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THESIS

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By:
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Theme:

Contributions to Adaptive Control of Multivariable Nonlinear Systems With and Without Observers

Defended on: 08/12/2019 in front of the Examination Committee:

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Abstract:

The work of this thesis deals with two types of problems in adaptive fuzzy control for nonlinear multivariable systems: state-feedback control and output-feedback control. In all proposed adaptive control schemes, fuzzy logic systems are used to appropriately approximate uncertain nonlinear functions. A Lyapunov-based analysis is carried out to conclude about the stability of the closed-loop system as well as the convergence of the underlying (tracking and observation) errors. Numerical simulations are presented to demonstrate the effectiveness of theoretical results.

Key-words:

Multivariable systems, adaptive fuzzy control, observer, Lyapunov stability.

Résumé :

Les travaux de cette thèse traitent deux type de problèmes de la commande adaptative floue pour les systèmes multivariables non linéaires: la commande par retour d'état et la commande par retour de sortie. Dans tous les schémas de commande adaptative proposés, les systèmes à logique floue sont utilisés pour approcher correctement les fonctions non linéaires incertaines. Une analyse basée sur Lyapunov est réalisée pour conclure sur la stabilité du système en boucle fermée ainsi que sur la convergence des erreurs sous-jacentes (de poursuite et d'observation). Des simulations numériques sont présentées pour démontrer l'efficacité des résultats théoriques.

Mots-clés :

Systèmes multivariables, contrôle flou adaptatif, observateur, stabilité de Lyapunov.

ملخص:

يتعامل عمل هذه الأطروحة مع مشكلتين أساسيتين في التحكم التكيفي الغامض لأنظمة الغير خطية متعددة المتغيرات: تحكم رد فعل الحالة وتحكم مزود بمراقب. في جميع مخططات التحكم التكيفي المقترحة، يتم استخدام أنظمة المنطق الغامض لتقريب الوظائف غير الخطية غير المؤكدة بشكل صحيح. استعملنا طريقة ليابونوف لدراسة استقرار طرق التحكم التكيفي الغامض المقترحة. يتم تقديم المحاكاة العددية لإثبات فعالية النتائج النظرية.

كلمات مفتاحية

أنظمة متعددة المتغيرات، تحكم تكيفي غامض، مراقب، استقرار ليابونوف.

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الحمد لله والشكر لله على توفيقه لي لإتمام هذا العمل المتواضع ...

اهدي هذا العمل الى ذكرى والدي - رحمهما الله - لكل الحب والصبر والتشجيع الذي قدماه لي طوال مشواري الدراسي ...

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الى استاذي المحترم بولقرون عبد السلام ...

اهديه الى كل من ساعدني ودعمني وشجعني ولو بكلمة طيبة واحدة ...

لبنى مرازقة

Outline

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Chapter 3 : High-gain Observer-based Fuzzy Adaptive Controller of MIMO Nonlinear Systems

Chapter 4 : Unified Observer-based Adaptive Fuzzy Control For a Class of MIMO Nonlinear Systems

General Introduction

1. Background and motivation

In control engineering, the majority of systems are ***multivariable and nonlinear*** in nature. It is certain that the control theory for multivariable nonlinear systems will find a direct application in a wide variety of problems (electrical machines, robotics, chemical process, space technology, and so on). On one hand, designing a controller for multivariable systems is a difficult problem owing to the coupling that naturally exists between the control inputs and the outputs. In a multivariable system, an input does not only effect its output, but also one or more other outputs in the plant [SUJ13]. On the other hand, the superposition principle does not hold for nonlinear systems and this tends to make their stability analysis and the control design more difficult. Given a nonlinear problem, it would appear that the problem could be linearized in some significant way so that the problem may well be explored using linear techniques, which is unfortunately not always possible. For example, given a nonlinear system, a linear feedback control designed to its linearized model may in fact not work correctly (robustly) for the original system (i.e. for the actual nonlinear system). Hence, many important nonlinear systems' control problems must be analyzed from an inherently nonlinear point of view [MAH96].

Robust control strategies can yield valuable performances for nonlinear systems. It is possible to determine the bounds on the system uncertainties and design a robust controller guaranteeing good performances as long as these uncertainties remain within those bounds. However, the robust controllers (i.e. sliding mode controllers) suffer from certain disadvantages, namely: 1) they are high-gain controllers, therefore they are very sensitive to noise and they suffer from the inherent chattering problem. 2) The bounds of uncertainties should be known. 3) They do not perform well for systems with time-variable parameters. On the other hand, the use of ***adaptive control strategies*** can be an effective solution to these above problems. Adaptive control offers an important advantage; the bounds of the uncertainties are not required to be known, they are in fact cancelled online in an adaptive way. In an adaptive control scheme, the controller parameters (direct scheme) or the nonlinear function parameters (indirect scheme) are updated online using the available signals form the sensors. However, standard adaptive control strategies [SLO91, BOU05, BOU06, BOU07b] are limited to nonlinear systems that can be linearly parameterized. Unfortunately, it is often very difficult, even impossible, to obtain this form of linear parameterization, particularly for uncertain complex physical systems.

Fuzzy control has an important impact in the control community because the fuzzy controllers provide a systematic and efficient framework to incorporate linguistic fuzzy information from human expert [WAN92, WAN94]. An adaptive fuzzy system is a fuzzy logic system that is equipped with an appropriate learning algorithm. The nonlinear multivariable control incorporating the fuzzy systems as universal approximators has received a great deal of attention over the past few decades. Some fuzzy adaptive control approaches for a class of multivariable nonlinear uncertain systems have been proposed in [BOU10a, TON03, ESS06, ORD99, CHA00, CHE03] thanks to the universal approximation theorem [WAN94]. The stability of the closed-loop system, in these schemes, has been established according to Lyapunov’s theory. Unlike standard adaptive control strategies, adaptive fuzzy controllers present a valuable solution to the control for nonlinear uncertain systems, as these control systems do not require models with a linear parameterization. To cope with fuzzy approximation errors and external disturbances in these fuzzy control schemes, two approaches are generally used, namely:

- the main fuzzy control law is augmented by a robustifying control term (sliding mode control term and /or H_∞ control term), e.g. [TON00, LAB07, BOU08a, BOU14],
- the parameter update law is augmented by a proportional term, [BOU10b].

The most of the adaptive fuzzy control schemes devoted to multivariable systems need the complete state to calculate the control signals. Whereas, in the practice, only the output vector could be obtained by a direct measurement. In this case, an observer is needed and a ***fuzzy adaptive output feedback control*** (i.e. an observer-based adaptive fuzzy control) should be designed [SHI15, ARE13, SHA16].

2. Contributions of this thesis

This thesis contributes to the theory of adaptive control of nonlinear multivariable systems in the following:

1. Design of an adaptive fuzzy state-feedback control scheme of uncertain nonlinear multivariable systems having a control gain matrix which is not necessarily symmetric or with a definite sign.

2. Proposition of a novel proportional-integral (PI) update law to improve the adaptive parameters convergence. To overcome the famous problem of the algebraic-loop in this update law and to make it robust against the disturbances, the latter is equipped respectively by a low-pass filter and a robust term (either $\sigma - modification$ or $e - modification$).

3. Design of a fuzzy approximation-based output-feedback control scheme for a class of uncertain nonlinear multi-input multi-output mechanical systems using a high-gain observer.

4. Proposition of a unified observer-based adaptive fuzzy control for a special class of MIMO nonlinear systems, which consists of N interconnected subsystems. One will show that this unified design framework is valid for several types of observers.

3. Outline of the Thesis

This thesis is a compilation of several research works. Its chapters are based on journal or conference articles, which are either published or currently under review..It is organized as follows:

The first chapter is principally devoted to the recalling of the basic concepts of fuzzy systems, adaptive control and high-gain observers. Some stability definitions, lemmas and theorems, which are essential for the control laws' synthesis, are also given.

In the second chapter, we design a novel fuzzy adaptive state-feedback control scheme for a class of uncertain nonlinear multivariable systems whose control gain matrix is assumed to be characterized by non-zero leading principle minors. In fact, this control gain matrix can be non-symmetric and but with an indefinite sign (i.e., it can be neither positive definite nor negative definite). Adaptive fuzzy systems are used to online learn the uncertainties. A proper decomposition property of the control gain matrix is exploited in the control design process as well as in the stability analysis of the closed-loop system. A PI type adaptation mechanism is proposed to improve the adaptive parameter convergence. An appropriate quadratic Lyapunov function is built to analyze the stability of the closed-loop control system as well as to design the parameters update laws. A set of numerical results are also given to support the theoretical aspects.

In the third chapter, a fuzzy approximation based output-feedback adaptive control scheme is presented for a class of uncertain nonlinear multi-input multi-output mechanical systems. Just like the previous chapter, the control-gains matrix is supposed to be characterized by non-zero leading principal minors as well as a non-symmetric structure. This control methodology uses a linearly parameterized fuzzy system to approximate the uncertain nonlinear functions in the system as well as a high-gain observer to accurately and quickly construct the unmeasured states. A suitable Lyapunov function is constructed to analyze the stability of the associated closed-loop system as well as to design the update laws. Extensive simulation results are also given to demonstrate that the proposed control methodology is well effective.

In the fourth chapter, we present a novel fuzzy adaptive output-feedback control for a special class of multivariable nonlinear systems with uncertainties and bounded external disturbances. We use a unified observer (high-gain observer, smooth sliding mode observer, non-smooth sliding mode observer, …) to estimate the immeasurable states. A PI update law augmented by the $\sigma - modification$ and a low-pass filter is designed to online estimate the unknown fuzzy parameters. A Lyapunov's approach is exploited to prove the uniform ultimate boundedness of all signals involved in the control loop as well as the convergence of observation and tracking errors. The performances of the proposed controller are also demonstrated in a realistic simulation framework involving two practical nonlinear systems.

A general conclusion is given at the end for the contents synthesis and contributions of this thesis with some important perspectives that can fill the aspects not covered in this thesis.

Chapter 1

Background on adaptive control, high-gain observer and fuzzy systems

1.1 Introduction

In this chapter, we will exhibit some useful notions of fuzzy systems, adaptive control and high-gain observers. Also, we will briefly recall some mathematical background necessary for the design of the control schemes developed in this thesis.

1.2 Stability of dynamical systems

Definitions, lemmas, and theorems given in this section are particularly useful in stability analysis and adaptive control synthesis.

1.2.1 Lyapunov stability

The stability of dynamical systems characterizes the behavior of their trajectories around equilibrium points. The stability of an equilibrium point determines whether or not solutions in the neighborhood of this equilibrium point remain nearby, get closer, or get further away [ZEM07].

Consider the following class of nonlinear systems described by the dynamical equations:

$$\dot{x}(t) = f(x(t), t), \quad x(t_0) = x_0 \tag{1.1}$$

where $x(t) \in R^n$ and $f: R^n \times R^+ \to R^n$ is continuous. One denotes by x_e an equilibrium point of (1.1) such that $f(x_e, t) = 0,\ \forall t \geq t_0$, and by $x(t, t_0, x_0)$ the solution at time $t \geq t_0$ of the system (1.1) initialized by x_0 at time t_0.

One assumes that the system (1.1) has a unique equilibrium point $x_e = 0$. One presents now some stability definitions of the system (1.1) around the origin.

Definition 1.1 (Stability): *The origin is a stable equilibrium point in Lyapunov's sense for (1.1), if* $\forall\, \varepsilon > 0,\ \forall\, t_0 \geq 0$, *there exists a positive scalar* $\delta(\varepsilon, t_0)$ *such that:*

$$\|x_0\| < \delta(\varepsilon, t_0) \Rightarrow \|x(t, t_0, x_0)\| < \varepsilon, \qquad \forall t \geq t_0.$$

We say that the origin is unstable in the opposite case.

Definition 1.2 (Uniform stability): *The origin is a uniformly stable equilibrium point for* (1.1), *if* $\forall\, \varepsilon > 0$, *there exists a positive scalar* $\delta(\varepsilon)$ *such that :*

$$\|x_0\| < \delta(\varepsilon) \Rightarrow \|x(t, t_0, x_0)\| < \varepsilon, \qquad \forall t \geq t_0.$$

Definition 1.3 (Uniformly Ultimately Bounded (UUB) stability): *The solutions of the system (1.1) are said UUB, if there exist constants b and c such that, for all* $\alpha \in [0, c]$, *there is a positif time* $T = T(\alpha)$ *(independent of* t_0*) such that, for all* $x_0 \in B_0$ *and* $\forall\, t_0 \geq 0$, *one has:*

$$\|x_0\| < \alpha \Rightarrow \|x(t, t_0, x_0)\| < b, \qquad \forall t \geq t_0 + T.$$

When this is true for any positive constant α, the solutions of (1.1) are globally UUB (GUUB).

Definition 1.4 (Attractiveness): *The origin is an attractive equilibrium point for (1.1), if* $\forall\, \varepsilon > 0$, *there exists a positive scalar* $\delta(t_0)$ *such that :*

$$\|x_0\| < \delta(t_0) \Rightarrow \lim_{t\to\infty}\big(x(t, t_0, x_0)\big) = 0\,, \qquad \forall t \geq t_0.$$

When $\delta(t_0) = +\infty$, *we say that the origin is globally attractive.*

Definition 1.5 (Asymptotical stability): *The origin is an asymptotically (respectively, globally asymptotically) stable equilibrium point for (1.1), if it is stable and attractive (respectively, globally attractive).*

Definition 1.6 (Exponential stability): *The origin is a locally exponentially stable equilibrium point for (1.1), if there are two strictly positive constants α and β such that:*

$$\|x(t, t_0, x_0)\| \leq \alpha \exp(-\beta(t - t_0)), \;\; \forall t \geq t_0, \forall x_0 \in B_r$$

when $B_r = R^n$, *we say that the origin is globally exponentially stable*.

The utilization of the above definitions to study the stability of (1.1) around its equilibrium point requires the explicit resolution of the differential equation (1.1), which is often very difficult or even impossible in most cases. The use of the Lyapunov's direct method (also called the Lyapunov's second method) makes it possible to circumvent this obstacle by defining a particular function whose existence guarantees stability.

1.2.2 Lyapunov's direct method

The Lyapunov's direct method allows us to conclude about the stability of a nonlinear system without the analytical resolution of the nonlinear differential equations (1.1).

Definition 1.7: *let* $V\ (x,t)$: $R^n \times R^+ \rightarrow R^+$ *be a continuous function.* V *is proper and positive definite, if:*

1. $\forall t \in R^+, \forall x \in R^n, x \neq 0: V(x,t) > 0;$
2. $\forall t \in R^+, V(x,t) = 0 \Rightarrow x = 0;$
3. $\forall t \in R^+, \lim_{\|x\| \to \infty} V(x,t) = \infty.$

Definition 1.8 (Lyapunov function): *A function* $V(x,t)$ *of the class* C^1 *is a local (respectively, global) large Lyapunov function for the system* (1.1), *if it is proper and definite positive and if there is a neighborhood of the origin* V_0 *such that* $\forall x \in V_0$ *(respectively,* $\forall x \in V_0 = R^n$*):*

$$\dot{V}(x,t) = \frac{\partial V(x,t)}{\partial t} + \left(\frac{\partial V(x,t)}{\partial x}\right) f(x(t),t) \leq 0.$$

If $\dot{V}(x,t) < 0$, *then* V *is called the strict Lyapunov function.*

Theorem 1.1 (Lyapunov's direct method): *If the system (1.1) admits a local large Lyapunov function (respectively, a local strict Lyapunov function), then the origin is a locally stable (respectively, asymptotically stable) equilibrium point.*

This result can be globally validated in the case where $\forall x \in R^n$.

Theorem 1.2 (Exponential stability): *The origin of (1.1) is locally exponentially stable, if there are constants* $\alpha, \beta, \gamma > 0$, $p \geq 0$ *and a function* $V\ (x,t) : V_0 \times R^+ \rightarrow R^+$ *of class* C^1 *such that,* $\forall x \in V_0$*:*

1. $\alpha \|x\|^p \leq V(x,t) \leq \beta \|x\|^p;$
2. $\dot{V}(x,t) < -\gamma V(x,t).$

If the ball $V_0 = R^n$, *then the origin is globally exponentially stable.*

Remark 1.1: *Choosing a quadratic Lyapunov function* $V(x,t) = x^T P x,\ with\ P = P^T > 0$, *the linear system* $\dot{x}(t) = Ax(t)$ *is globally exponentially stable, if and only if* P *is the solution of the equation* $A^T P + PA = -Q$, *for a given positive definite matrix Q.*

1.2.3 Barbalat's lemma

The following lemma will be used in this thesis to demonstrate the convergence of the tracking error to zero.

Lemma 1.1: *if* $f, \dot{f} \in L_\infty$, *and* $f \in L_p$, *for* $p \in [1, \infty[$, *then* $f \to 0$, *when* $t \to \infty$.

The result of Lemma 1.1 is a special case of a more general result given by the lemma of Barbalat indicated below.

Lemma 1.2 (Barbalat's lemma [KHA96]): *if* $\lim_{t\to\infty} \int_0^t f(\tau)d\tau$ *exists and is finite, and* $f(t)$ *is uniformly continuous function, then* $\lim_{t\to\infty} f(t) = 0$.

1.3 Matrix decomposition

Motivated by [BOU08a, COS03, ZHA04, CHE06, HSU07], one will use later a useful matrix decomposition lemma, in the control design for multivariable systems.

Lemma 1.3 [COS03, BOU08a]**:** *Any matrix* $G_p \in R^{p\times p}$ *having nonzero leading-principal minors can be decomposed as a product of three matrices*:

$$G_p = SDT \tag{1.2}$$

with $S \in R^{p\times p}$ *is a positive definite symmetric matrix,* $D \in R^{p\times p}$ *is a diagonal matrix, and* $T(x) \in R^{p\times p}$ *is a unity upper triangular matrix.*

Proof of Lemma 1.3 [STR80, COS03]**:** Since the principal minors of G_p are different from zero, there is a single factorization:

$$G_p = L_1 D_p L_2^T \tag{1.3}$$

where L_1 and L_2 are lower triangular matrices with ones on the diagonal, and

$$D_p = diag\left\{\Delta_1, \frac{\Delta_2}{\Delta_1}, \dots, \frac{\Delta_m}{\Delta_{m-1}}\right\} \tag{1.4}$$

where Δ_i are the principal minors of G_p. By decomposing the matrix D_p as follows:

$$D_p = D_+ D \tag{1.5}$$

where D_+ is a free diagonal matrix with positive inputs.

From equations (1.5) and (1.3), one can write $G_p = L_1 D_+ D L_2^T = L_1 D_+ L_1^T L_1^{-T} D L_2^T = (L_1 D_+ L_1^T) D \left(D^{-1} L_1^{-T} D L_2^T\right)$, so that (1.2) is satisfied by:

$$S = L_1 D_+ L_1^T, \;\; T = D^{-1} L_1^{-T} D L_2^T$$

This ends the proof of this lemma.

Illustrative example [BOU08a]**:** To illustrate the characteristics of each matrix in this decomposition, S, D and T, one can consider the following matrix:

$$G_p = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

The factorization (1.3), so-called LDU, gives

$$L_p = \begin{bmatrix} 1 & 0 \\ l_1 & 1 \end{bmatrix}, \qquad D_p = \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_2/\Delta_1 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 1 & 0 \\ l_2 & 1 \end{bmatrix},$$

where $l_1 = g_{21}/\Delta_1$ and $l_2 = g_{12}/\Delta_1$, and for :

$$D_+ = \begin{bmatrix} d_1^+ & 0 \\ 0 & d_2^+ \end{bmatrix},$$

Finally, the decomposition (1.2) gives:

$$D = D_+^- D_p,$$

$$S = \begin{bmatrix} d_1^+ & d_1^+ l_1 \\ d_1^+ l_1 & d_2^+ + d_1^+ l_1^2 \end{bmatrix},$$

$$T = \begin{bmatrix} 1 & l_2 - \dfrac{d_1^+ l_1 \Delta_2}{d_2^+ + \Delta_1^2} \\ 0 & 1 \end{bmatrix}.$$

Remark 1.2: *In [MOR93], the matrix D is chosen diagonal with +1 or -1 on its main diagonal. However, the decomposition $G_p = SDT$ (1.2) is not unique, because the positive definite diagonal matrix D_+ introduced in (1.5) is a free choice matrix.*

From Remark 1.2 and Lemma1.3, the following lemma can be directly obtained:

Lemma 1.4 [COS03, BOU08a]**:** *Any matrix* $G(x) \in R^{p\times p}$ *having nonzero leading-principle minors can be decomposed as a product of three matrices*:

$$G(x) = S(x)DT(x) \tag{1.6}$$

with $S(x) \in R^{p\times p}$ *is a positive definite symmetric matrix*, $D \in R^{p\times p}$ *is a diagonal constant matrix, and* $T(x) \in R^{p\times p}$ *is a unity upper triangular matrix.* ***The diagonal elements of the matrix D (which are +1 or -1) represent the ratios of the signs of the leading-principal minors of*** $G(x)$.

