_2)^2 - m_t^2] = \delta^+(M_{Z'}^2 - 2M_{Z'} \sqrt{|\overrightarrow{p_2}|^2 + m_q^2}) \tag{4.114}$$

for simplifying , we use :

$$\delta[g(x)] = \sum_i \frac{\delta(x - x_i)}{|g\prime(x_i)|} \tag{4.115}$$

the relation (4.114) becomes :

$$\delta^+\left[M_{Z'}^2 - 2M_{Z'} \sqrt{|\overrightarrow{p_2}|^2 + m_q^2}\right] = \frac{\delta^+\left(|\overrightarrow{p_2}|^2 - \frac{M_{Z'}}{2}\sqrt{1 - \frac{4m_q^2}{M_{Z'}^2}}\right)}{2M_{Z'}\sqrt{1 - \frac{4m_q^2}{M_{Z'}^2}}} \tag{4.116}$$

we use the spherical coordinate :

$$d^3\overrightarrow{p_2} = |\overrightarrow{p_2}|^2 d\overrightarrow{p_2} \sin\theta d\theta d\phi \tag{4.117}$$

$\Gamma_{Z\prime}$ becomes :

$$\Gamma_{Z\prime} = \frac{1}{4(2\pi)^2 M_{Z'}} \int_0 \frac{|\overrightarrow{p_2}|^2 d|\overrightarrow{p_2}|}{\sqrt{|\overrightarrow{p_2}|^2 + m_q^2}} \int_0^\pi \sin\theta d\theta \frac{\delta^+\left(|\overrightarrow{p_2}|^2 - \frac{M_{Z'}}{2}\sqrt{1 - \frac{4m_q^2}{M_{Z'}^2}}\right)}{2M_{Z'}\sqrt{1 - \frac{4m_q^2}{M_{Z'}^2}}} \int_0^{2\pi} d\phi \bar{\sum} |M|^2 \tag{4.118}$$

integrate over θ and ϕ , we get :

$$\Gamma_{Z\prime} = \frac{1}{8\pi M_{Z'}^2} \int_0 \frac{|\overrightarrow{p_2}|^2 d|\overrightarrow{p_2}|}{\sqrt{|\overrightarrow{p_2}|^2 + m_q^2}} \frac{\delta^+\left(|\overrightarrow{p_2}|^2 - \frac{M_{Z'}}{2}\sqrt{1 - \frac{4m_q^2}{M_{Z'}^2}}\right)}{\sqrt{1 - \frac{4m_q^2}{M_{Z'}^2}}} \bar{\sum} |M|^2 \tag{4.119}$$

with

$$|\overrightarrow{p_2}|^2 = \frac{M_{Z'}^2}{4}(1 - \frac{4m_q^2}{M_{Z'}^2}) \tag{4.120}$$

The partial decay rate,

$$\Gamma_{Z\prime} = \frac{\sqrt{1 - \frac{4m_q^2}{M_{Z'}^2}}}{16\pi M_{Z'}} \bar{\sum} |M|^2 \tag{4.121}$$

we substitute the amplitude squared, we find :

$$\Gamma_{Z\prime} = \frac{\sqrt{1 - \frac{4m_q^2}{M_{Z'}^2}}}{4\pi M_{Z'}} \left[a_{Z'}^{t2}(2m_q^2 + M_{Z'}^2) + b_{Z'}^{2t}(-4m_q^2 + M_{Z'}^2) \right] \tag{4.122}$$

$$g_{Z'} = -\frac{1}{\sqrt{3}} M_{Z'} \tag{4.123}$$

Finally, we get

$$\Gamma_{Z\prime} = \frac{g_{Z'}^2 M_{Z'}}{12\pi} \sqrt{1 - \frac{4m_q^2}{M_{Z'}^2}} \left[a_{Z'}^{t2}(1 + \frac{2m_q^2}{M_{Z'}^2}) + b_{Z'}^{2t}(1 - \frac{2m_q^2}{M_{Z'}^2}) \right] \tag{4.124}$$

CHAPTER 5

NLO QCD Correction to the top pair production at the LHC

In this chapter, we present the numerical calculation of the hadronic cross section for top pair production at leading order (LO) and next to leading order (NLO) in QCD corrections at LHC in the presence of the Z' gauge boson.

5.1 Factorization

5.1.1 Factorization Theorem

The factorization theorem helps to express the inclusive cross section in a hadron collisions as two different parts, short distance (or hard) part and long (or soft) distance part. The first part is represented by the partonic cross section that describes the interaction between partons without interest of hadrons structure, it can be calculated in perturbative QCD. The second part of hadron collisions depends on hadron structure, called "parton distribution functions". It can not be calculated perturbatively such as the partonic cross section, but it can be determined from experiment[4, 43, 44].