1.4 Adaptive control

The term "***adaptive control***" refers to the set of methods for the automatic real-time adjustment of controller parameters implemented in a control loop, in order to achieve or maintain a desired level of performance when the controlled process is not well known or has significant nonlinearity or time-varying parameters [BOD89, CHA87, BOH05].

Motivating example: Let's consider the following scalar system [LIB13]:

$$\dot{x} = \theta x + u \tag{1.7}$$

where x is the system state, u is the control input and θ is an unknown fixed parameter.

The main goal of the control is to determine a bounded function $u = f(x,t)$ such that the state $x(t)$ is bounded and asymptotically converges to zero (stabilization) whatever the initial condition x_0.

a) Case 1: If θ is already known (unlike the case of interest).

If $\theta < 0$, then $u \equiv 0$ already works well.

If $\theta > 0$ but known, then the following feedback control law

$$u = -(\theta + 1)x \tag{1.8}$$

gives the following dynamics:

$$\dot{x} = -x, \tag{1.9}$$

which means that x tends to zero. It is worth noting that instead of +1 in (1.8), one can use any other positive number.

But if the parameter θ is unknown (as in our case), the previous feedback law (1.8) cannot be implemented.

b) Case 1: If θ is unknown.

Now, let's suggest the following adaptive control law:

$$\dot{\hat{\theta}} = x^2 \tag{1.10}$$

$$u = -(\hat{\theta} + 1)x \tag{1.11}$$

where (1.10) is the tuning law, it "tunes" the feedback law (1.11).

The closed-loop system can be written as follows:

$$\dot{x} = (\theta - \hat{\theta} - 1)x \tag{1.12}$$

$$\dot{\hat{\theta}} = x^2 \tag{1.13}$$

Intuitively, the growth of $\hat{\theta}$ dominates the linear growth of *x*, and eventually the feedback gain $\hat{\theta} + 1$ becomes large enough to overcome the uncertainty and stabilizes the system.

To analyze the stability of the closed-loop system, let's choose the following Lyapunov function:

$$V = \frac{x^2}{2} \tag{1.14}$$

Taking its derivative along the closed-loop system gives:

$$\dot{V} = (\theta - \hat{\theta} - 1)x^2 \tag{1.15}$$

which is not guaranteed to be negative.

Besides, V should be a function of both states of the closed-loop system (1.12)-(1.13), i.e. x and $\hat{\theta}$.

Let's take

$$V(x,\hat{\theta}) = \frac{x^2}{2} + \frac{(\hat{\theta}-\theta)^2}{2} \tag{1.16}$$

By using (1.16), we get:

$$\dot{V} = (\theta - \hat{\theta} - 1)x^2 + (\hat{\theta} - \theta)x^2 = -x^2 \tag{1.17}$$

$\dot{V}$ in (1.17) is semi-negative definite. Then, one can conclude that $x,\ (\hat{\theta}-\theta) \in L_\infty$. The boundedness of the term $(\hat{\theta}-\theta)$ implies that of $\hat{\theta}$. From (1.17), one can obtain $\int_0^\infty x^2 dt = V(0) - V(\infty)$, that implies $x \in L_2$. Now, as x and $\dot{x} \in L_\infty$ and $x \in L_2$, and by using the Barbalt's Lemma 1.2, one can conclude that $\lim_{t\to\infty} x(t) = 0$. However the convergence of $x(t)$ to zero does not imply the convergence of $\hat{\theta}$ to θ.

1.5 High-gain observers

The majority of the control schemes designed for nonlinear systems necessitate the complete measurability of the system states. Hence, if some of the states are immeasurable; these results have no practical usage. Therefore, the use of observer-based control is very desirable.

In the design of output feedback control, high-gain observers have gained in popularity due to their ability to accurately estimate the unmeasured states of a system while rejecting disturbances. Recently, high-gain observers and their asymptotic properties have been studied by many authors [ATA99, ESF92, MAH02, ATA00, DAB01].

Motivating example: Consider the following nonlinear second order system [KHA13]

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= f(x,u,\omega,d)\\ y &= x_1\end{aligned} \tag{1.18}$$

where $x = [x_1, x_2]^T$ is the state vector, u and y are respectively the control input and the measured output, d is a disturbances' vector and ω is a vector of known exogenous signals. f is a locally Lipschitz function in (x,u) and continuous in (d,ω). $d(t)$ and $\omega(t)$ are assumed to be measurable and bounded time functions [KHA13].

It is supposed that the state feedback control signal $u = \gamma(x, \omega)$ stabilizes the origin $x = 0$ of the closed-loop system,

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x, \gamma(x, \omega), \omega, d) \end{aligned} \tag{1.19}$$

uniformly in (d, ω) where $\gamma(x, \omega)$ is locally Lipschitz in x and continuous in ω. To make this state-feedback control practically implementable by using only the measureable signals (i.e., the output y), we can design the following state observer:

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + h_1(y - \hat{x}_1) \\ \dot{\hat{x}}_2 &= \hat{f}(\hat{x}, u, \omega) + h_2(y - \hat{x}_1) \end{aligned} \tag{1.20}$$

where $\hat{f}(\hat{x}, u, \omega)$ is a model of $f(x, u, \omega, d)$, and then we take the control law as $u = \gamma(\hat{x}, \omega)$.

If f is a known function of $f(x, u, \omega)$, we can set $\hat{f} = f$. Otherwise, we can simply take $\hat{f} = 0$ (if no model of f is available). Let's define observation (estimation) errors as

$$\tilde{x} = \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} = \begin{bmatrix} x_1 - \hat{x}_1 \\ x_2 - \hat{x}_2 \end{bmatrix}, \tag{1.21}$$

whose dynamics are given by,

$$\begin{aligned} \dot{\tilde{x}}_1 &= -h_1\tilde{x}_1 - \tilde{x}_2 \\ \dot{\tilde{x}}_2 &= -h_2\tilde{x}_1 + \delta(x, \tilde{x}, \omega, d) \end{aligned} \tag{1.22}$$

where $\delta(x, \tilde{x}, \omega, d) = f(x, \gamma(\hat{x}, \omega), \omega, d) - \hat{f}(x, \gamma(\hat{x}, \omega), \omega)$. It is worth noting that, in the absence of δ, asymptotic error convergence is achieved when the matrix

$$\begin{bmatrix} -h_1 & 1 \\ -h_2 & 0 \end{bmatrix}$$

is Hurwitz (i.e., stable). This matrix is always stable, if the constants h_1 and h_2 are positive. On the other hand, in the presence of δ, we should design h_1 and h_2 with the additional aim of rejecting the effect of δ on $\tilde{x}$. This is perfectly achieved if the transfer matrix

$$G_0(s) = \frac{\frac{1}{\sqrt{h_2}}}{(\frac{s}{\sqrt{h_2}})^2 + \frac{h_1}{\sqrt{h_2}}\frac{s}{\sqrt{h_2}} + 1} \begin{bmatrix} \frac{1}{\sqrt{h_2}} \\ \frac{s}{\sqrt{h_2}} + \frac{h_1}{\sqrt{h_2}} \end{bmatrix} \tag{1.23}$$

from δ to $\tilde{x}$ is identically null. Although this is not possible, we can try only to make $sup_{w \in R}\|G_0(jw)\|$ arbitrary small. This objective is realized when the ratio $\frac{h_1}{\sqrt{h_2}}$ is chosen as some fixed positive real number, and we let h_2 go to infinity. That motivates us to choose h_1 *and* h_2 as:

$$h_1 = \frac{\alpha_1}{\varepsilon}, \qquad h_2 = \frac{\alpha_2}{\varepsilon^2} \tag{1.24}$$

for some positive constants α_1 and α_2, and with ε arbitrarily small. By this way, the observer eigenvalues are assigned at $1/\varepsilon$ times the roots of the polynomial $s^2 + \alpha_1 s + \alpha_2$. Therefore, by choosing ε small, we make the observer dynamics much faster than the dynamics of the closed-loop system under state feedback (1.19).

The disturbance rejection property of the high-gain observer, and its fast dynamics, can be also seen in the time domain using the following useful scaling

$$\eta_1 = \frac{\tilde{x}_1}{\varepsilon}, \qquad \eta_2 = \tilde{x}_2 \tag{1.25}$$

which satisfy the singularly perturbed equation

$$\begin{aligned} \varepsilon\dot{\eta}_1 &= -\alpha_1\eta_1 + \eta_2 \\ \varepsilon\dot{\eta}_2 &= -\alpha_1\eta_1 + \varepsilon\delta(x, \tilde{x}, \omega, d) \end{aligned} \tag{1.26}$$

This equation shows that reducing ε diminishes the effect of δ and makes the dynamics of η much faster than those of x. However, the scaling (1.25) shows that the transient response of the high-gain observer suffers from the so-called "***peaking phenomenon***". The initial condition $\eta_1(0)$ could be $O\left(\frac{1}{\varepsilon}\right)$ when $x_1(0) \neq \hat{x}_1(0)$. Consequently, the transient response of (1.26) could contain a term of the form $\frac{1}{\varepsilon}\ e^{-at/\varepsilon}$ for some $a > 0$. Although this exponential mode decays rapidly, it exhibits an impulsive-like behavior where the transient peaks to $O\left(\frac{1}{\varepsilon}\right)$ values before it decays rapidly towards zero. In fact, the function $\frac{1}{\varepsilon}\ e^{-at/\varepsilon}$ approaches an impulse function as ε tends to zero. In addition to inducing unacceptable transient response, the peaking phenomenon could destabilize the closed-loop nonlinear system [KHA13]. This phenomenon is an artifact of the high-gain observer. This is why in practice, we should add saturation functions to the observer state.

1.6 Fuzzy approximators

The main structure of a fuzzy logic system (FLS), as presented in Figure 1.1, consists in a fuzzifier, certain fuzzy IF-THEN rules, a fuzzy inference engine and a defuzzifier [BUH94, WAN94, JAN95, MEN95, LAB05a].

A mapping from an input vector $\underline{x}^T = [x_1, x_2, \dots, x_n] \in R^n$ to an output $\hat{f} \in R$ is performed by the fuzzy inference engine using a set of fuzzy If-Then. The *i*th fuzzy rule can be expressed as

$$R^{(i)}\text{: IF } x_1 \text{ is } A_1^i \text{ and} \dots \text{and } x_n \text{ is } A_n^i \;\; Then\ \hat{f} \text{ is } f^i,$$

where $A_1^i, A_2^i, \dots,$ and A_n^i are fuzzy sets and f^i is the fuzzy singleton for the i^{th} rule. The use of the singleton fuzzifier, product inference, and center-average defuzzifier allows to express simply the fuzzy system output as follows:

$$\hat{f}(\underline{x}) = \frac{\sum_{i=1}^{M} f^i (\prod_{j=1}^{n} \mu_{A_j^i}(x_j))}{\sum_{i=1}^{M} (\prod_{j=1}^{n} \mu_{A_j^i}(x_j))} = \theta^T \psi(x) \tag{1.27}$$

where $\mu_{A_j^i}(x_j)$ represent the degree of membership of x_j to A_j^i, *M* is the number of fuzzy rules, $\theta^T = [f^1, f^2, \dots, f^M]$ stands for the adaptable parameter vector (the consequent parameters), and $\psi^T = [\psi^1, \psi^2, \dots, \psi^M]$,

where

$$\psi^i(\underline{x}) = \frac{(\prod_{j=1}^{n} \mu_{A_j^i}(x_j))}{\sum_{i=1}^{M} (\prod_{j=1}^{n} \mu_{A_j^i}(x_j))} \tag{1.28}$$

is the *fuzzy basis function* (FBF).

It is worth noting that the FLS (1.27), which is broken down into the product of two main terms θ and (x) , allows us to show well the concept of fuzzy adaptive control and helps us in the control design and stability analysis. Via an update law usually determined based on Lyapunov stability theorem, all the elements of the vector θ are prone to be changed in order to ameliorate the fuzzy approximation, [WAN94, LIU17]. However, the FBF vector $\psi(x)$ should be adequately specified by the designer.

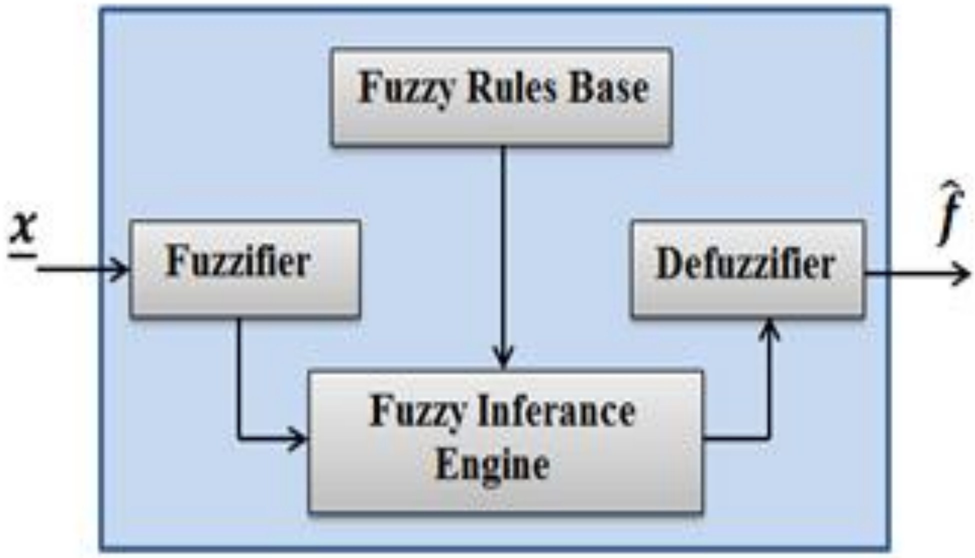

Figure 1.1 : *The basic configuration of a fuzzy logic system..*

1.6.1 Universal approximation theorem

The fuzzy control schemes designed in this thesis are principally based on the following useful theorem (the so-called *universal approximation theorem*).

Theorem 1.5: *let* $f(x)$ *be a continuous nonlinear function defined on a compact set* Ω_x*, and for any positive constant* ε*, there exists a fuzzy system* $y(x) = \theta^T \psi(x)$ *such that :*

$$\sup_{x \in \Omega_x} |f(x) - \theta^T \psi(x)| < \varepsilon \tag{1.29}$$

The proof of this theorem is given in [WAN94].

1.7 Conclusion

In this chapter, we have principally recalled some basic concepts of adaptive control, high-gain observers and fuzzy systems. Some useful stability definitions, lemmas and theorems, which are essential for the fuzzy adaptive control schemes' design, have been also given.

Chapter 2

Fuzzy Adaptive State-feedback Control Scheme of Uncertain Nonlinear Multivariable Systems

2.1 Introduction

Over the past two decades, many adaptive fuzzy control strategies have been developed for uncertain nonlinear multivariable systems [BOU10a, TON00, BOU08b, BOU10b, BOU12a], by exploiting the universal approximation feature of fuzzy systems [WAN94]. In these control strategies, the stability study associated with the closed-loop system is established using a Lyapunov's approach. To handle the fuzzy approximation (reconstruction) errors and eventual disturbances, the main controller (which is the fuzzy control term) is generally improved by adding a robust control term to the latter, namely a sliding mode control term and /or H_∞ control term [TON03, TON05, LAB05b, BOU08b, BOU12a]. In [BOU10b], this issue has been differently solved by adding to the integral update laws a proportional term derived from Lyapunov stability theory. In spite of this important extension and improvement, the control scheme in [BOU10b] presents unfortunately some drawbacks:

- ❖ In the adaptive laws, the proportional term has been designed as a non-derivable function, while it should be derivable.
- ❖ The presence of the well-known algebraic-loop problem in these updates laws.

Although the control schemes in [TON00, TON03, TON05, LAB05b, ESS06] can give, in general, good performances, they are unfortunately limited to MIMO systems which are characterized by an input-gains matrix being positive definite (PD) or negative definite (ND). It is worth noting that this assumption is paramount in the control design as well as in the stability study, thereby it limits the scope of applications of these fundamental results. How to extend these results to MIMO systems with an input-gains matrix which is not PD or ND is an important issue. Motivated by the early works in [MOR93, XIA04, CHE06], this problem has been solved in [BOU10b], by exploiting a so-called matrix SDT-factorization lemma given in section 1.3, where the matrix S can be incorporated in the Lyapunov's function candidate, as to the matrix T, it allows us to sequentially design the control signals by avoiding algebraic-loop problem.

Motivated by the aforementioned discussions, the purpose of this chapter is to design a new fuzzy adaptive state-feedback controller for multivariable nonlinear systems with uncertainties and eventual dynamical external disturbances. The matrix factorization lemma 1.4 is used when designing the control scheme. A new proportional-integral (PI) update law is proposed to estimate the unknown fuzzy parameters. It is theoretically demonstrated that our controller can ensure the tracking errors' convergence asymptotically to the origin. Compared to the available results [BOU10b], our contributions can be summarized as:

- By designing a low-pass filter, new PI update laws are introduced, in which the proportional part is designed as a derivable function and the famous algebraic-loop problem is avoided.
- A detailed comparison study is conducted to investigate the influence of each term in both the main control law and the adaptation law.

2.2 Notation and problem statement

Consider the uncertain multivariable nonlinear systems

$$y^{(n)} = F(x) + G(x)u + P(x,t), \tag{2.1}$$

where $x = [y^T\ \dot{y}^T \ldots \left(y^{(n-1)}\right)^T]^T \in R^r$ denotes the state vector with $r = m * n$, $u(t) \in R^m$ is the input, $y(t) \in R^m$ is the output. The functions $F(x) \in R^m$ and $G(x) = [g_{ij}] \in R^{m\times m}$ are smooth and uncertain, and $P(x,t) \in R^m$ is an unknown disturbance.

Our goal is to design a state-feedback control scheme, which ensures the boundedness of all variables in the closed-loop system and guarantees the output's tracking to the desired trajectory $y_d(t) \in R^m$, in spite of the presence of uncertainties in the model and eventual disturbances. The next assumptions will be helpful to design our control system.

Assumption 2.1: *The vector of the desired trajectories,* $x_d = [{y_d}^T\ {\dot{y}_d}^T \ldots \left(y_d^{(n-1)}\right)^T]^T \in R^{m*n}$, *is continuous and bounded .*

Assumption 2.2:

a) *The signs of the leading principal minors of input-gains matrix $G(x)$, are known.*
b) $\partial g_{ij}(x)/\partial y_j^{(n-1)} = 0, \forall\ i = 1, \ldots, m$ *and* $j = 1, \ldots, m$.

Assumption 2.3: *The disturbance vector, $P(x,t) \in R^m$, is bounded and uncertain.*

Remark 2.1: *Assumptions 2.1 and 2.3 are not restrictive. They are standard assumptions for adaptive control theory [BOU10a, LAB05b, WAN94].*

Remark 2.2: *Assumption 2.2 can be considered as a condition of controllability and is not restrictive as it is fulfilled by numerous practical systems: e.g., robotic systems, chaotic systems, mechanical systems, electrical machines, and so on.*

By using Lemma 1.4, we can rewrite the system (2.1) as:

$$S_1(x)y^{(n)} = H(x) + DT(x)u \tag{2.2}$$

where $S_1(x) = S^{-1}(x)$ is PD and symmetric, $H(x) = S^{-1}(x)(F(x) + P(x,t)) \in R^m$.

Let's define the tracking error of the system's output, $e_1(t) = [e_{11}, e_{12}, \dots, e_{1m}]^T \in R^m$, as

$$e_1 = y_d - y \tag{2.3}$$

In addition, we introduce the following error variables $e_i \in R^m, i = 2, \dots, n$, [CHE06]:

$$\begin{aligned} e_2 &= \dot{e}_1 + e_1, \\ e_3 &= \dot{e}_2 + e_2 + e_1, \\ &\vdots \\ e_n &= \dot{e}_{n-1} + e_{n-1} + e_{n-2}. \end{aligned} \tag{2.4}$$

From (2.4), the errors, $e_i, i = 2, \dots, n$, can be simply expressed as :

$$e_i = \sum_{j=0}^{i-1} a_{ij} e_1^{(j)} \quad \forall\, i = 2,3, \dots, n. \tag{2.5}$$

where the constants a_{ij} can be determined via the well-known Fibonacci number series [XIA04].

Define the novel error signal E as:

$$E = e_n + e_{n-1} \tag{2.6}$$

From (2.5) and (2.6), the time-derivative of E can be expressed as:

$$\dot{E} = \dot{e}_n + \dot{e}_{n-1} = \sum_{j=0}^{n-1} a_{nj} e_1^{(j+1)} + \dot{e}_{n-1} = \sum_{j=0}^{n-2} a_{nj} e_1^{(j+1)} + e_1^{(n)} + \dot{e}_{n-1}$$

$$= \sum_{j=0}^{n-2} a_{nj} e_1^{(j+1)} + y_d^{(n)} - y^{(n)} + \dot{e}_{n-1} \qquad (2.7)$$

Multiplying (2.7) by S_1 and using (2.3), we get:

$$S_1 \dot{E} = S_1 \left(\sum_{j=0}^{n-2} a_{nj} e_1^{(j+1)} + y_d^{(n)} + \dot{e}_{n-1} \right) - S_1 y^{(n)}$$

$$= S_1 \left(\sum_{j=0}^{n-2} a_{nj} e_1^{(j+1)} + y_d^{(n)} + \dot{e}_{n-1} \right) - H(x) - DT(x)u$$

$$= S_1 \left(\sum_{j=0}^{n-2} a_{nj} e_1^{(j+1)} + y_d^{(n)} + \dot{e}_{n-1} \right) - H(x) - Du - D(T(x) - I_m)u \qquad (2.8)$$

We can also write (2.8) as follows:

$$D^{-1} S_1 D \, D^{-1} \dot{E} = D^{-1} S_1 \left(\sum_{j=0}^{n-2} a_{nj} e_1^{(j+1)} + y_d^{(n)} + \dot{e}_{n-1} \right) - D^{-1} H(x) - (T(x) - I_m)u - u$$

Or equivalently as

$$\overline{S}_1 \dot{\overline{E}} = D^{-1} S_1 \left(\sum_{j=0}^{n-2} a_{nj} e_1^{(j+1)} + y_d^{(n)} + \dot{e}_{n-1} \right) - D^{-1} H(x) - (T(x) - I_m)u - u \qquad (2.9)$$

where $\overline{S}_1 = D^{-1} S_1 D$ and $\overline{E} = \left[\overline{E}_1, \dots, \overline{E}_m\right]^T = D^{-1} E$.

Finally, by posing

$$\alpha(z) = [\alpha_1(z_1), \dots, \alpha_m(z_m)]^T = D^{-1} S_1 \left(y_d^{(n)} + \sum_{j=0}^{n-2} a_{nj} e_1^{(j+1)} + \dot{e}_{n-1} \right) - D^{-1} H(x)$$

$$+ \frac{1}{2} \dot{\overline{S}}_1 \overline{E} - (T(x) - I_m)u + De_{n-1} \qquad (2.10)$$

Equation (2.9) becomes

$$\overline{S}_1 \dot{\overline{E}} + \frac{1}{2} \dot{\overline{S}}_1 \overline{E} = \alpha(z) - u - De_{n-1} \qquad (2.11)$$

where $z = [z_1^T, z_2^T, \dots, z_m^T]^T$. Due to the special structure of $\alpha_i(z_i)$, we can select the argument vector z_i as follows [BOU08b, BOU10b]

$$
\begin{aligned}
z_1 &= [x^T, u_2, \ldots, u_m]^T, \\
z_2 &= [x^T, u_3, \ldots, u_m]^T, \\
&\vdots \\
z_{m-1} &= [x^T, u_m]^T, \\
z_m &= x^T,
\end{aligned} \tag{2.12}
$$

Note that this helpful triangular structure of z_i permits us to sequentially design control inputs $u_i, \forall\, i = 1,2, \ldots, m,$ without any algebraic loop problem.
Now, let's denote the following compact sets,

$$
\Omega_{z_i} = \{[x^T, u_{i+1}, \ldots, u_m]^T \;\; | x \in \Omega_x \subset R^r\}, \forall\, i = 1, \ldots, m-1,
$$
$$
\Omega_{z_m} = \{x \;\; | x \in \Omega_x \subset R^m\}.
$$

In what follows, adaptive fuzzy-logic systems will be employed to model the uncertain function $\alpha_i(z_i)$.

2.3 Adaptive fuzzy controller disign

The uncertain function $\alpha_i(z_i)$ may be expressed in terms of the fuzzy system (1.27) as follows:

$$
\hat{\alpha}_i(z_i, \theta_i) = \theta_i^T \psi_i(z_i), \quad i = 1, \ldots, m \tag{2.13}
$$

where $\psi_i(z_i)$ is the FBF vector. The latter should be properly determined by the designer. And θ_i is the vector of the updated online parameters.
Now, denote the parametric estimation error as follows:

$$
\tilde{\theta}_i = \theta_i - \theta_i^* \text{ with } i = 1, \ldots, m, \tag{2.14}
$$

and the fuzzy reconstruction error as [WAN94, LIU16, XU17, HSU12, FU11, LIU 15]:

$$
\varepsilon_i(z_i) = \alpha_i(z_i) - \hat{\alpha}_i(z_i, \theta_i^*) \tag{2.15}
$$

with $\hat{\alpha}_i(z_i, \theta_i^*) = \theta_i^* \psi_i(z_i)$.

According to the universal approximation theorem [WAN94], we have:

$$|\varepsilon_i(z_i)| \leq \bar{\varepsilon}_i, \qquad \forall\, z_i \in \Omega_{z_i},$$

where $\bar{\varepsilon}_i$ is an uncertain constant. Also, let's denote

$$\alpha(z) = \theta^T\psi(z) = [\alpha_1(z_1,\theta_1),\dots,\alpha_m(z_m,\theta_m)]^T$$
$$\varepsilon(z) = [\varepsilon_1(z_1),\dots,\varepsilon_m(z_m)]^T,\ \bar{\varepsilon} = [\bar{\varepsilon}_1,\dots,\bar{\varepsilon}_m]^T.$$

From the above considerations, we can get

$$\begin{aligned}\hat{\alpha}(z_i,\theta) - \alpha(z_i) &= \hat{\alpha}(z_i,\theta_i) - \hat{\alpha}(z_i,\theta_i^*) + \hat{\alpha}(z_i,\theta_i^*) - \alpha(z_i)\\ &= \hat{\alpha}(z_i,\theta_i) - \hat{\alpha}(z_i,\theta_i^*) - \varepsilon_i(z_i)\\ &= \tilde{\theta}^T\psi(z_i) - \varepsilon_i(z_i) \qquad (2.16)\end{aligned}$$

with $\tilde{\theta}^T\psi(z) = \left[\tilde{\theta}_1^T\psi_1(z_1),\dots,\tilde{\theta}_m^T\psi_m(z_m)\right]^T$, and $\tilde{\theta}_i = \theta_i - \theta_i^*$ where $i = 1,\dots,m$.

Using (2.14), (2.15) and (2.16), (2.11) can be expressed as:

$$\begin{aligned}\bar{E}^T\bar{S}_1\dot{\bar{E}} + \frac{1}{2}\bar{E}^T\dot{\bar{S}}_1\bar{E} &= -\sum_{i=1}^{m}\bar{E}_i\,\tilde{\theta}_i^T\psi_i(z_i) + \sum_{i=1}^{m}\bar{E}_i\,\varepsilon_i(z_i)\\ &\quad + \sum_{i=1}^{m}\bar{E}_i\,\theta_i^T\psi_i(z_i) - \bar{E}^T u - \bar{E}^T De_{n-1} \qquad (2.17)\end{aligned}$$

The control law *u* can be determined based on (2.13) as follows:

$$u = \theta^T\psi(z) + K_r sign(\bar{E}) + K_p\bar{E} + K_i\int_o^t \bar{E}\,d\tau \qquad (2.18)$$

with $K_i = diag\{k_{i1}\}$, $K_p = diag\{k_{pi}\}$ and $K_r = diag\{k_{ri}\}$. K_i, K_p, and $K_r \in R^{m\times m}$ are positive definite design matrices.

Remark 2.3: *Note that the control input (2.18) is essentially comprised of four terms, namely:*

- ***a fuzzy term,*** $\theta_i^T\psi_i(z_i)$, *employed to model the uncertain functions* $\alpha_i(z_i)$,

- ***a robust control term,*** $K_r sign(\bar{E})$, *designed to deal with the uncertain term* $\frac{\sigma_i}{2}\|\theta^*\|^2 + \bar{\varepsilon}_i$.
- ***a linear control term***, $K_p\bar{E}$, *introduced for stability purposes, and finally*
- ***an integral control term,*** $K_i\int_o^t \bar{E}d\tau$, *added to compensate for the steady-state error.*

The PI update laws associated to (2.18) are given by

$$\dot{\theta}_i + \gamma_{2i}\dot{\delta}_{fi} = -\sigma_i\gamma_{1i}|\bar{E}_i|\theta_i + \gamma_{1i}\bar{E}_i\psi_i(z_i), \quad (2.19)$$

$$\dot{\delta}_{fi} = -\gamma_{2i}\delta_{fi} + \gamma_{2i}\,\delta_i, \quad (2.20)$$

where $\delta_i = -(-\sigma_i|\bar{E}_i|\theta_i + \bar{E}_i\psi_i(z_i))$, with $\gamma_{1i}, \gamma_{2i}, \sigma_i > 0$ are design parameters.

Remark 2.4: *The adaptation law mechanism (2.19) can be also written as follows:*

$$\theta = \theta_I + \theta_P = \int_0^t \left(-\sigma_i\gamma_{1i}|\bar{E}_i|\theta_i + \gamma_{1i}\bar{E}_i\psi_i(z_i)\right)d\tau - \gamma_{2i}\delta_{fi}$$

where $\theta_I = \int_0^t \left(-\sigma_i\gamma_{1i}|\bar{E}_i|\theta_i + \gamma_{1i}\bar{E}_i\psi_i(z_i)\right)d\tau$ *is an adaptation integral term, and* $\theta_P = -\gamma_{2i}\delta_{fi}$ *is an adaptation proportional term. Note that the added proportional term* $(-\gamma_{2i}\delta_{fi})$ *allows a rapid convergence to adaptive fuzzy parameters and tracking errors. In fact, the latter introduces, in the time-derivative of the Lyapunov function, a significant negative term (see (2.23)).*

Remark 2.5:

- *The adaptation law mechanism (2.19) is also equipped by a useful robust term, namely: an* ***e-modification term***, $\sigma_i\gamma_{1i}|\bar{E}_i|\theta_i$, *which guarantees the uniform boundedness of the adaptive parameters.*
- *Contrary to [BOU10b], because the proportional term (-*$\gamma_{2i}\delta_{fi}$*) is obtained via the low-pass filter (2.20), the algebraic-loop problem and the time-differentiability requirement of the proportional term are indeed solved in the modified adaptive law (2.19). Recall that the PI adaptive law in [BOU10b] uses only the term* $\delta_i = -(-\sigma_i|\bar{E}_i|\theta_i + \bar{E}_i\psi_i(z_i))$ *as a proportional term.*

By considering (2.18), the dynamics (2.17) become

$$\bar{E}^T \bar{S}_1 \dot{\bar{E}} + \frac{1}{2}\bar{E}^T \dot{\bar{S}}_1 \bar{E} = -\sum_{i=1}^{m} \bar{E}_i\, \tilde{\theta}_i^T \psi_i(z_i) - \bar{E}^T K_i \left[\int_o^t \bar{E} dt\right] - \bar{E}^T K_p \bar{E}$$
$$- \sum_{i=1}^{m} k_{ri} |\bar{E}_i| + \sum_{i=1}^{m} \bar{E}_i\, \varepsilon_i(z_i) - \bar{E}^T D e_{n-1} \qquad (2.21)$$

Theorem 2.1 : *Let's consider the system (2.1) subject to Assumptions 2.1-2.3, and with the control law (2.18) and its update law given by (2.19) and (2.20), we can ensures the following properties:*

- *The boundedness of all signals involved in the closed-loop control system.*
- *The underlying tracking errors, e_1 , asymptotically vanish at the origin.*

Proof of Theorem 2.1 : Consider the following candidate quadratic Lyapunov function:

$$V = \frac{1}{2}\sum_{i=1}^{n-1} e_i^T e_i + \frac{1}{2}\bar{E}^T S_1 \bar{E} + \frac{1}{2\gamma_{1i}}\sum_{i=1}^{m} \left\|\tilde{\theta}_i + \gamma_{2i}\delta_{fi}\right\|^2$$
$$+ \sum_{i=1}^{m} \left\|\delta_{fi}\right\|^2 + \frac{1}{2}\left[\int_o^t \bar{E} d\tau\right]^T K_i \left[\int_o^t \bar{E} d\tau\right] \qquad (2.22)$$

By considering (2.18)-(2.22), the derivative of V is

Table 2.1: *Comparison between the control scheme of this chapter and that of [BOU10b].*

Comparison	*[BOU10a]*	*This chapter*
Control law	- It is composed of three terms: a fuzzy term, a linear term and a robust term.	- It is comprised ***of four terms***: a fuzzy term, a linear term, ***an integral term*** and a robust term. -This integral term is added to compensate for the steady-state error and to enhance then the tracking performances.
Update law	- It is of type PI -The proportional (P) part has been designed as a non-derivable function, while it should be derivable. -It suffers from the well-known algebraic loop problem.	- It is of type PI, augmented by a linear filter. ***-The proportional (P) part has been designed as a derivable function.*** ***- The well-known algebraic loop problem has been resolved by using this linear filter.***

$$\dot{V} \leq -\sum_{i=1}^{n-1} e_i^T e_i \ -\bar{E}^T K_p \bar{E} - \sum_{i=1}^{m} \gamma_{2i} \|\delta_{fi}\|^2 \ -\sum_{i=1}^{m} k_{ri} |\bar{E}_i|$$

$$-\sum_{i=1}^{m} \sigma_i |\bar{E}_i| \tilde{\theta}_i^T \theta_i + \sum_{i=1}^{m} |\bar{E}_i| \, \bar{\varepsilon}_i \qquad (2.23)$$

Using the inequality

$$-\sum_{i=1}^{m} \sigma_i |\bar{E}_i| \tilde{\theta}_i^T \theta_i \leq -\sum_{i=1}^{m} \frac{\sigma_i}{2} |\bar{E}_i| \|\tilde{\theta}_i \ \|^2 \ + \sum_{i=1}^{m} \frac{\sigma_i}{2} |\bar{E}_i| \|\theta_i^*\|^2 \, , \qquad (2.24)$$

and if we select $k_{ri}^* \geq \frac{\sigma_i}{2} \|\theta_i^*\|^2 + \bar{\varepsilon}_i$, (2.23) becomes

$$\dot{V} \leq -\sum_{i=1}^{n-1} e_i^T e_i - \bar{E}^T K_p \bar{E} \qquad (2.25)$$

Then, from (2.25), we can easily conclude about the boundedness of all signals in the closed-loop control system, including the control input u. By the Barbalat's lemma [KHA01], we can establish that the errors $e_1, \bar{E}_j$ (for $j = 1, \dots, m$, and e_i (for $i = 2, \dots, n$), can vanish asymptotically at the origin.

Remark 2.6: *To highlight the theoretical contributions of this chapter, a comparison between this work and that of Boulkroune et al. in [BOU10b] is given in Table 2.1.*

2.4 Simulation results and Comparative study

In this section, we will give some simulation examples to show the efficiency of the above theoretical results and will conduct a detailed comparison to investigate the influence of each term in both main control law and adaptation law.

2.4.1 Simulation examples

2.4.1.1 Example 1: Consider a 2 DOF polar manipulator robot (see Figure 2.1). Its mathematical model is given by [VIT16]:

$$\begin{bmatrix} 1+q_2^2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} q_2\dot{q}_2 & 0 \\ -\dot{q}_1 q_2 & 1 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} (1+q_2)\cos(q_1) \\ \sin(q_1) \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad (2.26)$$

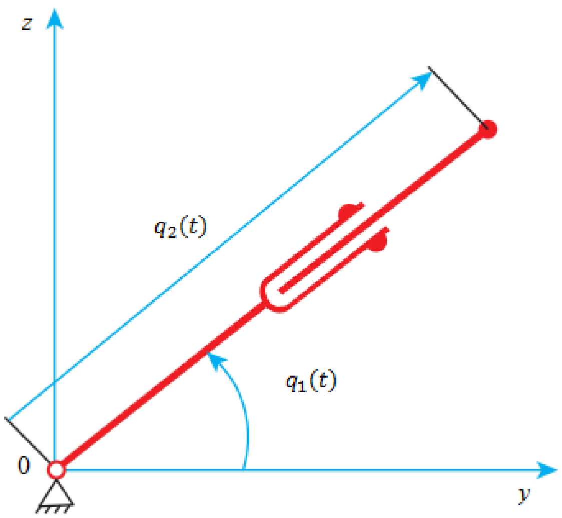

Figure 2.1: *A two DOF polar manipulator robot.*

The system (2.26) can be transformed to (2.1), as follows:

$$\ddot{y} = F_1(x) + G_1(x)u + P_1(x,t), \tag{2.27}$$

with

$$F_1(x) = -\begin{bmatrix} 1+q_2^2 & 0 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} q_2\dot{q}_2 & 0 \\ -\dot{q}_1 q_2 & 1 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} - \begin{bmatrix} 1+q_2^2 & 0 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} (1+q_2)\cos(q_1) \\ \sin(q_1) \end{bmatrix},$$

$$G_1(x) = \begin{bmatrix} 1+q_2^2 & 0 \\ 0 & 1 \end{bmatrix}^{-1},$$

where $y = [x_1, x_2]^T = [q_1, q_2]^T$, $x = [x_1, x_2, x_3, x_4]^T = [q_1, q_2, \dot{q}_1, \dot{q}_2]^T$, $u = [u_1, u_2]^T$ is the system input, and $P_1(x,t) = [p_{11}(x,t)\,,\ p_{12}(x,t)]^T$ is an added disturbance.

It is assumed that the model (2.26) includes the dynamics of both drives and the desired trajectories are given by: $q_{d1} = 0.5\sin(2t) + 0.5$ and $q_{d2} = 0.2\sin(4t) + 1$, [VIT16].

For the proposed controller (2.18), two adaptive fuzzy systems, namely ${\theta_1}^T\psi_1(z_1)$ and ${\theta_2}^T\psi_2(z_2)$, have been constructed, where $z_1 = [x^T, u_2]^T$ and $z_2 = x$. we define for each input variable, as in [BOU08b], three membership functions (one triangular and two trapezoidal) being uniformly distributed on the following selected intervals: [-2, 2] for $\dot{q}_2$ *and* $\dot{q}_1$; [0.5, 2] for q_2; [-0.5, 1.5] for q_1; and [-10, 10] for u_2.

The control parameters are chosen as follows: $\gamma_{11} = \gamma_{12} = 100$, $\gamma_{21} = \gamma_{22} = 35$, $\sigma_1 = \sigma_2 = 0.1$, $\lambda_1 = \lambda_2 = 2$, $k_{r1} = k_{r2} = 0.2$, $k_{i1} = k_{i2} = 0.2$, $k_{p1} = k_{p2} = 0.2$. The initial conditions are taken as : $x(0) = [0,\ 0.5,\ 0,\ 0]^T$.

The external disturbances $p_{11}(x,t)$ and $p_{12}(x,t)$ are square signals with an amplitude of ∓ 1 and a frequency of $1/2\pi\ Hz$.

The simulation results obtained are presented in Figure 2.2. It can be seen from this figure that the system effectively tracks the desired reference signals and also the control signals are bounded and admissible.

2.4.1.2 Example 2: Consider the dynamical equations of a two-link rigid manipulator robot moving on a horizontal plane shown in Figure 2.3, this MIMO system is given by [BOU10b]

$$\begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}^{-1} \left\{ \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} - \begin{pmatrix} -h\dot{q}_2 & -h(\dot{q}_1 + \dot{q}_2) \\ h\dot{q}_1 & 0 \end{pmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} \right\} \tag{2.28}$$

where $M_{11} = a_1 + 2a_3\cos(q_2) + 2a_4\sin(q_2)$, $M_{22} = a_2$, $M_{21} = M_{12} = a_2 + a_3\cos(q_2) + a_4\sin(q_2)$, and $h = a_3\sin(q_2) - a_4\cos(q_2)$, with $a_1 = I_1 + m_1 l_{c1}^2 + I_e + m_e l_{ce}^2 + m_e l_1^2$, $a_2 = I_e + m_e l_{ce}^2$, $a_3 = m_e l_1 l_{ce}\cos(\delta_e)$, and $a_4 = m_e l_1 l_{ce}\sin(\delta_e)$,

The robot's parameter values used are: $m_1 = 1$, $m_e = 2$, $l_1 = 1$, $l_{c1} = 0.5$, $l_{ce} = 0.6$, $I_1 = 0.12$, $I_e = 0.25$, $\delta_e = 30$. Let $y = [q_1, q_2]^T$, $u = [u_1, u_2]^T$, $x = [q_1, q_2, \dot{q}_1, \dot{q}_2]^T$.