According to the factorization theorem, the inclusive cross section for the collision of two Hadron h_1 and h_2 can be expressed as:

$$\sigma_{h_1 h_2 \to X} = \sum_{a,b \in \{q,g\}} \int dx_a \int dx_b f_a^{h_1}(x_a, \mu_F^2) f_b^{h_b}(x_b, \mu_F^2) \int d\hat{\sigma}_{ab \to Y}(\frac{m_t^2}{s}, \mu_R, \mu_F, \alpha_S(\mu_R)) \tag{5.1}$$

where μ_F and μ_R are the factorization and renormalization scales. a and b are the initial state partons, quark or gluon where X are the partons produced in the final state. For the top-quark pair production, $Y = \{t\bar{t}\}$, see figure (5.1). We observe that there is a similarity between this expression and the expression in the partons model. However, the parton distribution functions $f_a^h(x, \mu_F^2)$ and the cross section $d\hat{\sigma}_{ab \to Y}$ depend on the factorization and renormalization scale factors .

The partonic cross section in relation (5.1) is renormalizable after subtracting the collinear divergences. It can be written in terms of renormalization scale μ_R, which is introduced to conserve the mass dimension of the coupling constant $\alpha_S(\mu_R)$.

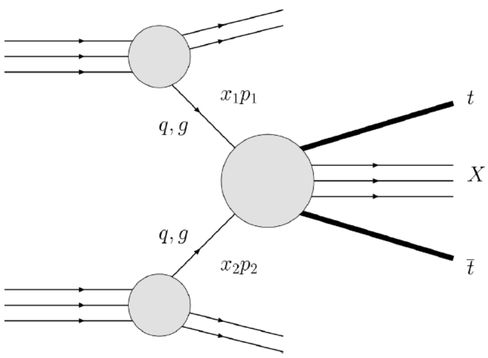


Figure 5.1: Top Quark Pair Production process in factorization picture

The theorem introduces the factorization scale μ_F to cure the collinear divergence absorbed by the PDF in the initial state. In this scale, the partonic cross section and the parton distribution function are separated, and they will become dependent on μ_F. The partons emmitted with $P^2 < \mu_F^2$ are considred as a part of the partonic distribution function while the partons emmitted with $\mu_F^2 < P^2$ are considred as a part of the partonic cross section, which are produced from the Hard scattering process [38, 43, 45].

The factorization and renormalization scales have no physical meaning, they are arbitrary and we are free to choose their values. However, the total cross section is a physical observable, so that it cannot depend on non physical scales.

$$\frac{d\sigma(s,m_t)}{d\mu_F} = \frac{d\sigma(s,m_t)}{d\mu_R} = 0 \tag{5.2}$$

the value of μ_F and μ_R can be chosen independently, but usually we keep them in their formula or choosing a common value $\mu = \mu_F = \mu_R$.

In order to determine the dependence of the PDFs on the factorization scale μ_F, we have to introduce the renormalization group equation, which is given by:

$$\mu_F^2 \frac{df_a(x,\mu_F^2)}{d\mu_F^2} = \sum_{b\in\{q,g\}} \int_x^1 \frac{dz}{z}\frac{\alpha_S}{2\pi}\hat{P}_{ba}(z) f_b(\frac{x}{z},\mu_F^2) \tag{5.3}$$

This is the evolution equation of the pdf with the factorization scale μ_F, which is used to evolve the probability of the distribution of quarks in the proton, it is also called the “DGLAP” equation. The $\hat{P}_{ba}(z)$ are the regularized Altarelli-Parisi splitting functions, which describe the probability of a parton **b** transformed into a parton **a** with a momentum fraction z via emission a gluon (a and b are gluon or quark) as shown in the figure (5.2).

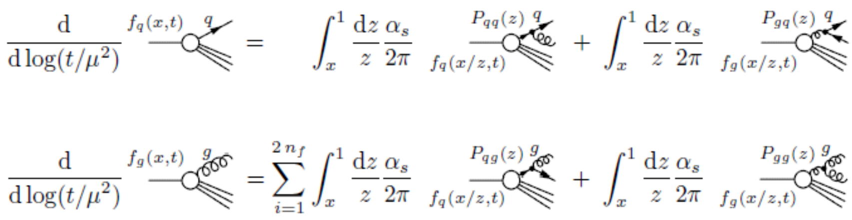


Figure 5.2: Pictorial representation of the DGLAP evolution of PDFs [44]

5.1.2 Parton Distribution Functions

The parton distribution functions (PDFs) are parameterizations of hadrons in term of its components, quarks and gluons. It depends on the structure of hadrons and the factorization scale μ_F. These functions describe the probability of finding a parton i with a momentum fraction $x_i = \frac{p_i}{p_A}$ in an incoming hadron A, where p is the longitudinal momentum of the particle ($P_T = 0$).