System (2.28) can be rewritten in the form of system (2.1) as follows:

$$\ddot{y} = F_2(x) + G_2(x)u + P_2(x,t) \tag{2.29}$$

where

$$F_2(x) = \begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} = -M^{-1} \begin{pmatrix} -h\dot{q}_2 & -h(\dot{q}_1 + \dot{q}_2) \\ h\dot{q}_1 & 0 \end{pmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix},$$

$$G_2(x) = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} = M^{-1} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}, \text{ and } P_2(x,t) = [p_{21}(x,t)\ p_{22}(x,t)]^T.$$

The desired trajectories are $y_{d1} = q_{d1} = \sin(t)$ and $y_{d2} = q_{d2} = \sin(t)$.

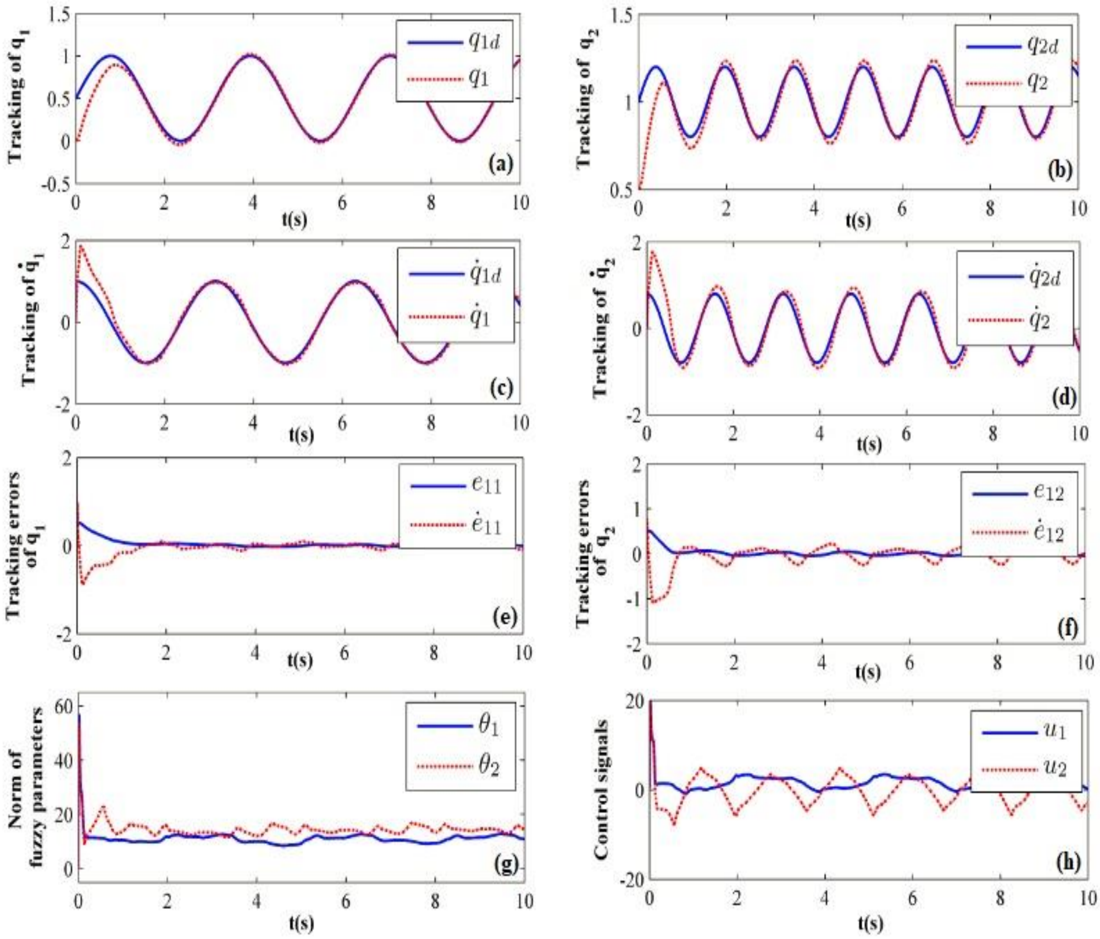

Figure 2.2: *Simulation results for a 2 DOF polar manipulator robot (example 1) : (a) Tracking of* q_1*, (b) Tracking of* q_2*, (c) Tracking of* $\dot{q}_1$*, (d) Tracking of* $\dot{q}_2$*, (e) Tracking errors* e_{11} *and* $\dot{e}_{11}$*, (f) Tracking errors* e_{12} *and* $\dot{e}_{12}$*, (g) Norm of fuzzy parameters:* $\|\theta_1\|$ *and* $\|\theta_2\|$*, (h) Control signals:* u_1 *and* u_2*.*

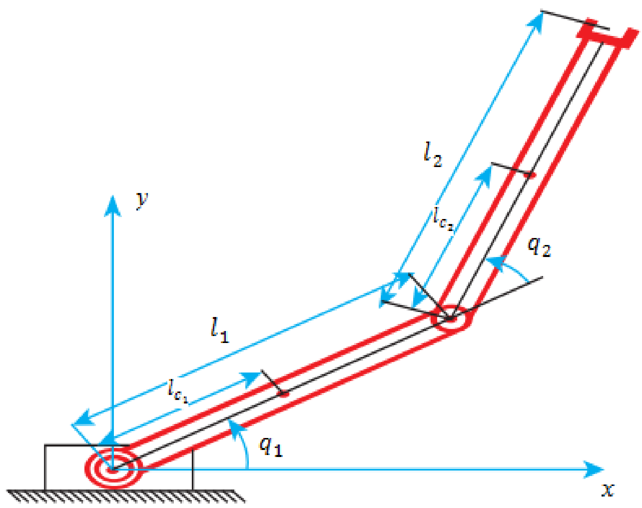

Figure 2.3: *A two links manipulator robot.*

For the proposed controller (2.18), two adaptive fuzzy systems, namely ${\theta_1}^T\psi_1(z_1)$ and ${\theta_2}^T\psi_2(z_2)$, have been constructed, where $z_1 = [x^T, u_2]^T$ and $z_2 = x$. we define for each input variable, as in [BOU08b], three membership functions (one triangular and two trapezoidal) being uniformly distributed on the following selected intervals: [-2, 2] for q_1 *and* $\dot{q}_1$; [-2, 2] for $q_2, \dot{q}_2$; and [-10, 10] for u_2.

The controller parameters are selected as follows: $\gamma_{11} = \gamma_{12} = 100$, $\gamma_{21} = \gamma_{22} = 100$, $\sigma_1 = \sigma_2 = 0.005$, $\lambda_1 = \lambda_2 = 2$, $k_{p1} = k_{p2} = 0.2$, $k_{i1} = k_{i2} = 2$, $k_{r1} = k_{r2} = 0.1$. The initial conditions are given as $x(0) = [1, 1,0,\ 0]^T$. The external disturbances $p_{21}(x,t)$ and $p_{22}(x,t)$ are square signals with an amplitude of ∓ 1 and a frequency of $1/2\pi\ Hz$.

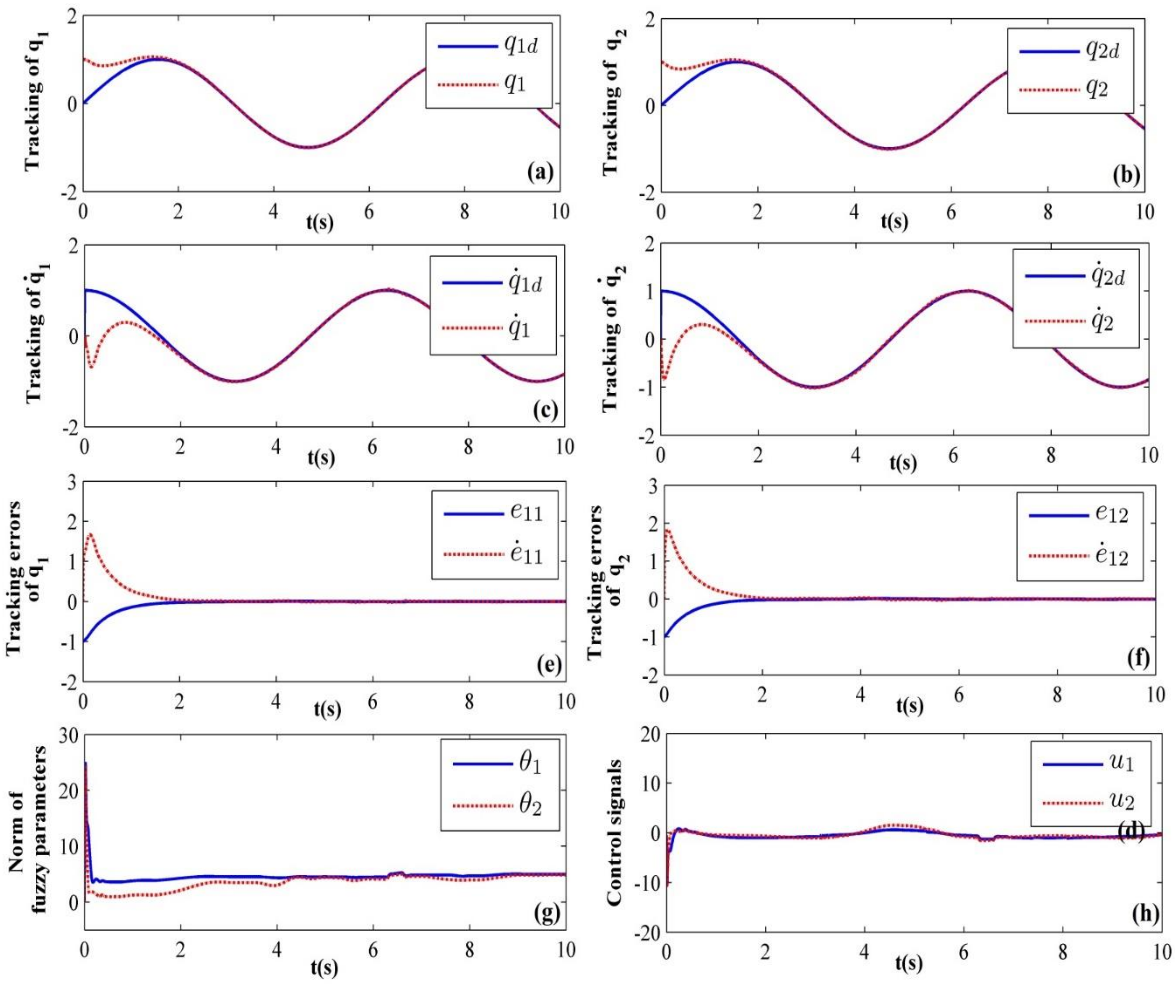

Figure 2.4: *Simulation results for a 2 links manipulator robot (example 2): (a) Tracking of* q_1, *(b) Tracking of* q_2, *(c) Tracking of* $\dot{q}_1$, *(d) Tracking of* $\dot{q}_2$, *(e) Tracking errors* e_{11} *and* $\dot{e}_{11}$ *, (f) Tracking errors* e_{12} *and* $\dot{e}_{12}$, *(g) Norm of fuzzy parameters:* $\|\theta_1\|$ *and* $\|\theta_2\|$, *(h) Control signals:* u_1 *and* u_2.

Simulation results of this example are displayed in Figure 2.4. Despite the presence of disturbances and uncertainties, it is clear from this figure, that this MIMO system successfully follows the desired trajectories. The control signals associated are also bounded and admissible.

2.4.1.3 Example 3: Let's consider in this example a 2 DOF helicopter (CE150) shown in Figure 2.5. The dynamics of this MIMO nonlinear system are given by [HUM89, MER16]:

$$M(\phi,\vartheta)\begin{bmatrix}\ddot{\phi}\\ \ddot{\vartheta}\end{bmatrix}+C(\phi,\vartheta,\dot{\phi},\dot{\vartheta})\begin{bmatrix}\dot{\phi}\\ \dot{\vartheta}\end{bmatrix}+G(\phi,\vartheta)=u \tag{2.30}$$

with $M(\phi,\vartheta)=\begin{bmatrix}cos(\phi I_{l2})^2 & 0\\ 0 & I_{l2}\end{bmatrix}$, $G(\phi,\vartheta)=\begin{bmatrix}0\\ mgI_c cos\vartheta\end{bmatrix}$,

$$C(\phi,\vartheta,\dot{\phi},\dot{\vartheta})=\begin{bmatrix}-cos\vartheta sin\vartheta\dot{\vartheta}I_{l2} & -cos\vartheta sin\vartheta\dot{\phi}I_{l2}\\ cos\vartheta sin\vartheta\dot{\phi}I_{l2} & 0\end{bmatrix}, \text{ and } u=\begin{bmatrix}u_1\\ u_2\end{bmatrix}.$$

where $I_{l2}=\frac{m_l}{3}\frac{(l_1^3+l_2^3)}{(l_1+l_2)}+m_1l_1^2+m_2l_2^2$ and $l_c=(m_l(l_1-l_2)+m_1l_1-m_2l_2)/m$

The helicopter given by (2.30) can be expressed in the following form:

$$\ddot{y}=F_3(x)+G_3(x)u+P_3(x,t), \tag{2.31}$$

where

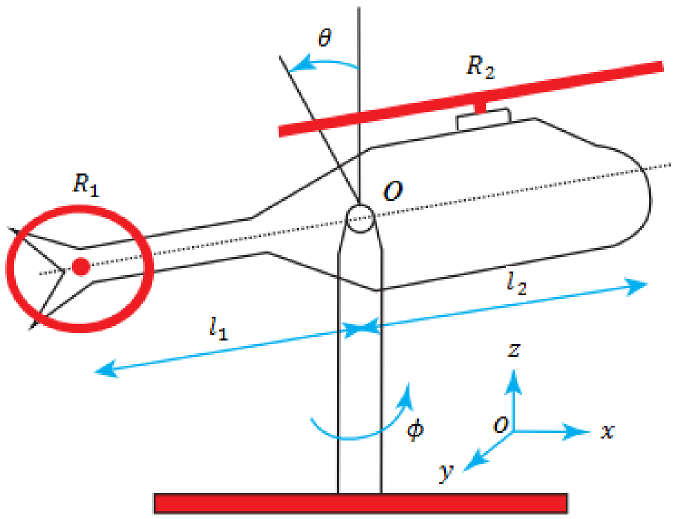

Figure 2.5: *A two DOF helicopter (CE150).*

$$F_3(x) = -M^{-1}\begin{bmatrix} -\cos\vartheta\ sin\vartheta\dot{\vartheta}I_{l2} & -\cos\vartheta\ sin\vartheta\dot{\phi}I_{l2} \\ \cos\vartheta\ sin\vartheta\dot{\phi}I_{l2} & 0 \end{bmatrix}\begin{bmatrix} \dot{\phi} \\ \dot{\vartheta} \end{bmatrix} - M^{-1}G,$$

$G_3(x) = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} = M^{-1}$, and $P_3(x,t) = [p_{31}(x,t)\ ,\ p_{32}(x,t)]^T$.

The desired trajectories are $y_{d1} = \phi_d = \sin(t)$ and $y_{d2} = \vartheta_d = \sin(t)$.

For the proposed controller (2.18), two adaptive fuzzy systems, namely $\theta_1{}^T\psi_1(z_1)$ and $\theta_2{}^T\psi_2(z_2)$, have been constructed, where $z_1 = [x^T, u_2]^T$ and $z_2 = x$. we define for each input variable, as in [BOU08b], three membership functions (one triangular and two trapezoidal) being uniformly distributed on the following selected intervals: [-2,2] for $\phi\ and\ \dot{\phi}$; [-2,2] for $\vartheta, \dot{\vartheta}$; and [-10,10] for u_2.

The helicopter's parameters are: $m_1 = 0.42\ (kg)$, $m_2 = 0.16\ (kg)$, $m_l = 0.35\ (kg)$, $m = m_l + m_1 + m_2$, $l_1 = 0.198\ (m)$, $l_2 = 0.174\ (m)$, $g = 9.8$.

The controller parameters used are selected as follows: $\gamma_{11} = \gamma_{12} = 100, \gamma_{21} = \gamma_{22} = 100$, $\sigma_1 = \sigma_2 = 0.005$, $\lambda_1 = \lambda_2 = 2$, $k_{p1} = k_{p2} = 0.2, k_{i1} = k_{i2} = 2, k_{r1} = k_{r2} = 0.1$. The initial conditions are chosen as: $[0.5, -0.5, 0,0]^T$. The external disturbances, $p_{31}(x,t)$ and $p_{32}(x,t)$ are square signals with an amplitude of ∓ 1 and a frequency of $1/2\pi\ Hz$.

In this third example, the simulation results are presented in Figure 2.6. These results show good tracking as well as the efficiency of our proposed control strategy.

2.4.2 Comparative studies

a) Comparative study 1:

In this sub-section, we will conduct a series of tests to evaluate the performances and the contribution of each term in the main control law and in the adaptive law by applying the developed control law in different situations on our robotic system (2.26).

Firstly, we evaluate the effect of each term in the main control law (2.18). The latter can be expressed as:

$$u = u_{PI} + u_r + u_f \tag{2.32}$$

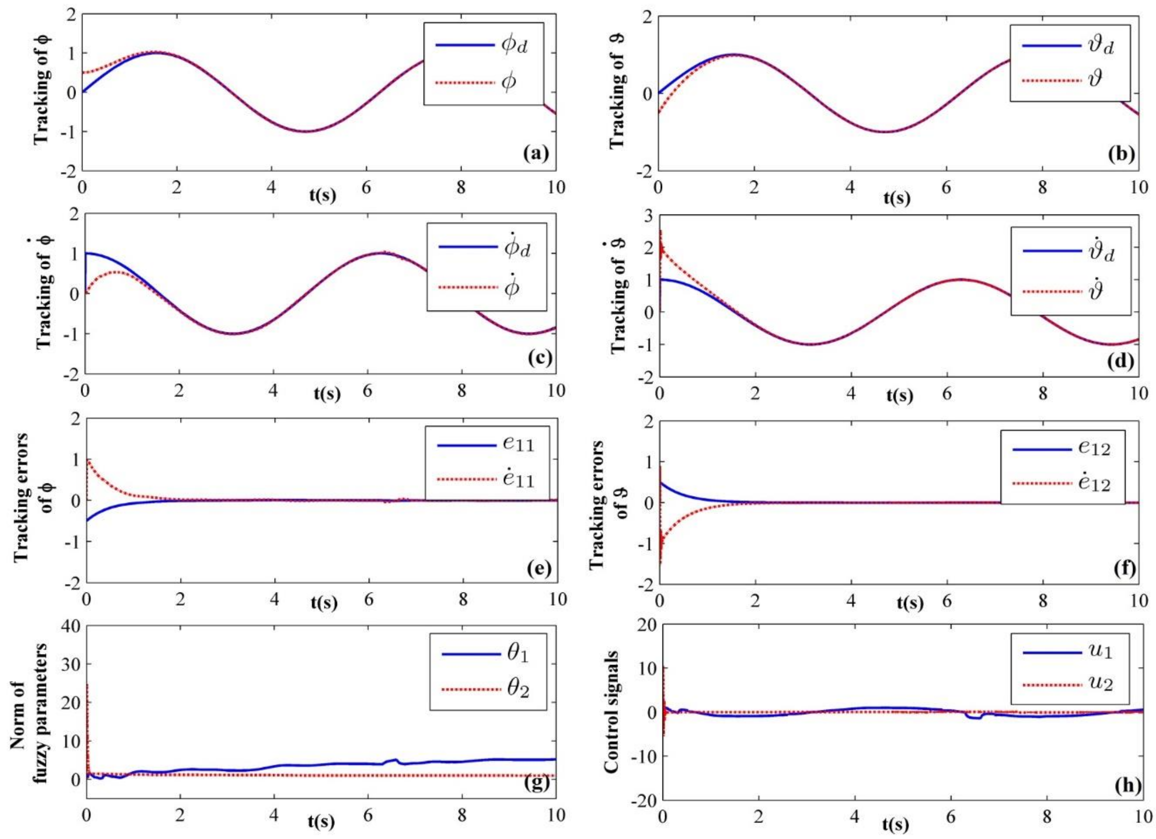

Figure 2.6: *Simulation results for a 2 DOF helicopter (example 3): (a) Tracking of ϕ, (b) Tracking of ϑ, (c) Tracking of $\dot{\phi}$, (d) Tracking of $\dot{\vartheta}$, (e) Tracking errors e_{11} and $\dot{e}_{11}$, (f) Tracking errors e_{12} and $\dot{e}_{12}$, (g) Norm of fuzzy parameters: $\|\theta_1\|$ and $\|\theta_2\|$, (h) Control signals: u_1 and u_2.*

where $u_{PI} = K_p\bar{E} + K_i\int_o^t \bar{E}d\tau$ is a PI control term, $u_r = K_r sign(\bar{E})$ is a robust control term, and $u_f = \theta^T\psi(z)$ is a fuzzy control term.

Secondly, we investigate the influence of both main terms in the adaptation laws (2.19) (i.e. the adaptation integral term and the adaptation proportional term). From Remark 2.4, we have :

$$\theta = \theta_I + \theta_P, \tag{2.33}$$

where $\theta_I = \int_0^t \left(-\sigma_i\gamma_{1i}|\bar{E}_i|\theta_i + \gamma_{1i}\bar{E}_i\psi_i(z_i)\right)d\tau$ is an adaptation integral term, and $\theta_P = -\gamma_{2i}\delta_{fi}$ is an adaptation proportional term.

Moreover, to ***quantify the tracking performances*** in all tests, the following performance index (the integral of absolute value of the tracking error (IAE)) is used

$$I_{IAE} = \int_0^T |y_{di} - y_i| \, dt. \tag{2.34}$$

The obtained results are given in Table 2.2 and Table 2.3. From Table 2.2, in the third test (i.e. when the robust term is added to the control law), we can clearly observe a notable improvement in the tracking performances. From Table 2.3, the superiority of the PI adaptation law over other adaptation laws (i.e. in the test 6) is clearly shown in these results.

b) Comparative study 2:

In this sub-section, we will conduct a test of our PI update law (2.18) and (2.19), with three well-known update laws, namely $\sigma - modification$ [IOA84], $e - modification$ [NAR87], and standard adaptive law [WHI61]. For that, for all update laws, the control law given by (2.18) is applied to the system (2.26).

Simulation results for all update laws are displayed in Figure 2.7 and Figure 2.8. From Figure 2.7, the PI adaptation law proposed in this chapter allows for a rapid convergence of adaptive fuzzy parameters and the final values of the steady state are well acceptable. However, the behavior of fuzzy parameters tuned with $\sigma - modification$ is well similar to that of the of fuzzy parameters adjusted only with $e - modification$. For these two update laws, the fuzzy parameters are bounded and their final values are quite acceptable. However, for the standard adaptation law, we can remark that the adaptive parameters tend to diverge, due to the pure integral action.

Also, Figure 2.8, which shows the underlying tracking errors obtained by applying the four update laws, confirms well the superiority of our PI update laws. The standard adaptation law also gives a satisfactory tracking performance, but at the cost of a great value for fuzzy parameters (i.e. an important control effort).

Table 2.2: *The effect of each term in the control law (2.32)*

Tests	**Control law**	$I_{IAE} = \begin{bmatrix} I_{IAE1} \\ I_{IAE2} \end{bmatrix}$
Test N°1	$\boldsymbol{u = u_f}$, with $\theta = \theta_I + \theta_P$	$\begin{bmatrix} 0.06508 \\ 0.06694 \end{bmatrix}$
Test N°2	$\boldsymbol{u = u_{PI} + u_f}$ with $\theta = \theta_I + \theta_P$	$\begin{bmatrix} 0.06315 \\ 0.06671 \end{bmatrix}$
Test N°3	$\boldsymbol{u = u_{PI} + u_r + u_f}$ with $\theta = \theta_I + \theta_P$	$\begin{bmatrix} \mathbf{0.03021} \\ \mathbf{0.04006} \end{bmatrix}$

Table 2.3: *The effect of the PI terms in the adaptation law (2.33)*

Tests	**Control law**	$I_{IAE} = \begin{bmatrix} I_{IAE1} \\ I_{IAE2} \end{bmatrix}$
Test N°4	$\boldsymbol{u = u_f}$, **with** $\boldsymbol{\theta = \theta_P}$	$\begin{bmatrix} 0.8289 \\ 0.6980 \end{bmatrix}$
Test N°5	$\boldsymbol{u = u_f}$ **with** $\boldsymbol{\theta = \theta_I}$	$\begin{bmatrix} 0.16690 \\ 0.6980 \end{bmatrix}$
Test N°6	$\boldsymbol{u = u_f}$ **with** $\boldsymbol{\theta = \theta_I + \theta_P}$	$\begin{bmatrix} \mathbf{0.06508} \\ \mathbf{0.06694} \end{bmatrix}$

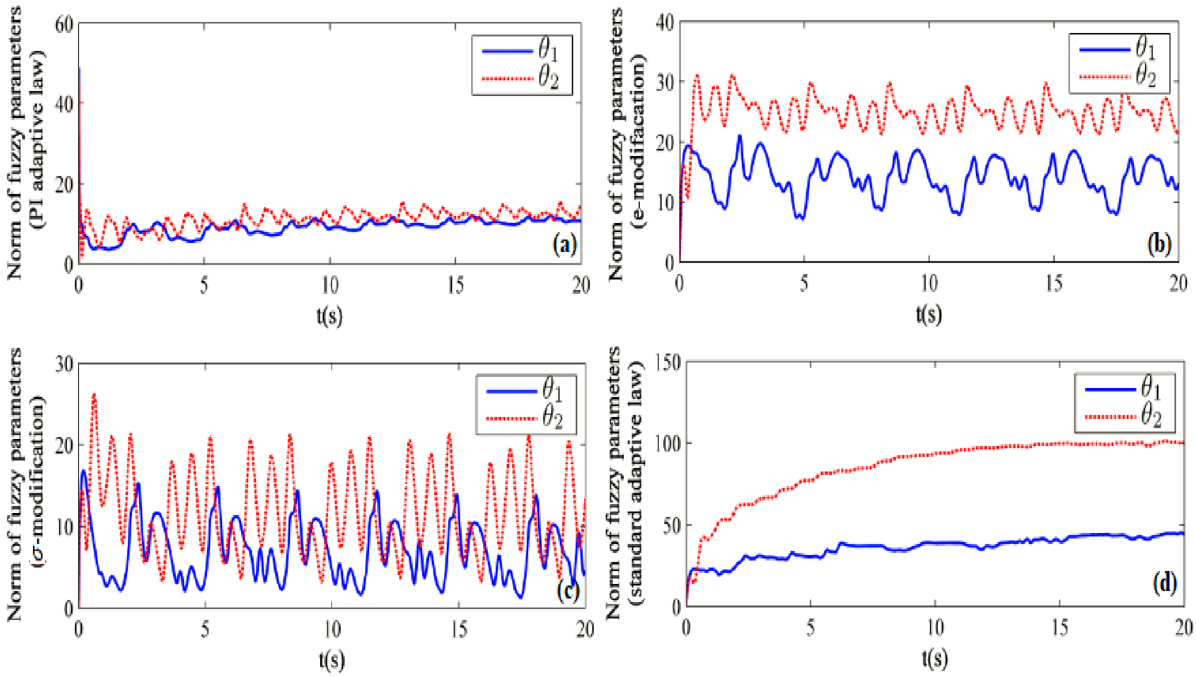

Figure 2.7: *Comparative study 2: Norm of fuzzy parameters: $\|\theta_1\|$ and $\|\theta_2\|$, obtained by using (a) the proposed PI adaptation law ; (b) the e-modification adaptive law; (c) the σ-modification adaptive law; (d) the standard adaptation law.*

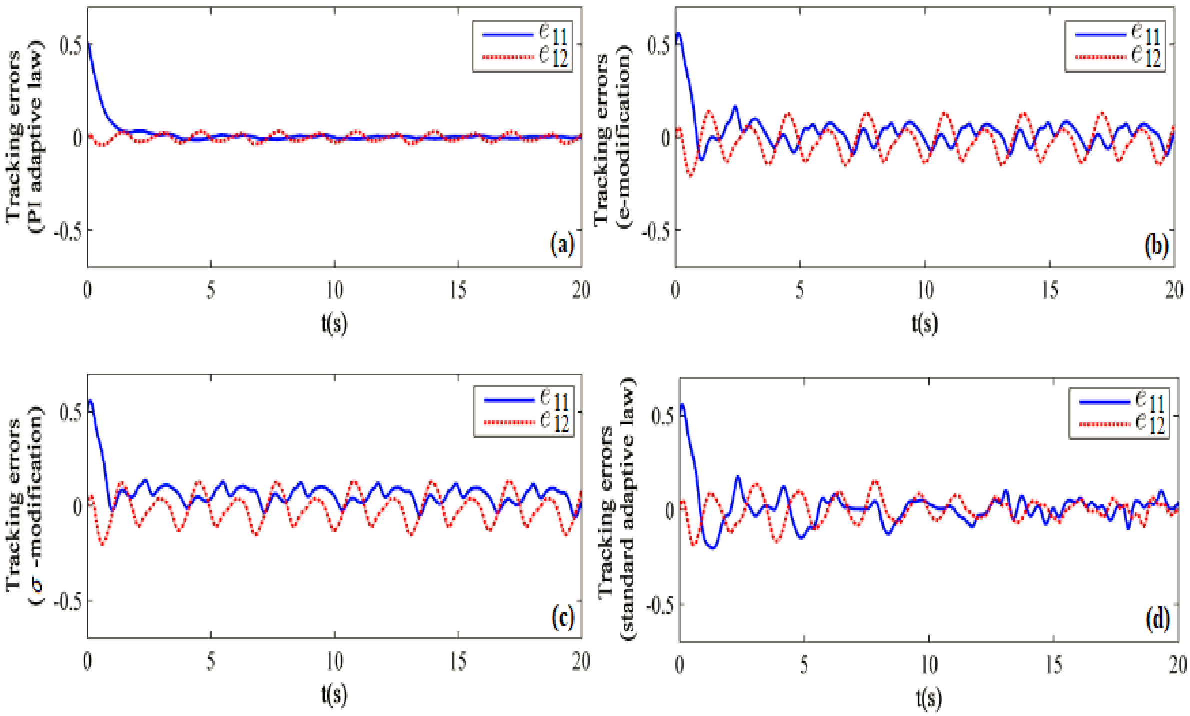

Figure 2.8: *Comparative study 2: the underlying tracking errors e_1 and e_2, obtained by using (a) the proposed PI adaptation law ; (b) the e-modification*

2.5 Conclusion

A fuzzy state-feedback controller for a class of uncertain multivariable nonlinear systems has been proposed in this chapter. Adaptive fuzzy systems have been used to online model uncertain functions. To enhance the convergence of the updated fuzzy parameters as well as the tracking performances, a PI update law has been introduced. Simulation results of three practical examples have been given to emphasize the performance of the proposed controller. A comparative study has been conducted also to evaluate the effect of each terms in the main control and PI update laws.

Chapter 3

High-gain Observer-based Fuzzy Adaptive Controller of MIMO Nonlinear Systems

3.1 Introduction

Most of the adaptive fuzzy control schemes devoted to multivariable systems need the complete state to calculate the control signals. Whereas, in the practice, only the output vector could be obtained by a direct measurement. In this case, an observer is needed and a fuzzy adaptive output feedback control (i.e. an observer-based adaptive fuzzy control) should be designed [CHI13, SHA15, HUA14, PEN14, TON05, TON03, BOU08d]. In the control literature, two design approaches for the adaptive fuzzy output feedback control have been employed:

a) The utilization of *a linear observer* to estimate either the system state or the tracking error dynamics' state, for which the strict positive real (SPR) property is required (see e.g. [SHI15, ARE13, SHA16, SHI12]). To have this restrictive property, an appropriate SPR low-pass filter [BOU08d] in general augments the observation error dynamics and this will probably cause the filtering of the fuzzy basis functions' vector. In this case, the order of the control system can become very large. In addition to these disadvantages, as stated in [BOU08d], most of these works are questionable.

b) The utilization of *a high-gain observer* to build the states of the tracking error dynamics [TON03], for which the so-called SPR property is not required. Unfortunately, as stated in [LAB06], this work is also questionable.

On the other hand, all observer-based adaptive control approaches designed in [SHI12, CHI13, SHA15, HUA14, TON03] requires another common assumption, viz., that the high-frequency gain matrix is positive (or negative) definite. The case where the sign of the high-frequency gain matrix is not definite has not treated in these designs. Therefore, how to extend the results [SHI15, ARE13, HUA14, PEN14, TON05] to more general case, i.e. multivariable systems with indefinite high-frequency gain matrix, is an important problem.

Motivated by the above observations, in this chapter, a novel fuzzy adaptive control methodology based on high-gain observer for a class of multivariable uncertain nonlinear systems is considered. It is assumed that the system model is unknown (except the sign of the leading principal minors of the high-frequency gain matrix) and with uncertain dynamical disturbances and the state vector are not available for measurement (except the output vector). In addition, in order to consider a more general case, this high-frequency gain matrix can be with an indefinite sign. Fuzzy logic systems are employed to online approximate the uncertain dynamics and an HG observer is designed to robustly and quickly construct the unmeasured states. The SDT factorization is appropriately utilized to factorize the high-frequency gain matrix. The fuzzy adaptive control and its update laws are derived based on Lyapunov approach. Simulations carried out on many multivariable systems show that a precise tracking and a fast estimation of the states can be accomplished in the closed-loop systems in spite of uncertainties and external disturbances using the derived observer-based control.

Bearing in mind the previous results dealing with multivariable adaptive output-feedback control [SHA16, SHI12, CHI13, TON05, TON03], the main contributions of this chapter can be listed as:

- The considered class of multivariable systems is quite large as its high-frequency gain matrix can be with indefinite sign.
- A high-gain observer is designed to robustly estimate the non-measured states. Further, the stability analysis of the overall system (system + controller + observer) is rigorously demonstrated.

3.2 Notation and problem statement

In this chapter, we are going to study the uncertain multivariable nonlinear systems tranformable into the following form:

$$y^{(n)} = F(x) + G(x)u + P(x,t), \tag{3.1}$$

where $x = [y^T\ \dot{y}^T \ldots \left(y^{(n-1)}\right)^T]^T$ denotes the system state vector, $u(t) \in R^m$ is the control input vector, $y(t) \in R^m$ is the measured output vector. The function $F(x) \in R^m$ and the matrix $G(x) = [g_{ij}(x)] \in R^{m\times m}$ are unknown and smooth, and $P(x,t) \in R^m$ is the vector of the unknown but bounded input disturbances.

This study aims to synthesize a fuzzy adaptive output-feedback control u such that the output $y(t)$ ultimately converge to desired trajectory $y_d(t) \in R^m$, despite the presence of uncertain dynamics, immeasurable states and external disturbances.

Let us make the following mild assumptions.

Assumption 3.1: *The state vector x is not available for measurement.*

Assumption 3.2: *The vector of the desired trajectories* $x_d = [{y_d}^T \ {\dot{y}_d}^T \ldots \left(y_d^{(n-1)}\right)^T]^T \in R^{m*n}$ *is continuous and bounded.*

Assumption 3.3: *The leading principal minors of the input gain matrix* $G(x)$ *are all nonzero. The sign of each minor is known.*

$$\partial g_{ij}(x)/\partial y_j^{(n-1)} = 0, \forall\, i = 1, \ldots, m \text{ and } j = 1, \ldots, m.$$

Assumption 3.4: *The disturbances' vector,* $P(x,t)$, *is assumed to be unknown but bounded.*

Evoking Lemma 1.4, the system (3.1) can be expressed as follows:

$$S_1(x)y^{(n)} = H(x) + DT(x)u \tag{3.2}$$

where $S_1(x) = S^{-1}(x) \in R^{m\times m}$ is a positive definite symmetric matrix, and $H(x) = S^{-1}(x)(F(x) + P(x,t)) \in R^m$.

The output tracking error, $e_1(t) \in R^m$, is defined as

$$e_1 = y_d - y \tag{3.3}$$

For the subsequent analysis, we introduce filtered error systems, $e_i \in R^m, i = 2, \ldots, n$, as [MOR93, MER16, MER17a, MER17b]:

$$e_2 = \dot{e}_1 + e_1,$$

$$e_3 = \dot{e}_2 + e_2 + e_1,$$

$$\vdots$$

$$e_n = \dot{e}_{n-1} + e_{n-1} + e_{n-2}. \tag{3.4}$$

These errors, $e_i, i = 2, \ldots, n$, can be simply rewritten as

$$e_i = \sum_{j=0}^{i-1} a_{ij} e_1^{(j)} \quad \forall\, i = 2,3, \ldots, n. \tag{3.5}$$

where a_{ij} are known coefficients, which can be generated through the well-known Fibonacci number series [XIA04].

To facilitate also the control design and stability analysis, Let us define a novel error variable E as :

$$E = e_n + e_{n-1} \tag{3.6}$$

According to (3.2), (3.5) and (3.6), we can obtain the following dynamics

$$\overline{S}_1\dot{\overline{E}} = D^{-1}S_1\left(y_d^{(n)} + \sum_{j=0}^{n-2} a_{nj}e_1^{(j+1)}(t) + \dot{e}_{n-1}\right) - D^{-1}H(x) - T(x)u \tag{3.7}$$

where $\overline{S}_1 = D^{-1}S_1D$ and $\overline{E} = \left[\overline{E}_1, \dots, \overline{E}_m\right]^T = D^{-1}E$.

Then, we can simply rewrite (3.7) as

$$\overline{S}_1\dot{\overline{E}} + \frac{1}{2}\dot{\overline{S}}_1\overline{E} = \alpha(z) - u - De_{n-1} \tag{3.8}$$

with

$$\alpha(z) = [\alpha_1(z_1), \dots, \alpha_m(z_m)]^T = D^{-1}S_1\left(y_d^{(n)} + \sum_{j=0}^{n-2} a_{nj}e_1^{(j+1)} + \dot{e}_{n-1}\right) - D^{-1}H(x)$$

$$+\frac{1}{2}\dot{\overline{S}}_1\overline{E} - (T(x) - I_m)u + De_{n-1} \tag{3.9}$$

where $z = [z_1^T, z_2^T, \dots, z_m^T]^T$.

The vector z_i can be defined as in equation (2.12).

Assumption 3.5: *The matrix* $\overline{S}_1$ *should satisfy :*

$$\left\|\overline{S}_1^{-1}\right\| \le \rho_1 \ \ and \ \left\|\tfrac{1}{2}\overline{S}_1^{-1}\dot{\overline{S}}_1\right\| \le \rho_2 \tag{3.10}$$

where ρ_1*and* ρ_2 *are some unknowm constants.*

Denote the subsequent compact sets:

$$\Omega_{z_i} = \left\{[x^T, u_{i+1}, \dots, u_m]^T \middle| x \in \Omega_x \subset R^m, x_d \in \Omega_{x_d}\right\},$$
$$i = 1,2, \dots, m-1,$$
$$\Omega_{z_m} = \left\{x^T \middle| x \in \Omega_x \subset R^m, x_d \in \Omega_{x_d}\right\},$$

Since the continuous nonlinear functions $\alpha_i(z_i)$ are unknown and only the system's output is available for measurement, later we will resort to use a fuzzy adaptive system to online estimate the functions $\alpha_i(z_i)$, and a high-gain observer to accurately reconstruct the unmeasurable state variables.

3.3 Output-feedback controller based on adaptive fuzzy system

We now turn out to the development of the observer-based adaptive control for the MIMO system (3.1). We first design a high-gain observer to indirectly estimate the immeasurable states. Then, we will show how this observer can be incorporated in an adaptive fuzzy control system, while ensuring the stability of the closed-loop system as well as the accuracy of the tracking.

3.3.1 Design of the HG observer

The error variable system (3.4) can be rewritten as follows:

$$\begin{aligned}
\dot{e}_1 &= e_2 - e_1, \\
\dot{e}_2 &= e_3 - e_2 - e_1, \\
&\vdots \\
\dot{e}_{n-2} &= e_{n-1} - e_{n-2} - e_{n-3}, \\
\dot{e}_{n-1} &= E - 2e_{n-1} - e_{n-2}, \\
\dot{E} &= \dot{E}.
\end{aligned} \tag{3.11}$$

Now, consider the following High-gain observer whose structure is corresponding to the previous system equation (3.11)

$$\begin{aligned}
\dot{\hat{e}}_1 &= \hat{e}_2 - \hat{e}_1 + \frac{\beta_1}{\epsilon}(e_1 - \hat{e}_1) \\
\dot{\hat{e}}_2 &= \hat{e}_3 - \hat{e}_2 - \hat{e}_1 + \frac{\beta_2}{\epsilon^2}(e_1 - \hat{e}_1) \\
&\vdots \\
\dot{\hat{e}}_{n-1} &= \hat{E} - 2\hat{e}_{n-1} - \hat{e}_{n-2} + \frac{\beta_{n-1}}{\epsilon^{n-1}}(e_1 - \hat{e}_1) \\
\dot{\hat{E}} &= \frac{\beta_n}{\epsilon^n}(e_1 - \hat{e}_1)
\end{aligned} \tag{3.12}$$

where $\beta_i > 0,\ \forall\, i = 1,2,\dots,n$, represent design parameters, and $\epsilon > 0$ is a small scalar. Adjusting the observer parameters, β_i and ϵ, could affect the performances of this high-gain observer.