The PDFs are universal, we cannot calculate them perturbatively as the partonic cross section, but it can be determined from experiments, such as: deep inelastic scattering $e(k) + N(p) \to e(k') + X$(DIS), Drell-Yan (DY) and jet production at current energy ranges [18, 38, 43]. The PDF also called ”structure function ” which is the sum of the PDF of each proton:

$$F_2 = \sum_{a=q,\bar{q},g} f_{a/p} \tag{5.4}$$

we can write the structure function at NLO as:

$$F_2(x,Q^2) = x\sum_q e_q^2 \int_x^1 \frac{d\varepsilon}{\varepsilon} f_q(\varepsilon,\mu_F^2)\Big[\delta(1-\frac{x}{\varepsilon}) + \frac{\alpha_S}{2\pi}P_{qq}(\frac{x}{\varepsilon}) ln\frac{Q^2}{\kappa^2} + C(\frac{x}{\varepsilon})\Big] \tag{5.5}$$

where $f_q(\varepsilon, \mu_F^2)$ is the renormalized PDF, given by:

$$f_q(x, \mu_F^2) = f_q(x) + \frac{\alpha_S}{2\pi} \int_x^1 \frac{d\varepsilon}{\varepsilon} f_q(\varepsilon) \left[P\left(\frac{x}{\varepsilon}\right) ln \frac{Q^2}{\mu_F^2} \right] \tag{5.6}$$

we can calculate the evolution of the PDFs perturbatively from DGLAP equations[29], we consider all possible quark and gluon splittings:

$$t \frac{\partial}{\partial t} \begin{pmatrix} f_{q_i}(x,t) \\ f_g(x,t) \end{pmatrix} = \frac{\alpha_S}{2\pi} \sum_j \int_x^1 \frac{d\varepsilon}{\varepsilon} \begin{pmatrix} p_{q_i q_j}(\frac{x}{\varepsilon}, \alpha_S(t)) & p_{q_i g}(\frac{x}{\varepsilon}, \alpha_S(t)) \\ p_{g q_j}(\frac{x}{\varepsilon}, \alpha_S(t)) & p_{gg}(\frac{x}{\varepsilon}, \alpha_S(t)) \end{pmatrix} \begin{pmatrix} f_{q_i}(x,t) \\ f_g(x,t) \end{pmatrix} \tag{5.7}$$

the $P_{q_i q_j}$ functions have a perturbative expansion in α_S

$$P_{q_i q_j (z, \alpha_S)} = \delta_{ij} P^0_{qq}(z) + \frac{\alpha_S}{2\pi} P^1_{q_i q_j}(z) + .. \tag{5.8}$$

The splitting functions $\hat{P}_{ba}(z)$ are given by:

$$\begin{aligned}
\hat{P}_{gg}(z) &= 2C_A \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) \right] + \delta(1-z) \left(\frac{11}{6} C_A - \frac{2}{3} n_f T_R \right) \\
\hat{P}_{qq}(z) &= C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right] \\
\hat{P}_{gq}(z) &= T_R \left[z^2 + (1-z)^2 \right] \qquad \hat{P}_{qg}(z) = C_F \left[\frac{1+(1-z)^2}{z} \right]
\end{aligned} \tag{5.9}$$

5.2 Next to leading order calculation

We have seen that it is necessery to go beyond the leading order in perturbative theory and include the higher orders cross section in order to reduce the non physical scales (the factorization and renormalization scales) dependence on the partonic cross section and thus to make the calculation more precise [4, 38, 45, 46].

We can write the perturbation expansion of leading order cross section as:

$$\hat{\sigma} = \alpha_S^a \sigma_0 + \alpha_S^{a+1} \sigma_1 + \alpha_S^{a+2} \sigma_2 + ... \tag{5.10}$$

where the coupling and the cross section depend on the non physical scales:

$$\alpha_S = \alpha_S(\mu_R) \quad , \quad \sigma_i = \sigma_i(\mu_F, \mu_R) \tag{5.11}$$

The σ_0 represents the LO cross section and σ_1 is the NLO cross section. If we include the first-order correction in the perturbative expansion. The cross section σ_1 will have less dependence on the factorization scale. However, these scale dependencies will entirely eliminated if we include all high-orders.