Remark 3.2: The observer (3.12) is a copy of the error systems (3.11) with a correction term (i.e. with feedback from measurements, $e_1 - \hat{e}_1$). Note that this observer can accurately estimate the error vector $r(t) = [e_1^T\ e_2^T \dots e_{n-1}^T\ E^T]^T \in R^{m*n}$. From this estimate $\hat{r}$, we can indirectly estimate the system states.

In order to put the observation error dynamics into a convenient form for further analysis, the following transformation is introduced.

$$\eta_i(t) = \frac{1}{\epsilon^{n-i}}(e_i - \hat{e}_i) \quad i = 1,2,\dots,n-1$$

$$\eta_n(t) = E - \hat{E} \tag{3.13}$$

Let us denote $\eta\ (t) \triangleq [\eta_1^T\ \ \eta_2^T \dots \eta_n^T] \in R^{m*n}$.

By considering (3.11)-(3.13), we can obtain

$$\epsilon\dot{\eta}_1 = -\beta_1\eta_1 + \eta_2 - \epsilon\eta_1$$

$$\epsilon\dot{\eta}_i = -\beta_i\eta_1 + \eta_{i+1} - \epsilon^2\eta_{i-1} - \epsilon\eta_i\ , \quad i = 2,\dots,n-1 \tag{3.14}$$

$$\epsilon\dot{\eta}_n = -\beta_n\eta_1 + \epsilon\dot{E}$$

Then we can rewrite (3.14) as follows:

$$\dot{\eta}(t) = \frac{1}{\epsilon}A_0\eta(t) + h(.) \tag{3.15}$$

with $h(.) \in R^{m*n}$ being

$$h = -\left[\eta_1^T \quad \epsilon\eta_1^T + \eta_2^T \quad \dots \quad \epsilon\eta_{n-2}^T + 2\eta_{n-1}^T \quad -\dot{E}\right]^T \tag{3.16}$$

and

$$A_0 = \begin{bmatrix} -\beta_1 I_m & I_m & \cdots & 0_m \\ \vdots & \vdots & \ddots & \vdots \\ -\beta_{n-1} I_m & 0_m & \dots & I_m \\ -\beta_n I_m & 0_m & \cdots & 0_m \end{bmatrix}, \tag{3.17}$$

where $\beta_i, \forall\, i = 1,2, \dots, n$, should be selected such as this matrix A_0 is Hurwitz .

3.3.2. Output-feedback controller design

The uncertain smooth function $\alpha_i(z_i)$ can be estimated by the FLS (1.27) as follows :

$$\hat{\alpha}_i(z_i, \theta_i) = \theta_i^T \psi_i(z_i), \quad i = 1, \dots, m \tag{3.18}$$

where $\psi_i(z_i)$ is a vector containing the FBFs, which should be appropriately fixed by the designer. θ_i is a vector containing all the parameters to be online estimated.

The unknown ideal values of θ_i can be defined as follows [BOU08b, BOU12c, BOU12a]:

$$\theta_i^* = arg_{\theta_i} min \left[\sup_{x \in D_x} |\alpha_i(z_i) - \hat{\alpha}_i(z_i, \theta_i)| \right], \tag{3.19}$$

Now, let us denote

$$\tilde{\theta}_i = \theta_i - \theta_i^*, \tag{3.20}$$

$$\omega_i(z_i) = \alpha_i(z_i) - \hat{\alpha}_i(z_i, \theta_i^*), \qquad i = 1, \dots, m \tag{3.21}$$

where $\hat{\alpha}_i(z_i, \theta_i^*) = \theta_i^* \psi_i(z_i)$, as the parameter error vector and the approximation error, respectively. According to [WAN94], the latter is bounded on a compact set by $|\omega_i(z_i)| \leq \overline{\omega}_i$, $\forall\, z_i \in \Omega_{z_i}$, with $\overline{\omega}_i$ being an unknown bound.

By replacing z_i by its estimate $\hat{z}_i$ into (3.18), we get:

$$\hat{\alpha}_i(\hat{z}_i, \theta_i) = \theta_i^T \psi_i(\hat{z}_i), \quad i = 1, \dots, m \tag{3.22}$$

with $\hat{z}_i$ being given by

$$\begin{aligned} \hat{z}_1 &= [\hat{x}^T, u_2, \dots, u_m]^T, \\ &\vdots \\ \hat{z}_{m-1} &= [\hat{x}^T, u_m]^T, \\ \hat{z}_m &= \hat{x}^T, \end{aligned} \tag{3.23}$$

Denote

$$\alpha(\hat{z}) = \theta^T \psi(\hat{z}) = [\alpha_1(\hat{z}_1, \theta_1), \dots, \alpha_m(\hat{z}_m, \theta_m)]^T$$

$$=[\theta_1^T \psi_1(\hat{z}_1), \dots, \theta_m^T \psi_m(\hat{z}_m)]^T,$$

and

$$\omega(\hat{z}) = [\omega_1(\hat{z}_1), \dots, \omega_m(\hat{z}_m)]^T \tag{3.24}$$

From (3.18) and (3.20)-(3.21), we have

$$\begin{aligned}\hat{\alpha}_i(\hat{z}_i, \theta_i) - \alpha_i(z_i) &= \hat{\alpha}_i(\hat{z}_i, \theta_i) - \hat{\alpha}_i(\hat{z}_i, \theta_i^*) + \hat{\alpha}_i(\hat{z}_i, \theta_i^*) - \hat{\alpha}_i(z_i, \theta_i^*) + \hat{\alpha}_i(z_i, \theta_i^*) - \alpha_i(z_i) \\ &= \hat{\alpha}(\hat{z}_i, \theta_i) - \hat{\alpha}(\hat{z}_i, \theta_i^*) + \omega_{ai}(z_i, \hat{z}_i) \\ &= \theta_i^T \psi_i(\hat{z}_i) - \theta_i^{*T} \psi_i(\hat{z}_i) + \omega_{ai}(z_i, \hat{z}_i) \\ &= \tilde{\theta}_i^T \psi_i(\hat{z}_i) + \omega_{ai}(z_i, \hat{z}_i) \end{aligned} \tag{3.25}$$

with

$$\begin{aligned}\omega_{ai}(z_i, \hat{z}_i) &= \hat{\alpha}_i(\hat{z}_i, \theta_i^*) - \hat{\alpha}_i(z_i, \theta_i^*) - \omega_i(z_i) \\ &= \theta_i^{*T} \psi_i(\hat{z}_i) - \theta_i^{*T} \psi_i(z_i) - \omega_i(z_i)\end{aligned} \tag{3.26}$$

Denote

$\tilde{\theta}^T \psi(\hat{z}) = \left[\tilde{\theta}_1^T \psi_1(\hat{z}_1), \dots, \tilde{\theta}_m^T \psi_m(\hat{z}_m)\right]^T$, and $\omega_a(z, \hat{z}) = [\omega_{a1}(z_1, \hat{z}_1), \dots, \omega_{am}(z_m, \hat{z}_m)]^T$.

As the approximation error $\omega_i(z_i)$ and FBFs are bounded, we can prove the existence of the following upper bound [BOU08d]:

$$\|\omega_a(z, \hat{z})\| \le \bar{\omega}_a \tag{3.27}$$

with $\bar{\omega}_a$ being an unknown constant.

Substituting (3.25) into (3.8), we can get

$$\bar{S}_1 \dot{\bar{E}} + \frac{1}{2} \dot{\bar{S}}_1 \bar{E} = -\tilde{\theta}^T \psi(\hat{z}) + \theta^T \psi(\hat{z}) - \omega_a(z, \hat{z}) - u - De_{n-1} \tag{3.28}$$

To stabilize the dynamics (3.28), we design our output-feedback control and its update law as follows:

$$u = \theta^T \psi(\hat{z}) + K_p \hat{\bar{E}} \tag{3.29}$$

$$\dot{\theta}_i = -\sigma_i \gamma_i \theta_i + \gamma_i \hat{\bar{E}}_i \psi_i(\hat{z}_i) \tag{3.30}$$

with $K_p = diag\{k_{pi}\} \in R^{m\times m}$ being a positive definite design matrix, and γ_i and $\sigma_i > 0$ are the design scalars. $\hat{\overline{E}} = D\hat{E} = D^{-1}\hat{E} = D^T\hat{E}$ where $\hat{E}$ is the estimate of E.

Substituting the control law (3.29) into (3.28) gives

$$\begin{aligned}
\overline{E}^T \overline{S}_1 \dot{\overline{E}} + \frac{1}{2}\overline{E}^T \dot{\overline{S}}_1 \overline{E} &= -\overline{E}^T \tilde{\theta}^T \psi(\hat{z}) + \overline{E}^T \theta^T \psi(\hat{z}) - \overline{E}^T D e_{n-1} - \overline{E}^T \omega_a(z,\hat{z}\ ) \\
&\quad -\overline{E}^T \left(\theta^T \psi(\hat{z}) + K_p \hat{\overline{E}}\right) \\
&= -\overline{E}^T \tilde{\theta}^T \psi(\hat{z}) - \overline{E}^T D e_{n-1} - \overline{E}^T K_p \hat{\overline{E}} - \overline{E}^T \omega_a(z,\hat{z}\ ) \\
&= -\left(\hat{\overline{E}} + D\eta_n\right)^T \tilde{\theta}^T \psi(\hat{z}) - e_n^T e_{n-1} - e_{n-1}^T e_{n-1} \\
&\quad - \overline{E}^T K_p \left(\overline{E} - D\eta_n\right) - \overline{E}^T \omega_a(z,\hat{z}\ )
\end{aligned} \tag{3.31}$$

with $\overline{E}^T D e_{n-1} = \overline{E}^T D^{-1} D e_{n-1} = e_n^T e_{n-1} + e_{n-1}^T e_{n-1}$ and $\overline{E} = \hat{\overline{E}} + D\eta_n$.

Based on the above considerations, we can obtain the following fundamental result.

Theorem 3.1: *For the plant (3.1) under Assumptions 3.1-3.5, if the high-gain observer (3.12) and the controller (3.29) together with adaptive law (3.30) are considered, then*

- *the corresponding closed-loop system is stable in the sense that all signals (including the error variables e_i, $\overline{E}$ and η, and the control input u) are UUB, and*
- *these error variables converge to a small enough region of the origin.*

Proof of Theorem 3.1: Let select a positive definite quadratic Lyapunov function as

$$L = \frac{1}{2}\sum_{i=1}^{n-1} e_i^T e_i + \frac{1}{2}\overline{E}^T S_1 \overline{E} + \sum_{i=1}^{m} \frac{1}{2\gamma_i} \tilde{\theta}_i^T \tilde{\theta}_i + \frac{1}{2}\eta^T P \eta \tag{3.32}$$

with the matrix P being a positive definite symmetric solution of (3.41).

Let us write (3.32) as follows:

$$L = L_1 + L_2 + L_3 \tag{3.33}$$

with

$$L_1 = \frac{1}{2}\sum_{i=1}^{n-1} e_i^T e_i + \frac{1}{2}\overline{E}^T S_1 \overline{E}, \tag{3.34}$$

$$L_2 = \sum_{i=1}^{m} \frac{1}{2\gamma_i} \tilde{\theta}_i^T \tilde{\theta}_i \tag{3.35}$$

$$L_3 = \frac{1}{2}\eta^T P \eta. \tag{3.36}$$

The time derivative of L_1 along with (3.31) and (3.11) is

$$\dot{L}_1 \leq -\sum_{i=1}^{n-1} \|e_i\|^2 - \overline{E}^T K_p \overline{E} - e_{n-1}^T e_{n-1} - \sum_{i=1}^{m} \widehat{\overline{E}}_i \tilde{\theta}_i^T \psi_i(\hat{z}_i) - \sum_{i=1}^{m} d_{ii} \eta_{n_i} \tilde{\theta}_i^T \psi_i(\hat{z}_i)$$
$$+ \overline{E}^T K_p D \eta_n - \overline{E}^T \omega_a(z, \hat{z}) \tag{3.37}$$

Taking also the time derivative of L_2 along with (3.30) yields

$$\dot{L}_2 = \sum_{i=1}^{m} \frac{1}{\gamma_i} \tilde{\theta}_i^T \dot{\theta}_i = \sum_{i=1}^{m} \tilde{\theta}_i^T \left[-\sigma_i \theta_i + \widehat{\overline{E}}_i \psi_i(\hat{z}_i) \right] \tag{3.38}$$

Using the property $-\sigma_i \tilde{\theta}_i^T \theta_i \leq -\frac{\sigma_i}{2} \left\| \tilde{\theta}_i \right\|^2 + \frac{\sigma_i}{2} \|\theta_i^*\|^2$, (3.38) becomes

$$\dot{L}_2 \leq \sum_{i=1}^{m} \widehat{\overline{E}}_i \tilde{\theta}_i^T \psi_i(\hat{z}_i) - \sum_{i=1}^{m} \frac{\sigma_i}{2} \left\| \tilde{\theta}_i \right\|^2 + \sum_{i=1}^{m} \frac{\sigma_i}{2} \|\theta_i^*\|^2 \tag{3.39}$$

Then, we can obtain

$$\dot{L}_1 + \dot{L}_2 \leq -\sum_{i=1}^{n-1} \|e_i\|^2 - \overline{E}^T K_p \overline{E} - \sum_{i=1}^{m} \frac{\sigma_i}{2} \left\| \tilde{\theta}_i \right\|^2 + \sum_{i=1}^{m} \frac{\sigma_i}{2} \|\theta_i^*\|^2$$
$$+ \sum_{i=1}^{m} d_{ii} \eta_{n_i} \tilde{\theta}_i^T \psi_i(\hat{z}_i) + \overline{E}^T K_p D \eta_n - \overline{E}^T \omega_a(z, \hat{z}) - e_{n-1}^T e_{n-1} \tag{3.40}$$

Since A_0 is a Hurwitz (stable) matrix, there exists a unique matrix $P \in R^{mn \times mn}$ positive definite symmetric, being the solution of the following matrix equation:

$$P A_0 + A_0^T P = -I \tag{3.41}$$

where $I \in R^{mn \times mn}$ is a unity matrix.

The time derivative of (3.36), i.e. L_3, along the trajectory of (3.15) is given by

$$\dot{L}_3 \leq -\frac{1}{2\epsilon}\|\eta\|^2 + \|P\|\|\eta\|\|h\|, \tag{3.42}$$

By (3.16), we can obtain

$$\|h\| \leq 3\|\eta\| + \left\|\dot{\bar{E}}\right\| \tag{3.43}$$

Using (3.43), (3.42) becomes:

$$\dot{L}_3 \leq -\frac{1}{2\epsilon}\|\eta\|^2 + 3\|P\|\|\eta\|^2 + \|P\|\|\eta\|\left\|\dot{\bar{E}}\right\| \tag{3.44}$$

By invoking (3.29), (3.28) can be rewritten as

$$\dot{\bar{E}} = -\frac{1}{2}\bar{S}_1^{-1}\dot{\bar{S}}_1\bar{E} - \bar{S}_1^{-1}\tilde{\theta}^T\psi(\hat{z}) - \bar{S}_1^{-1}\bar{E}^T\omega_a(z,\hat{z}\ \ ) - \bar{S}_1^{-1}De_{n-1} - \bar{S}_1^{-1}K_p\big(\bar{E} - D\eta_n\big) \tag{3.45}$$

From (3.45) and Assumption 3.5, $\dot{\bar{E}}$ can be bounded as follows:

$$\left\|\dot{\bar{E}}\right\| \leq \rho_2\|\bar{E}\| + \rho_1 c_1\left\|\tilde{\theta}\right\| + \rho_1\varpi_a + \rho_1\|e_{n-1}\| + \rho_1 k_{pM}\|\bar{E}\| + \rho_1 k_{pM}\|\eta_n\| \tag{3.46}$$

with $c_1 \geq \|\psi(\hat{z})\|$, and k_{pM} standing for the largest eigen-value of K_p .

From (3.40), we have

$$\dot{L}_1 + \dot{L}_2 \leq -\sum_{i=1}^{n-1}\|e_i\|^2 - \bar{E}^T K_p\bar{E} - \sum_{i=1}^{m}\frac{\sigma_i}{2}\left\|\tilde{\theta}_i\ \right\|^2 + \sum_{i=1}^{m}\frac{\sigma_i}{2}\|\theta_i^*\|^2 + c_1\|\eta_n\|\left\|\tilde{\theta}\right\|$$
$$+k_{pM}\|\bar{E}\|\|\eta_n\| - \|e_{n-1}\|^2 + \|\bar{E}\|\varpi_a \tag{3.47}$$

Then, the last term in (3.44), i.e. $\|P\|\|\eta\|\left\|\dot{\bar{E}}\right\|$, can be bounded as follows

$$\|P\|\|\eta\|\left\|\dot{\bar{E}}\right\| \leq \rho_2\|P\|\|\bar{E}\|\|\eta\| + \rho_1 c_1\|P\|\|\eta\|\left\|\tilde{\theta}\right\| + \rho_1\varpi_a\|P\|\|\eta\| + \rho_1\|P\|\|\eta\|\|e_{n-1}\|$$
$$+ \rho_1 k_{pM}\|P\|\|\eta\|\|\bar{E}\| + \rho_1 k_{pM}\|P\|\|\eta\|\|\eta_n\| \tag{3.48}$$

Also, from the completion of squares, the following inequality can be obtained

$$\rho_1\varpi_a\|P\|\|\eta\| + \|\bar{E}\|\|\varpi_a\| \leq \frac{1}{2}\|\eta\|^2 + \frac{1}{2}\rho_1^2\varpi_a^2\|P\|^2 + \frac{1}{4}\|\bar{E}\|^2 + \varpi_a^2, \tag{3.49}$$

$$\rho_2\|P\|\|\bar{E}\|\|\eta\| + \rho_1 k_{pM}\|P\|\|\eta\|\|\bar{E}\| + \rho_1 k_{pM}\|P\|\|\eta\|\|\eta_n\| \leq \frac{1}{4}\|\bar{E}\|^2 + \rho_2^2\|P\|^2\|\eta\|^2$$

$$+\frac{1}{4}k_{pM}\|\bar{E}\|^2 + k_{pM}\rho_1^2\|P\|^2\|\eta\|^2 + k_{pM}\rho_1 \|P\|\|\eta\|^2 \tag{3.50}$$

$$\rho_1\|P\|\|\eta\|\|e_{n-1}\|+k_{pM}\|\bar{E}\|\|\eta_n\| \le \|e_{n-1}\|^2 + \frac{1}{4}\rho_1^2\|P\|^2\|\eta\|^2 + k_{pM}\|\eta\|^2$$
$$+\frac{1}{4}k_{pM}\|\bar{E}\|^2 \tag{3.51}$$

$$\rho_1 c_1\|P\|\|\eta\|\|\tilde{\theta}\|c_1\|\eta_n\|\|\tilde{\theta}\| \le \frac{1}{2}w_0c_1^2\rho_1^2\|P\|^2\|\eta\|^2 + \frac{1}{2w_0}\|\tilde{\theta}\|^2 + \frac{1}{2}w_0c_1^2\|\eta\|^2$$
$$+\frac{1}{2w_0}\|\tilde{\theta}\|^2 \tag{3.52}$$

where $w_0 > 0$ is a positive constant.