The NLO cross section is the first order that contribute to an additional physical terms present in the QCD correction to the quark distribution functions in which, the PDF is compensated by loop corrections. The total QCD cross section in Next to Leading Order can be written as:

$$\sigma^{NLO} = \int_m d\sigma^B + \int_{m+1} d\sigma^R + \int_m d\sigma^V \tag{5.12}$$

where $d\sigma^B$ is the LO (or Born) cross section, $d\sigma^R$ and $d\sigma^V$ are the real and virtual contributions to the cross section, respectively. They have the same structure as the born contribution. The integration is over the final state phase space,

5.2.1 Virtual Contribution

The virtual contributions (or loop) diagram have two external line in initial and final state as in born contributions (2 → 2 process). but it is distinguished from other diagrams by an additional internal line, which forms a single closed loop. We can also called " 1 - loop diagram ", see figure(5.3) [4, 43] .
In this contributions, the amplitude squared is obtained by interfering Born diagrams with real diagrams.

$$d\sigma^V = d\phi_n \sum 2Re(\mathcal{M}_1^V \mathcal{M}_0^*) \tag{5.13}$$

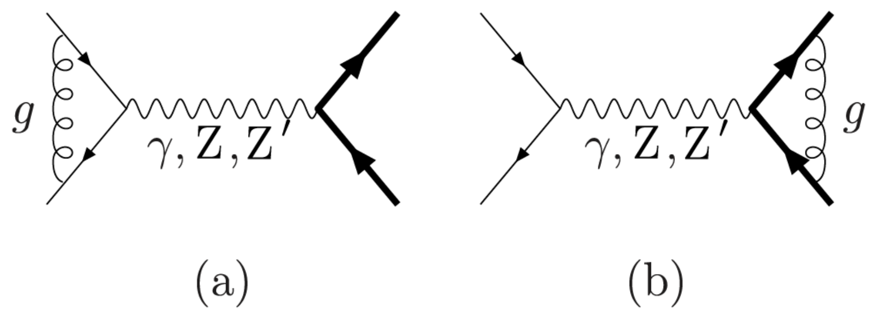


Figure 5.3: NLO virtual corrections to the top pair production at order $O(\alpha_S \alpha_W^2)$
.

But the calculation of the amplitude squared in eq (5.13) will lead to ultra-violet (UV) and infra-red (IR) divergences. The IR divergences will be canceled by the infrared singularities of the real contributions. The UV divergences can be eliminated by adding counterterms diagram (renormalization). This contribution can be calculated automatically by **MadGraph**.

5.2.2 Real Contribution

The real contributions diagram is represented by an additional external line (quark or gluon) in the initial or final state. It has different final state compared to the born and virtual diagrams as shown in the figure (5.4). The amplitude squared is obtained by interfering the real emission diagrams.

$$d\sigma^R = d\phi_{n+1} \sum |\mathcal{M}_1^R|^2 \tag{5.14}$$

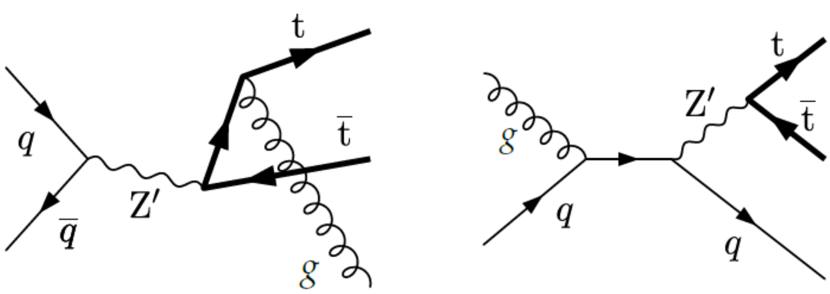


Figure 5.4: NLO real corrections to the top pair production at order $O(\alpha_S \alpha_W^2)$

We have seen that the NLO cross section contains more physical terms represented in the real and virtual contribution, which corresponds to adding gluons to the LO contribution cross section.

However, these contributions are infinite terms, they involve a collinear and soft divergences. According to KLN theorem, the divergences coming from the final state radiation can be cancelled, when we combine all possible real and virtual processes. But the initial state divergences do not cancel out. Since these divergences are absorbed in the PDFs by the virtue of the factorization theorem, and they can only be cancelled through the QCD corrections to the quark distribution functions “the collinear counterterms ” that are manifested in a new process with a new coupling to compensate these divergences. In order to cancel these divergences and obtain a finite cross section, we use the ”Substraction Method ”.