Using (3.44), (3.47) and (3.48)-(3.52) yields

$$\dot{L} \le -\frac{1}{2}\left(\frac{1}{\epsilon} - \left(\rho_4 + k_{pM}\rho_5\right)\right)\|\eta\|^2 - \sum_{i=1}^{n-1}\|e_i\|^2 - \left(\frac{\sigma_i}{2} - \frac{1}{w_0}\right)\|\tilde{\theta}_i\|^2$$
$$-\left(k_{pm} - \left(\frac{1}{2} + \frac{1}{2}k_{pM}\right)\right)\|\bar{E}\|^2 + \pi \tag{3.53}$$

with

$\pi = \frac{1}{2}\rho_1^2\bar{\omega}_a^2\|P\|^2 + \sum_{i=1}^{m}\frac{\sigma_i}{2}\|\theta_i^*\|^2 + \bar{\omega}_a^2,$

$\rho_5 = 2(\rho_1 \|P\|+\rho_1^2\|P\|^2 + 1),$

$\rho_4 = 1 + \rho_2^2\|P\|^2+\frac{1}{2}\rho_1^2\|P\|^2 + w_0c_1^2\rho_1^2\|P\|^2 + w_0c_1^2.$

Then, finally, (3.53) becomes

$$\dot{L} \le -\mu L + \pi \tag{3.54}$$

with $$\mu = min\left\{2, \frac{2}{\lambda_{max}(S_1)}\left(k_{pm} - \left(\frac{1}{2} + \frac{1}{2}k_{pM}\right)\right),\right.$$
$$\left.\frac{1}{\lambda_{max}(P)}\left(\frac{1}{\epsilon} - \left(\rho_4 + k_{pM}\rho_5\right)\right), min_i\left\{\gamma_i\left(\sigma_i - \frac{2}{w_0}\right)\right\}\right\}$$

where $\lambda_{max}(*)$ stands for the largest eigen-value of a matrix (*).

Solving the inequality (3.54) yields [MER17b]:

$$0 \leq L(t) \leq \frac{\pi}{\mu} + \left(L(0) - \frac{\pi}{\mu}\right) e^{-\mu t} \tag{3.55}$$

Then, we can infer from (3.55) that $L(t)$ eventually converges to $\frac{\pi}{\mu}$ as $t \to \infty$. Then, we can conclude that all the signals of the closed-loop system, in particularly the error variables e_i , $\overline{E}$ and η, are semi-globally UUB. These error variables also converge to a small enough region of the origin. This completes the proof.

3.4 Simulation results

To illustrate the effectiveness of the presented control strategy, a set of digital simulations are carried out in this section by considering three real multivariable nonlinear mechanical systems.

3.4.1 Example 1:

Consider again the 2 DOF helicopter considered in section 2.4.1 (see Figure 2.5). Let us choose the references as $y_{d1} = \phi_d = \sin(t)$ and $y_{d2} = \vartheta_d = \sin(t)$. To approximate the uncertain nonlinear functions, the presented controller (3.29) uses two adaptive fuzzy systems, namely ${\theta_1}^T \psi_1(\hat{z}_1)$ and ${\theta_2}^T \psi_2(\hat{z}_2)$, whose respective inputs are $\hat{z}_1$=$[\hat{x}^T, u_2]^T$ and $\hat{z}_2 = \hat{x}$. Three fuzzy sets are defined for the each input of these fuzzy systems over the following intervals: [-2,2] for $\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4$ and [-20,20] for u_2. The corresponding membership functions are selected as in [BOU08d].

The design parameters are set as: $\gamma_1 = 100$, $\gamma_2 = 35$, $\sigma_1 = \sigma_2 = 0.1$, $\lambda_1 = \lambda_2 = 2$, $k_{p1} = k_{p2} = 0.2$, $\epsilon = 0.1$, $\beta_1 = 8$, $\beta_2 = 16$. The initial conditions are selected as: $x(0) = [0.5,\ 0, -0.5,\ 0]^T$, $\theta_{1i}(0) = 0$, $\theta_{2i}(0) = 0$.

Figure 3.1 shows the simulation results of the proposed control scheme. It can be seen from Figure 3.1 (a)-(d) that the system states track well the corresponding reference signals , and the high-gain observer constructs well the system states. Figure 3.1 (e) illustrates the boundedness of the norm of the fuzzy adaptive parameters. Figure 3.1 (f) clearly shows that the generated control signals are bounded, smooth and admissible.

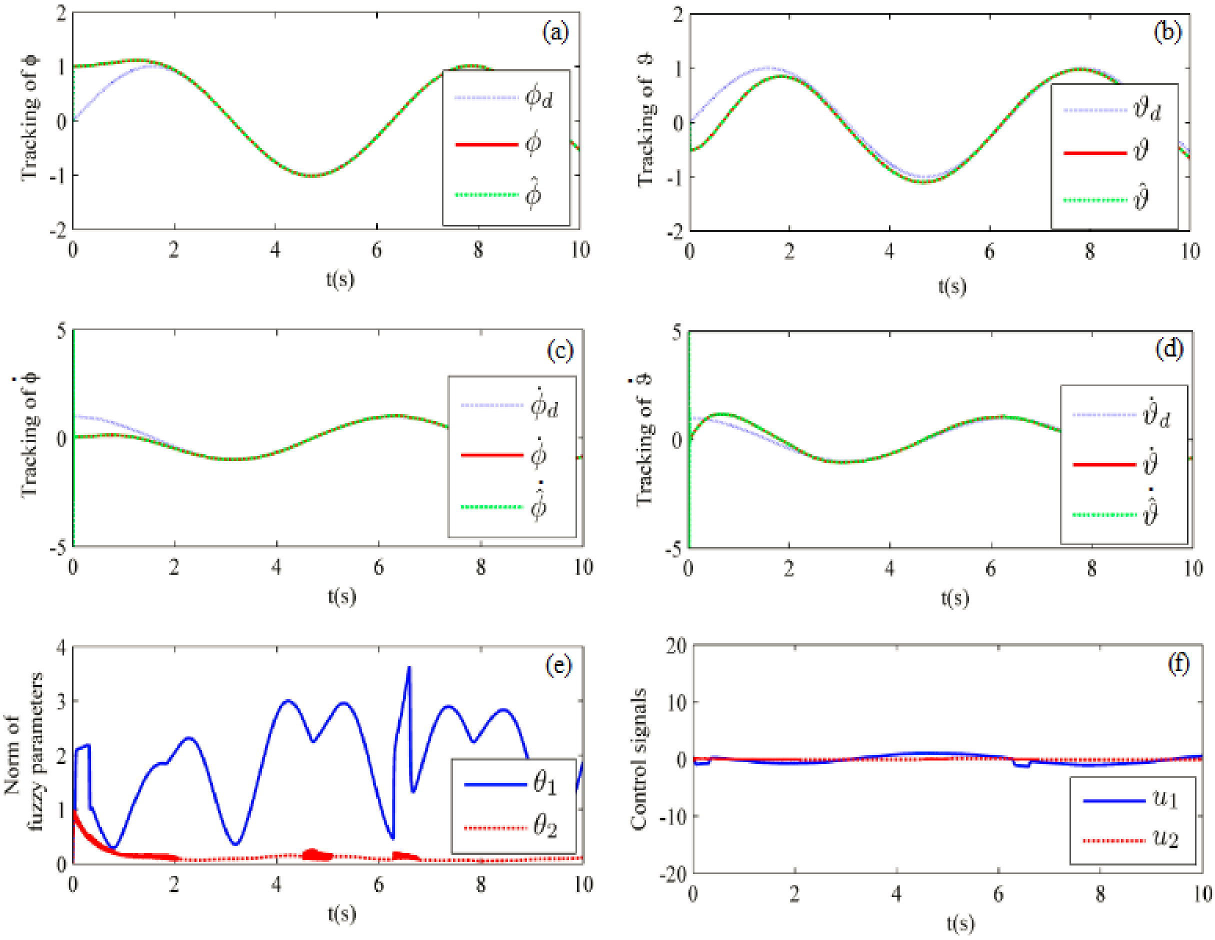

Figure 3.1: *Simulation results for a 2 DOF helicopter (example 1): (a) Trajectory of* ϕ_d, ϕ *and* $\hat{\phi}$ *. (b) Trajectory of* ϑ_d, ϑ *and* $\hat{\vartheta}$*. (c) Trajectory of* $\dot{\phi}_d$, $\dot{\phi}$ *and* $\dot{\hat{\phi}}$*, (d) Trajectory of* $\dot{\vartheta}_d$, $\dot{\vartheta}$ *and* $\dot{\hat{\vartheta}}$*. (e) Norm of fuzzy parameters:* $\|\theta_1\|$ *and* $\|\theta_2\|$*,(f) Control signals:* u_1 *and* u_2*.*

3.4.2 Example 2:

Consider again the 2 DOF polar manipulator robot presented in section 2.4.1 (see Figure 2.1).

Let us select the reference signals as $y_{d1} = q_{d1} = 0.5 + 0.5\sin(2t)$ and $y_{d2} = q_{d2} = 1 + 0.2\sin(4t)$. To approximate the uncertain nonlinear functions, the presented controller (3.29) uses two adaptive fuzzy systems, namely ${\theta_1}^T\psi_1(\hat{z}_1)$ and ${\theta_2}^T\psi_2(\hat{z}_2)$, whose respective inputs are $\hat{z}_1$=$[\hat{x}^T, u_2]^T$ and $\hat{z}_2 = \hat{x}$. Three fuzzy sets are defined for each input of these fuzzy systems over the following intervals: [-2,2] for $\hat{x}_1, \hat{x}_3,$ and $\hat{x}_4$; [0.5,2] for $\hat{x}_2$; and [-20,20] for u_2. The corresponding membership functions are selected as in [BOU08d].

The design parameters are set as: $\gamma_1 = 100$, $\gamma_2 = 35$, $\sigma_1 = \sigma_2 = 0.1$, $\lambda_1 = \lambda_2 = 2$, $k_{p1} = k_{p2} = 0.2$, $\epsilon = 0.1$, $\beta_1 = 8$, $\beta_2 = 16$. The initial conditions are taken as: $x(0) = [0.5,\ 1,\ 0,\ 0]^T$, $\theta_{1i}(0) = 0$, $\theta_{2i}(0) = 0$.

The numerical simulation results are presented in Figure 3.2. It is clear from this figure that the system tracks effectively and quickly their respective reference signals, despite the presence of unknown disturbances, uncertain dynamics and unmeasured states. The states are also correctly estimated by the designed observer. In addition, the generated control signals are smooth, bounded and admissible.

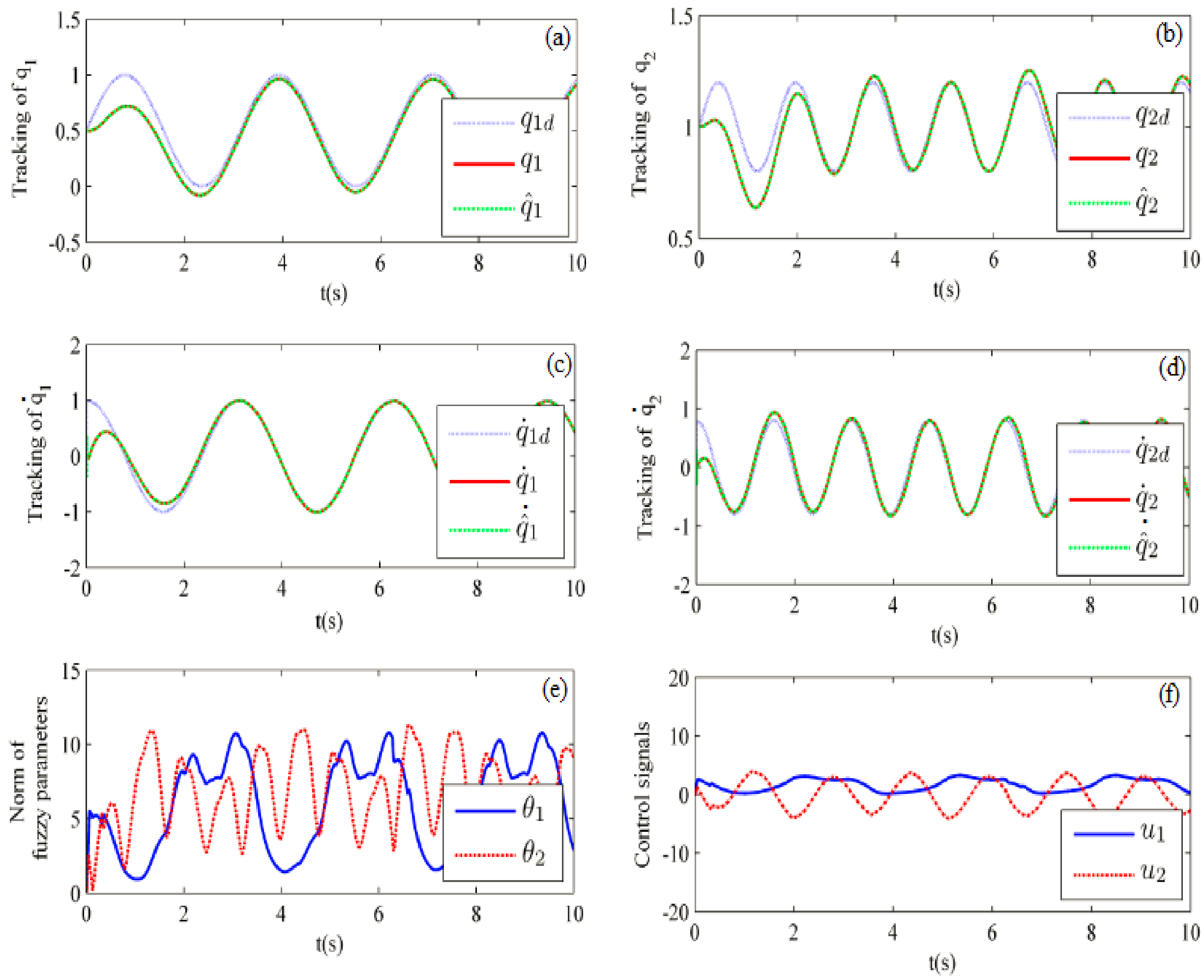

Figure 3.2: *Simulation results for a 2 DOF polar manipulator robot (example 2): (a) Trajectory of* q_{d1}, q_1*and* $\hat{q}_1$. *(b) Trajectory of* q_{d2}, q_2*and* $\hat{q}_2$ *(c) Trajectory of* $\dot{q}_{d1}$, $\dot{q}_1$ *and t* $\dot{\hat{q}}_1$. *(d) Trajectory of* $\dot{q}_{d2}$, $\dot{q}_2$ *and t* $\dot{\hat{q}}_2$, *(e) Norm of fuzzy parameters:* $\|\theta_1\|$ *and* $\|\theta_2\|$,*(f) Control signals:* u_1 *and* u_2.

3.4.3 Example 3:

Consider a system of mass–spring–damper being illustrated in Figure 3.3. Its simulation model is [LIU11]:

$$\begin{cases} M_1\ddot{y}_1 = u_1 - f_{K1}(x) - f_{B1}(x) + f_{K2}(x) + f_{B2}(x) - \\ \qquad f_{C1}(x)+f_{C2}(x) + p_{31}(x,t). \\ M_2\ddot{y}_2 = u_2 - f_{K2}(x) - f_{B1}(x) - f_{C2}(x) + p_{32}(x,t). \end{cases} \tag{3.56}$$

This model can be rewritten equivalently as:

$$\ddot{y} = F_3(x) + G_3(x)u + P_3(x,t), \tag{3.57}$$

with

$$M = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}, G_3(x) = M^{-1},$$

$$F_3(x)M^{-1}\begin{bmatrix} -f_{K1}(x) - f_{B1}(x) + f_{K2}(x) + f_{B2}(x) - f_{C1}(x)+f_{C2}(x) \\ -f_{K2}(x) - f_{B1}(x) - f_{C2}(x) \end{bmatrix}, \text{ and}$$

$$P_3(x,t) = [p_{31}(x,t), \qquad p_{32}(x,t)]^T,$$

where $y = [y_1, y_2]^T$ is the output vector, $x = [y_1, y_2, \dot{y}_1, \dot{y}_2]^T$ are the state vector, u_1 and u_2 are the control inputs, and y_1 and y_2 are the system outputs. $f_{K1}(x) = K_{10}y_1 + \Delta K_1 y_1^3$ and $f_{K2}(x) = K_{10}(y_2 - y_1) + +\Delta K_2(y_2 - y_1)^3$ denote the spring forces, $f_{B1}(x) = 2\dot{y}_1 + 0.2\dot{y}_1^2$ and $f_{B2}(x) = 2.2(\dot{y}_2 - \dot{y}_1) + 0.15(\dot{y}_2 - \dot{y}_1)^2$ denote the friction forces, $f_{C1}(x) = 0.02sgn(\dot{y}_1)$ and $f_{C1}(x) = 0.02sgn(\dot{y}_2 - \dot{y}_1)$ are the coulomb friction forces.

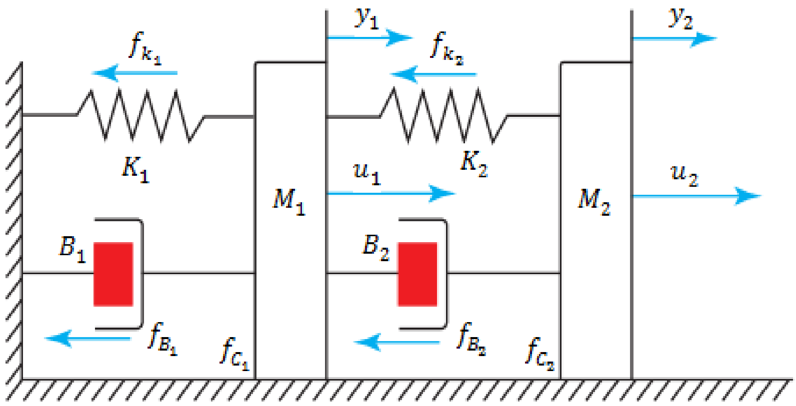

Figure 3.3: *The mass-spring-dumper system.*

The system parameters are given as follows: $M_1 = 0.25\ kg$, $M_2 = 0.2\ kg$, $K_{10} = 1\ N/m$, and $K_{20} = 2\ N/m$.

$p_{31}(x,t)$ and $p_{32}(x,t)$ are selected as a square signal, which is characterized by an amplitude ∓ 1 and a frequency $1/2\pi\ Hz$. Let us choose the reference signals as $y_{d1} = \sin(t)$ and $y_{d2} = \sin(t)$.

To approximate the uncertain nonlinear functions, the presented controller (3.29) uses two adaptive fuzzy systems, namely $\theta_1{}^T\psi_1(\hat{z}_1)$ and $\theta_2{}^T\psi_2(\hat{z}_2)$, whose respective inputs are $\hat{z}_1$=$[\hat{x}^T, u_2]^T$ and $\hat{z}_2 = \hat{x}$. Three fuzzy sets are defined for the each input of these fuzzy systems over the following intervals: [-2, 2] for $\hat{x}_1$ *and* $\hat{x}_2$; [-5, 5] for $\hat{x}_3, \hat{x}_4$; and [-10, 10] for u_2.