5.2.3 Substraction Method

The substraction method is a method used to cancel the divergences of the virtual and real contribution. The idea of this method is to introduce a fake cross section substracted from the real contribution and added back to the virtual contribution. In which, this term is constructed by summing over different contributions that describes soft and collinear divergences:

$$\sigma^{NLO} = \int_m d\sigma^B + \int_{m+1} [d\sigma^R - d\sigma_R^{sub}] + \int_{m+1} d\sigma_V^{sub} + \int_m d\sigma^V \tag{5.15}$$

where

$$\int_{m+1} d\sigma_V^{sub} + d\sigma_R^{sub} = 0 \tag{5.16}$$

the additional cross section $d\sigma^{sub}$ have the same pointwise singular behaviour (in D dimensions) as $d\sigma^R$. For the calculation of QCD correction, the MADGRAPH program use Madloop to calculate loop diagram and MadFKS to employ the substraction method.

5.2.4 NLO cross section

In this section, we will calculate the hadronic cross section of the top pair production $\sigma(pp \to t\bar{t})$ at LO and NLO orders. We use the same program as in the previous chapter.

For the top quark pair production, the the general perturbative expansion of partonic cross section in NLO is given by:

$$\sigma = \sigma_S^{LO}(\alpha_S^2) + \sigma_W^{LO}(\alpha_W^2) + \sigma(\alpha_S^2\alpha_W) + \sigma(\alpha_W^2\alpha_S) + \sigma(\alpha_S^3) + \sigma(\alpha_W^3) \tag{5.17}$$

where the first two term correspond to the LO partonic cross sections, the $\sigma(\alpha_W^2\alpha_S)$ and $\sigma(\alpha_S^3)$ terms correspond to the QCD correction by an additional QCD power α_S while the $\sigma(\alpha_W^3)$ and $\sigma(\alpha_S^2\alpha_W)$ terms correspond to the electroweak correction.

As we see in the previous chapter, the existence of Z' doesn't affect the cross section for the QCD process. In this case, the cross section at order $\sigma(\alpha_S^3)$ does not affected, while the cross section at order $\sigma(\alpha_W^2\alpha_S)$ will be affected. so, we will calculate the LO and the NLO QCD correction to the electroweak and QCD sub processes.

Electroweak (QED) processes

We will calculate the hadronic cross section for the electroweak processes. We consider a fixed Z' mass $M_{Z'} = 1000$ GeV and the center of mass energy $\sqrt{s} = 13 Tev$. The factorization and the renormalization scales are fixed to be equal to the mass $\mu = \mu_F = \mu_R$. We change the value of the scale factor each time for both cases. The results are shown in the table (5.1),

μ (Gev)	0.1	0.2	0.3	0.6	1	3	6	8	10
σ^{LO}(fb)	747	738	726	713	699	671	653	647	638
σ^{NLO} (fb)	9004	7114	6851	5770	5605	4535	4371	4064	3917

Table 5.1: Variation of the cross section at LO and NLO in term of scale factor μ.

Now we fix the value of the scale factor $\mu = 1$GeV and we we change the value of $M_{Z'}$ for $\sqrt{s} = 13Tev$. The results are shown in the table (5.2),

$M_{Z'}$ (Gev)	500	700	900	1000	2000	2500	3000
σ^{LO}(fb)	1761	1166	806	642	413	405	402
σ^{NLO} (fb)	7317	6469	5876	5259	4175	6322	6812

Table 5.2: Variation of cross section at LO and NLO in term of $M_{Z'}$.

We observe a significant difference in the values of the cross section between leading order and next to leading order, in which the values of the cross section at NLO are larger than LO. This difference is due to the NLO QCD correction that have more terms, see figure (5.5),

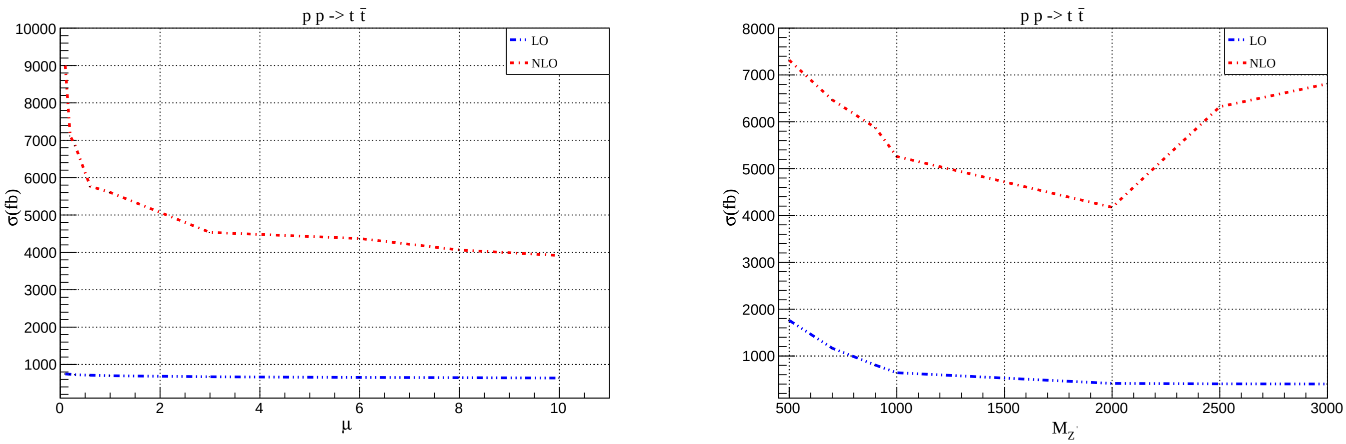


Figure 5.5: Variation of LO and NLO cross section for the EW processes in term of μ and $M_{Z'}$

The figure (5.5) shows the variation of LO and NLO cross section in term of the scale factor μ (left side) and the Z' mass $M_{Z'}$ (right). We observe that the NLO cross section decrease more than LO (LO is mostly constant) when the value of scale factor increases because the NLO QCD correction of the electroweak sub-processes are affected by the existence of Z' the new gauge boson and thus, the dependence of the NLO cross section on the non physical scale factor has reduced compared to LO. Since the NLO contains more physical term comparably to LO.

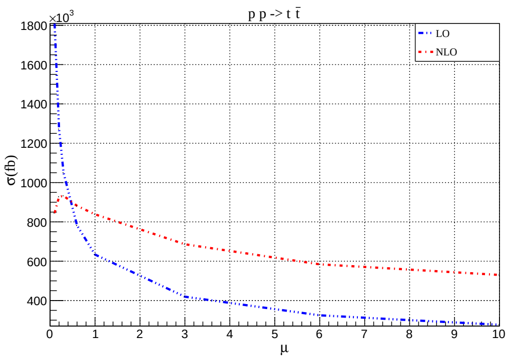
QCD processes

We calculate the hadronic cross section for the QCD process. The same as the previous work, we change the value of the scale factor for a fixed mass of $M_{Z'} = 1000$ GeV at $\sqrt{s} = 13$ energy, then we change the mass of Z' with the same value of the energy and scale factor $\mu = 1GeV$. The results are shown in the tables (5.3),

μ (Gev)	σ^{LO}(fb)	σ^{NLO} (fb)
0.1	1809000	845000
0.2	1272000	927800
0.3	1050000	932500
0.6	781700	881600
1	633800	838400
3	419400	686200
6	324800	584200
8	300300	557300
10	276800	530100

$M_{Z'}$ (Gev)	σ^{LO}(fb)	σ^{NLO} (fb)
500	641600	836100
700	631700	841300
900	636100	835000
1000	641200	837900
2000	638800	833700
2500	633400	829700
3000	639700	838400

Table 5.3: QCD contribution



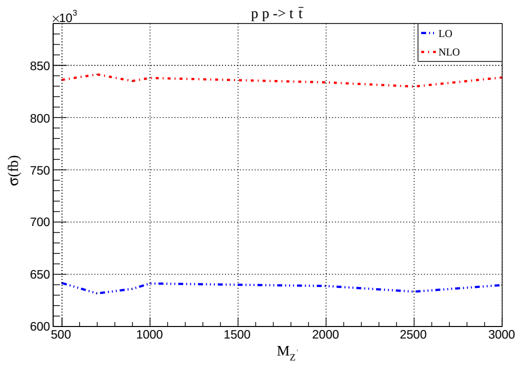


Figure 5.6: Variation of LO and NLO cross section for the QCD processes in term of μ and $M_{Z'}$

We observe that the variation of LO and NLO cross section in term of the Z' mass are mostly constant despite that the value of the NLO cross section is larger than LO order. Because the existence of Z' doesn't affect the cross section in both LO and NLO QCD correction.

5.3 Parton shower

Parton shower is an approximation of QCD, which describes the partons branching in the initial and final state in the hard scattering. The Parton shower approximation include all order contribution in the collinear limit[44, 47]. We use pythia8 to generate the Parton shower (all possibilities of splitting). We need the Parton shower if the calculation of cross section does not give a good description, although the Parton shower is only an approximation, it provides more events in the final and initial state. The figure (5.7) represents the production of top quark at high energy process.

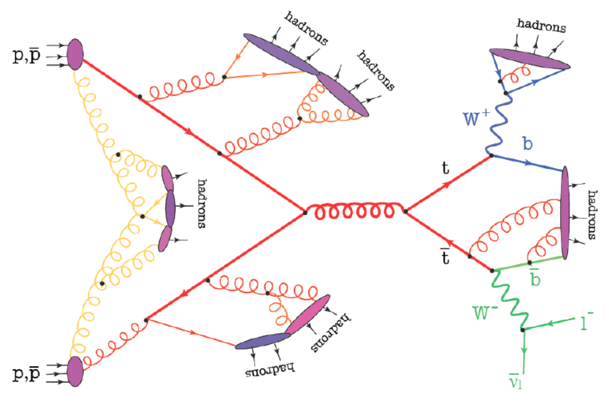


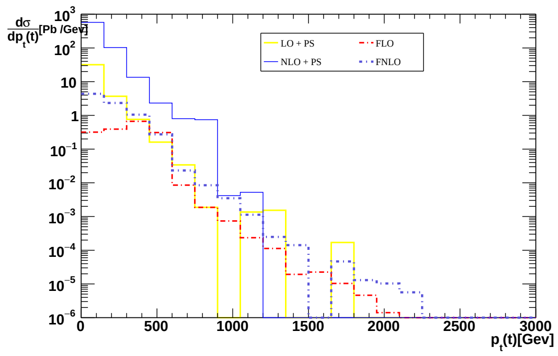
Figure 5.7: event generator in proton - proton collisions

The general steps are:

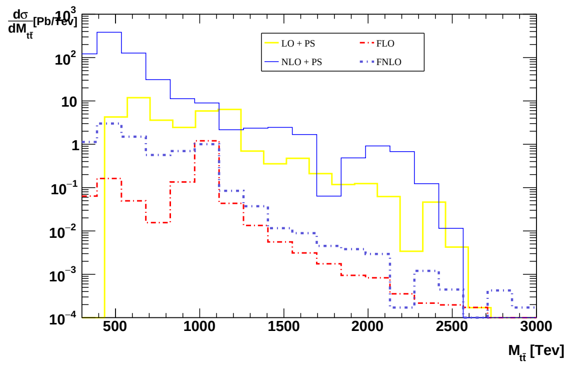
- **Hard process** the collision of proton-proton at high energy to produce the top quark pair in final state.
- **Radiation** the emission of gluon or quark from the initial and final state parton and then produce particles with less energy .
- **Hadronization** the confinement of the parton emitted into hadrons.
- **Underlying event** the decays of hadron in final state .

5.3.1 Differential distributions

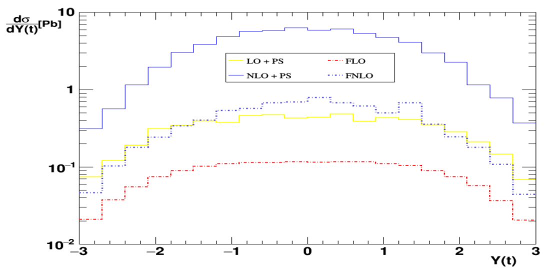
In this section, we will calculate the differential cross section for top pair production at LO , NLO , LO+PS and NLO+PS with a fixed mass $M_{Z'} = 2000Gev$ and energy $\sqrt{S} = 13Tev$,$\mu = 1$.



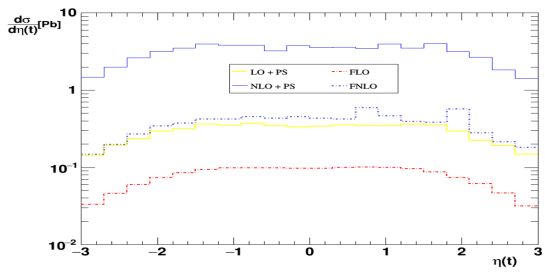
Transvere Momuntum distribution



Invariant-mass distributions



pseudo-Rapidity distributions



Rapidity distributions

Figure 5.8: differential cross sections of top-quark pairs at FLO , FNLO , LO + PS and NLO + PS

Chapter 6
Conclusion

In this master thesis, I studied the pair production of top quark in BSM models including an extra neutral gauge boson (Z') at fLO (fixed leading order), fNLO (fixed next-to-leading order), LO+PS (LO matched to parton shower) and NLO+PS (NLO matched to parton shower). I used MadGraph to compute numerically the hadronic cross section in all the approximation and MadAnalysis to produce the distributions and plot the Hisrtograms.

In the theoretical part, I studied many beyond standard model theories which predict the existence of Z' gauge boson, as the sequential standard model, GUT models, left right-symmetric models . . . etc. I presented also a phenomenological model (effective model) which allows us to study the physics of Z' gauge boson independently of the BSM model. I discussed how one can rearch for this particle at the LHC, how it can be produced and how it decays. I also studied a minimal U_{B-L} model which predicts the existence of a Z' gauge boson, in addition to new right handed neutrinos and a new scalar field "χ " to break the new symmetry in order to make a massive gauge boson (Z').

In the paractical part, I studied the effect of the Z' on the pair production of the top quarks at the LHC. I presented the analytical and numerical calculation of the LO cross section and its NLO QCD correction for the production of top quark pairs beyond the standard model in the existence of Z'. And then I discussed the variation of the hadronic cross section in terms of the mass of Z' and the renormalisation and factorisation scales. I produced many differential distributions in several observables in four approximations: fLO, fNLO, LO+PS and NLO+PS.

I showed how the total cross section and the top-pair invariant mass distribution can be strongly affected by the NLO QCD corrections. I showed that the NLO approximation does not fully cure the dependence of the cross section on the non-physical scale μ_F and μ_R, but it reduces them and still better than LO approximation.

I presented the LO and NLO approximation merging with the Parton shower effects. I showed that the differential distributions of the top quark at NLO+PS approximation are larger in comparaison with the other approximations. The parton shower approximation gives a good description of what happen in detectors and its results are more precise at low energy.

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Abstract:
The theories beyond standard model (BSM) predict the existence of many hypothetical particles that have not yet been detected in the experiment. Since the actual existence of such particles can cure the shortcomings of the standard model. In this work, we focus on the physics of one of the hypothetical gauge boson which is Z' Boson. The Z' is a neutral gauge boson related to the new $U'(1)$ gauge symmetry and there are many different types of this boson according to BSM models. The objective of this work is to study the effect of the Z' on the top quark pair production at the LHC. In order to know the most precise approximations, we calculate the hadronic cross section in the existence of the new Z' gauge boson at fLO , fNLO , LO+PS and NLO+PS. We use `hip` and `MadGraph` for the numerical calculations and `MadAnalysis` to produce the differential distributions.

Keywords: BSM physics, Z' Gauge Boson, Top Quark, Parton Shower, Z'.

Résumé :
Les théories au-delà du modèle standard (BSM) prédisent l'existence de nombreuses particules hypothétiques qui n'ont pas encore été détecté dans l'expérience. Étant donné que l'existence réelle de telles particules peut remédier aux lacunes du modèle standard. Dans ce travail, nous nous concentrons sur la physique d'un des bosons de jauge hypothétiques qui est le Z' Boson. Le Z' est un boson de jauge neutre lié à la nouvelle symétrie de jauge $U'(1)$, il existe de nombreux types différents de ce boson selon les modèles BSM.
L'objectif de ce travail est d'étudier l'effet du Z' sur la production des paires de quarks top au LHC. Afin de connaître les approximations les plus précises, on calcule la section efficace hadronique de la production d'une paire de quarks top dans les approximations fLO, fNLO, LO+PS et NLO+PS. Nous avons utilisé `hip` et `MadGraph` pour les calculs numériques de la section efficace et `MadAnalysis` pour produire les distributions différentielles.

Mots clés : physique BSM, boson de jauge Z', quark top, parton shower, LHC

ملـخّص:
نماذج الفيزياء ما وراء النّموذج القياسـي (BSM) تـتنبأ بوجود العديد من الجسيمات الافتراضيّة و الّتي لم يتم الكشف عنها تجريبيا بعد. نـنوه أنّ وجود مثـل هذه الجسيـمات يحلّ العـديد مـن مشاكل النّموذج القياسي.
في هذا العـمل نركّز على فيـزياء أحـد البوزونات المـعياريّة الافتراضيّة ألا و هـو البوزون Z' . البوزون Z' هــو بوزون معياري حيادي يرتبط بالتّناظر المـعياري الجديـد $U'(1)$ ، يوجـد أنواع مـتعدّدة مـن هذا البوزون و ذلك مـربط بالنّموذج BSM .
الـهدف مـن هذا العــمل هـو دراسة تاثيـر Z' على انتاج زوج مـن الكواركات توب فـي LHC . لـمعرفة التّقــريبات بـدقّة، نـحسب المـقطع الفعّال الهادروني لانتاج زوج مــن الكواركات توب فـي التّقريـبات fLO ، $fNLO$ ، $LO+PS$ و $NLO+PS$. اســتعملنا `hip` و `MadGraph` مـن أجــل الحسابات العدّيّة لـلمقطع الفعّال و `MadAnalysis` لانتاج التّوزيـعات التّفاضليّة.

كـلمات مـفتاحيّة: الفيـزياء BSM ، البوزون المـعياري Z' ، الكوارك تــوب، شـلال الجسيمات (PS)، LHC