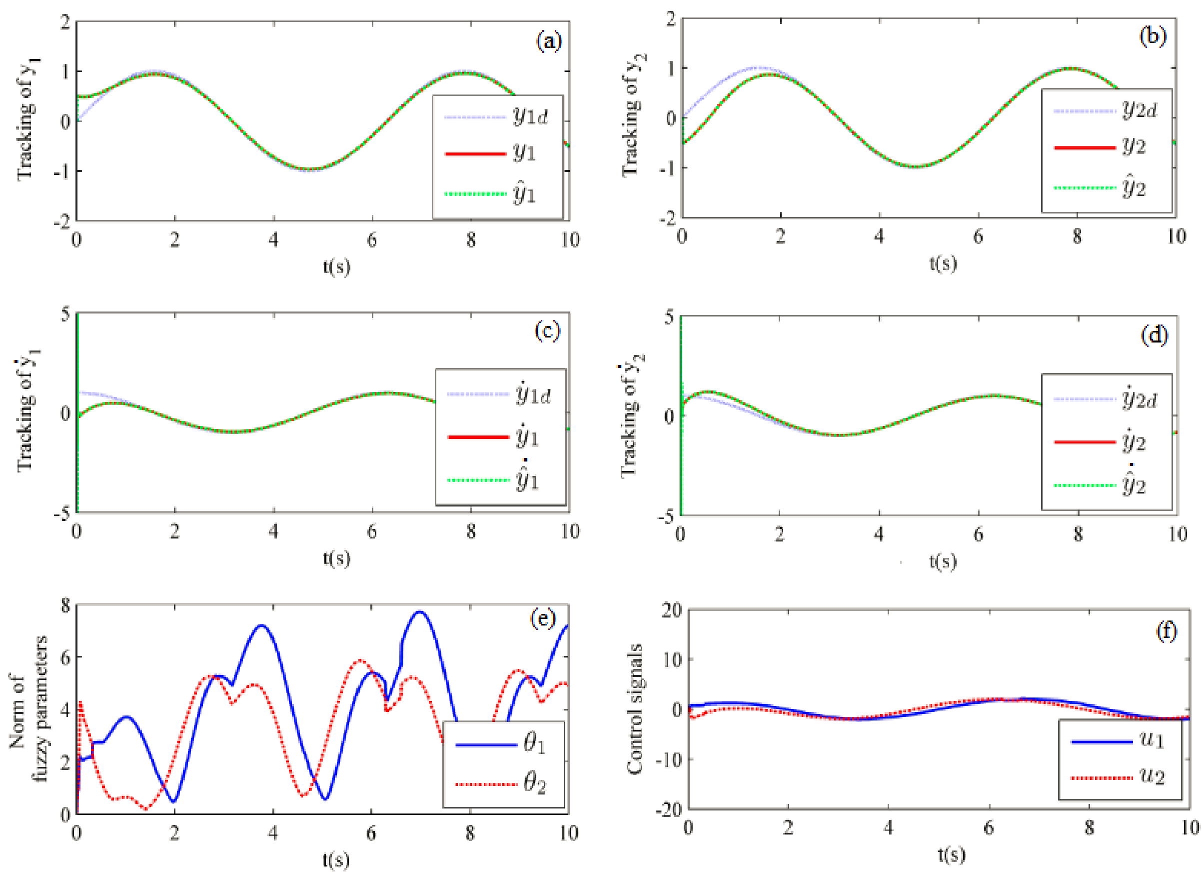

Figure 3.4: *Simulation results for a mass–spring–damper (example 3): (a) Trajectory of* y_{d1}*,* y_1 *and* $\hat{y}_1$ *. (b) Trajectory of* y_{d2}*,* y_2 *and* $\hat{y}_2$ *(c) Trajectory of* $\dot{y}_{d1}$*,* $\dot{y}_1$ *and* $\dot{\hat{y}}_1$*. (d) Trajectory of* $\dot{y}_{d2}$*,* $\dot{y}_2$ *and* $\dot{\hat{y}}_2$*, (e) Norm of fuzzy parameters:* $\|\theta_1\|$ *and* $\|\theta_2\|$*,(f) Control signals:* u_1 *and* u_2*.*

The corresponding membership functions are selected as in [BOU08d]. The design parameters are set as: $\gamma_1 = 100$, $\gamma_2 = 35$, $\sigma_1 = \sigma_2 = 0.1$, $\lambda_1 = \lambda_2 = 2$, $k_{p1} = k_{p2} = 0.2$, $\epsilon = 0.1$, $\beta_1 = 8$, $\beta_2 = 16$. The initial conditions are chosen as: $x(0) = [0.5, -0.5,\ 0, 0]^T$, $\theta_{1i}(0) = 0$, $\theta_{2i}(0) = 0$.

The simulation results are depicted in Figure 3.4. It can be seen from this figure that the system states effectively track their corresponding references signals and also the system states are effectively and quickly estimated by the proposed high-gain observer. The generated controls and the norm of the adaptive fuzzy parameters are bounded.

3.5 Conclusion

In this chapter, we have suggested a novel observer-based adaptive control methodology for a class of uncertain multi-input multi-output nonlinear MIMO systems. These systems under-consideration are characterized by a control gain matrix having non-zero leading principal minors as well as a non-symmetric structure. Adaptive fuzzy systems have been incorporated in the controller to tackle unknown nonlinear functions. A high-gain linear observer has been introduced to indirectly construct unmeasured states. Numerical simulations of three physical systems have been given to illustrate the effectiveness of the proposed control scheme.

Chapter 4

Unified Observer-based Adaptive Fuzzy Control For a Class of MIMO Nonlinear Systems

4.1 Introduction

In this chapter, one will present a novel fuzzy adaptive output-feedback control for a special class of multivariable nonlinear systems with uncertainties and external disturbances. Unlike the previous chapter, here we will design a unified observer to construct the immeasurable states. This unified design frame-work for the observer-based fuzzy adaptive control is valid for many observer types (high-gain observer, non-smooth sliding mode observer, smooth sliding mode observer,…). A PI adaptation law augmented by a $\sigma - modification$ term will be employed to robustly estimate the unknown fuzzy parameters. The uniformly ultimately boundness of all signals in the closed-loop system as well as the observation or tracking errors convergence will be analytically shown using a Lyapunov method.

Taking into consideration the earlier results dealing with multivariable adaptive output-feedback control [SHI15, ARE13, SHA16, HUA14, PEN14, TON05, TON03], the main contributions of this chapter can be listed as follows:

- A unified state observer will be designed to robustly estimate the non-measured system states. In fact, the observer correction term involves a well-defined design function derived from the Lyapunov stability analysis.
- On the basis of the tracking error estimate, a PI adaptation law equipped by the so-called $\sigma - modification$ will be proposed to robustly estimate the unknown fuzzy parameters. by incorporating a low-pass filter in this PI adaptation law, the algebraic-loop problem is sidestepped.

4.2 Notations and problem statement

Consider a class of uncertain nonlinear MIMO systems consisting of N interconnected subsystems. Each of subsystems is supposed to be described by [LIU11]

$$\begin{cases} \dot{x}_{i1} = x_{i2} \\ \quad\vdots \\ \dot{x}_{im_i-1} = x_{im_i} \\ \dot{x}_{im_i} = f_i(\underline{x}) + g_i(\underline{x})u_i + d_i(t,\underline{x}) \\ y_i = x_{i1}\,, \qquad for\ i = 1,\dots,N \end{cases} \tag{4.1}$$

with $\underline{x_i} = [\,x_{i1}, x_{i2}, \dots, x_{im_i}]^T \in R^{m_i}$, $u_i \in R$ and $y_i \in R$ being the state vector, the control input, and the output of the ith subsystem, respectively. $\underline{x} = [\underline{x_1}, \underline{x_2}, \dots, \underline{x_N}]^T$ is the over-all state vector. $f_i(\underline{x})$ is an unknown smooth function, $g_i(\underline{x})$ is an unknown control gain, and $d_i(t, \underline{x})$ stands for the external bounded disturbances for the ith subsystem.

Our main objective is to develop an observer-based fuzzy adaptive control in order to steer the outputs $y_1, \dots, y_N$ close to some specific desired trajectories, $y_{ir}(t), \dots, y_{ir}^{(m_i)}(t), i = 1, \dots, N$, while ensuring that all involved signals in the closed-loop system remain bounded.

Let us make the following mild assumptions.

Assumption 4.1: *The desired trajectories $y_{ir}(t), \dots, y_{ir}^{(m_i)}(t),\ for\ i = 1, \dots, N$, are supposed to be continuous, bounded and measurable.*

Assumption 4.2: *It is required that $g_i(\underline{x}) \neq 0,\ for\ i = 1, \dots, N$. Then, without losing of generality, it is assumed that $0 < g_i(\underline{x}) < g_i^*$, where g_i^* is a positive constant.*

Assumption 4.3: *There exist some positive constants ($d_i^* > 0$) such that the external disturbances are bounded as follows: $|d_i(t, \underline{x})| \leq d_i^*$, for $i = 1, \dots, N$.*

Assumption 4.4: *The state vector $\underline{x_i}$ is not measurable except its first component $y_i = x_{i1}$.*

The reference signal vector $\underline{y_{ir}}$ and the tracking error vector $\underline{e_i}$ are defined as follows

$$\underline{y_{ir}} = [y_{ir}(t), \dot{y}_{ir}(t), \dots, y_{ir}^{(m_i-1)}(t)]^T,$$

$$\underline{e_i} = \underline{x_i} - \underline{y_{ir}} = \left[e_i, \dot{e}_i, \dots, e_i^{(m_i-1)}\right]^T.$$

After adding and subtracting the term $b_i u_i$ to equation (4.1), the tracking error dynamics can be written as follows:

$$e_i^{m_i} = \underline{x}_i^{(m_i)} - y_{ir}^{(m_i)} = [f_i(\underline{x}) + (g_i(\underline{x}) - b_i)u_i] - y_{ir}^{(m_i)} + b_i u_i + d_i(t, \underline{x})$$

$$= \alpha_i(\underline{x}, u_i) - y_{ir}^{(m_i)} + b_i u_i + d_i(t, \underline{x}) \tag{4.2}$$

with $\alpha_i(\underline{x}, u_i) = f_i(\underline{x}) + (g_i(\underline{x}) - b_i)u_i$, where b_i is a positive design constant which should satisfy the following condition:

$$b_i > \frac{g_i(\underline{x})}{2}, \qquad \forall\, X_i \in \Omega_{\underline{x}_i} \tag{4.3}$$

The state space realization of the dynamics (4.2) is

$$\begin{cases} \underline{\dot{e}_i} = A_i \underline{e_i} + B_i \left[\alpha_i(\underline{x}, u_i) - y_{ir}^{(m_i)} + b_i u_i + d_i(t, \underline{x})\right] \\ e_{i1} = C_i^T \underline{e_i} \end{cases} \tag{4.4}$$

where

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{m_i \times m_i} ; \; B_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{m_i \times 1} ; \; C_i = \begin{bmatrix} 1 \\ \vdots \\ 0 \\ 0 \end{bmatrix}_{m_i \times 1} .$$

Since the functions $f_i(\underline{x})$ and $g_i(\underline{x})$ are unknown (i.e. the nonlinearity $\alpha_i(\underline{x}, u_i)$ is unknown) and the state vectors $\underline{x_i}$ are not available for measurement, subsequently one will design:

- *an adaptive fuzzy system in order to approximate these unknown nonlinearities $\alpha_i(\underline{x}, u_i)$, and*
- *a unified state observer to estimate the state vector (or the tracking error vector).*

From equation (4.4), the control input u_i can be determined as

$$u_i = \frac{1}{b_i}[-u_{ai} + v_i] \tag{4.5}$$

with u_{ai} being a fuzzy adaptive term designed to cancel the nonlinear function $\alpha_i(\underline{x}, u_i)$ and $v_i = -K_{ci}^T \underline{e_i} + y_{ir}^{(m_i)}$ being a linear control term employed to adequately stabilize the linearized dynamics.

As $\alpha_i(\underline{x}, u_i)$ is a function of the control input u_i, therefore the input vector of the adaptive fuzzy system should include the control input u_i. However, in this case, the problem of the well-known algebraic-loop can be occurred. How to circumvent this problem will be the objective of the following section.

4.3 Fuzzy approximation of $\boldsymbol{\alpha_i(\underline{x}, u_i)}$

The fuzzy system (1.27) will be used to online model the nonlinearity $\alpha_i(\underline{x}, u_i)$. However, the inputs to this fuzzy system (FS) should be $\underline{x}_i$ and u_i, since $\alpha_i(.)$ is a function of them. In this formulation the FS should be a recurrent one, as the output of the FS u_{ai} is directly feedback into the FS in order to yield the control input u_i. The Figure (4.1) illustrates this algebraic-loop situation. However, if one uses this recurrent configuration, a fixed-point problem should be solved at every time instant and this imposes a heavy computational burden [BOU08c]. To circumvent this problem, one will apply the well-known implicit function theorem [KHA95] to guarantee that u_{ai}^*, satisfying

$$h_i(\underline{x}, v_i, u_{ai}^*) = \alpha_i(\underline{x}, (-u_{ai}^* + v_i)/b_i) - u_{ai}^* = 0, \tag{4.6}$$

is a function only of $\underline{x}_i$ and v_i, therefore a non-recurrent FS can be employed to estimate $\alpha_i(\underline{x}, u_i)$, i.e. the feedback path in Figure (4.1) is not required and therefore the algebraic-loop problem is sidestepped. The following important lemma is introduced to show that u_{ai}^* satisfying (4.6) can be a function of $\underline{x}_i$ and v_i only.

Lemma 4.1 [PAR04]: If the constant b_i satisfies condition (4.3), then there exists a set $\Omega_{\underline{x}_i} \in R^{m_i}$ and a unique u_{ai}^* being a function of $\underline{x}_i$ and v_i only, such that $u_{ai}^*(\underline{x}, v_i)$ satisfies Equation (4.6) for all $(\underline{x}_i, v_i) \in \Omega_{\bar{x}_i} \times R$.

Proof of Lemma 4.1:

Firstly, we show that the solution u_{ai}^* to (4.6) exists. The sufficient condition for the existence of this solution u_{ai}^* is that the mapping $\alpha_i(.)$ is a contraction over the entire input domain, i.e. the following relation should hold [PAR04]:

$$\left|\frac{\partial \alpha_i}{\partial u_{ai}^*}\right| < 1 \tag{4.7}$$

The expression $\left|\frac{\partial \alpha_i}{\partial u_{ai}^*}\right|$ can be determined as follows:

$$\left|\frac{\partial \alpha_i}{\partial u_{ai}^*}\right| = \left|\frac{\partial (f_i + (g_i - b_i)u_i^*)}{\partial u_i^*}\frac{\partial u_i^*}{\partial u_{ai}^*}\right| = \left|(g_i - b_i)\left(\frac{-1}{b_i}\right)\right| = \left|\frac{g_i}{b_i} - 1\right| < 1 \tag{4.8}$$

where $u_i^* = (-u_{ai}^* + v_i)/b_i$. One can effortlessly show that the inequality (4.8) holds, if the condition (4.3) is satisfied.

Secondly, one will prove that the function $\partial h_i(.)/\partial u_{ai}^*$ is non-singular. Differentiating the left-hand side of equation (4.6) with respect to u_{ai}^* yields

$$\begin{aligned}\frac{\partial}{\partial u_{ai}^*} h_i(\underline{x}, v_i, u_{ai}^*) &= \frac{\partial}{\partial u_{ai}^*}\left[\alpha_i(\underline{x}, (-u_{ai}^* + v_i)/b_i) - u_{ai}^*\right] = \frac{\partial}{\partial u_{ai}^*}[f_i + (g_i - b_i)u_i^* - u_{ai}^*] \\ &= \frac{\partial}{\partial u_i^*}[f_i + (g_i - b_i)u_i^*]\frac{\partial u_i^*}{\partial u_{ai}^*} - 1 = (g_i - b_i)\left(\frac{-1}{b_i}\right) - 1 \\ &= -\frac{g_i}{b_i} \qquad (4.9)\end{aligned}$$

which is non-zero as $0 < g_i$. Thus, according to the implicit function theorem [KHA95], there exists a sole solution $u_{ai}^*(\underline{x}, v_i)$ satisfying (4.6) for all $(\underline{x}_i, v_i) \in \Omega_{\underline{x}_i} \times R$.

Lemma 4.1 allows us to use of a non-recurrent FS rather than a recurrent FS to estimate u_{ai}^* which enables the controller to avoid solving a fixed problem at every time instant. Then the input vector of the used FS is $\underline{\eta}_i = [\underline{x}^T, v_i]^T$ whose dimension is $M_i = m_i + 1$.

From (1.33), the fuzzy adaptive system can be described by:

$$u_{ai} = \theta_i^T \psi_i\left(\underline{\eta}_i\right) \qquad (4.10)$$

where $\underline{\eta}_i = [\underline{x}_i^T, v_i]^T$. The function u_{ai}^* satisfying (4.6) can be optimally approximated according to the universal approximation theorem as follows:

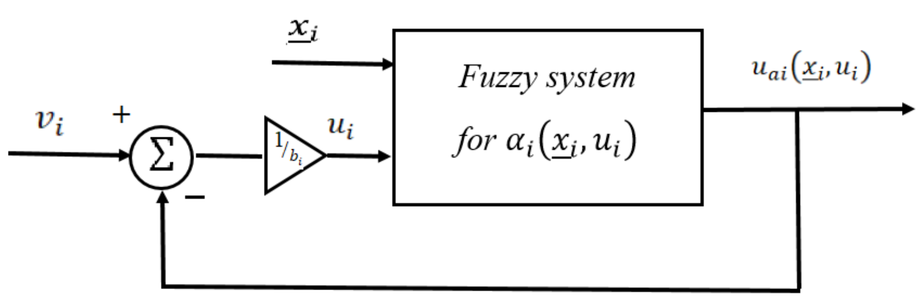

Figure 4.1: *Recurrent Fuzzy System.*

$$u_{ai}^{*} = u_{ai}^{*}\left(\underline{\eta}_i, \theta_i^{*}\right) + \varepsilon_i\left(\underline{\eta}_i\right) = \theta_i^{*T}\psi_i\left(\underline{\eta}_i\right) + \varepsilon_i\left(\underline{\eta}_i\right) \tag{4.11}$$

where θ_i^* is the fuzzy approximation error and θ_i^* is the optimal parameter vector which is assumed to be unknown and constant. Note that the optimal parameter vector θ_i^* is mainly introduced for analysis purposes and its value is not needed when implementing the controller.

Assumption 4.5

a) *The optimal parameter vector satisfies*

$$\|\theta_i^*\| \leq M_{\theta_i} \tag{4.12}$$

where M_{θ_i} is an unknown positive constant.

b) According to the universal approximation theorem, one can made

$$\left|\varepsilon_i\left(\underline{\eta}_i\right)\right| \leq \bar{\varepsilon}_i \tag{4.13}$$

with $\bar{\varepsilon}_i$ *being an unknown positive constant.*

4.4 Design of the observer-based fuzzy adaptive control

4.4.1 Fuzzy adaptive control Design

For brevity of presentation, one gives only the detailed design procedure for the *i*th subsystem.

One can rewrite the state-space realization of the system (4.4) as follows:

$$\begin{cases}\underline{\dot{e}}_i = A_i\underline{e}_i + B_i\left[\left(\alpha_i(\underline{x}, u_i) - \alpha_i(\underline{x}, u_i^*)\right) + \left(\alpha_i(\underline{x}, u_i^*) - u_{ai}^*\right) - y_{ir}^{(m_i)} + (b_i u_i + u_{ai}^*) + d_i(t, \underline{x})\right] \\ e_{i1} = C_i^T\underline{e}_i\end{cases} \tag{4.14}$$

As the control input (4.5) is not implementable (since it depends on the non-measurable states), it can be replaced by:

$$u_i = \frac{1}{b_i}\left[-\hat{u}_{ai} + \hat{v}_i\right] = \frac{1}{b_i}\left[-\theta_i^T\psi_i\left(\underline{\hat{\eta}}_i\right) - K_{ci}^T\underline{\hat{e}}_i + y_{ir}^{(m_i)}\right] \tag{4.15}$$

Considering (4.15) and (4.6), (4.14) can be rewritten as follows:

$$\begin{cases}\underline{\dot{e}}_i = A_i\underline{e}_i + B_i\left[-\theta_i^T\psi_i\left(\underline{\hat{\eta}}_i\right) - K_{ci}^T\underline{\hat{e}}_i + u_{ai}^* + \left(\alpha_i(\underline{x}, u_i) - \alpha_i(\underline{x}, u_i^*)\right) + d_i(t, \underline{x})\right] \\ e_{i1} = C_i^T\underline{e}_i\end{cases} \tag{4.16}$$

Now, let us define the observer error vector as $\underline{\tilde{e}_i} = \underline{e_i} - \underline{\hat{e}_i} = \underline{x_i} - \underline{\hat{x}_i}$. Therefore, by (4.11), the dynamics (4.16) become:

$$\begin{cases} \underline{\dot{e}_i} = (A_i - B_i K_{ci}^T)\underline{e_i} + B_i\left[-\tilde{\theta}_i^T \psi_i\left(\underline{\hat{\eta}}_i\right) + K_{ci}^T \underline{\tilde{e}}_i + w_i\right] \\ e_{i1} = C_i^T \underline{e_i} \end{cases} \tag{4.17}$$

where $\tilde{\theta}_i^T = \theta_i - \theta_i^*$, and $w_i = \left(\alpha_i(\underline{x}, u_i) - \alpha_i(\underline{x}, u_i^*)\right) + d_i(t, \underline{x}) + \varepsilon_i\left(\underline{\eta_i}\right) + \theta_i^{*T}\left[\psi_i\left(\underline{\eta_i}\right) - \psi_i\left(\underline{\hat{\eta}_i}\right)\right]$.

The following lemma is later required in the stability analysis of the closed-loop system.

Lemma 4.2: *There exist some positive constants ρ_{ij}, for $i = 1, \dots, N$, and $j = 1, \dots, 5$, such that:*

$$\left|\alpha_i(\underline{x}, u_i) - \alpha_i(\underline{x}, u_i^*)\right| \le \rho_{i1}\left\|\tilde{\theta}_i\right\| + \rho_{i2}\left\|\underline{\tilde{e}_i}\right\| + \rho_{i3} \tag{4.18}$$

$$|w_i| \le \rho_{i1}\left\|\tilde{\theta}_i\right\| + \rho_{i2}\left\|\underline{\tilde{e}_i}\right\| + \rho_{i4} \tag{4.19}$$

$$\left|-\tilde{\theta}_i^T \psi_i\left(\underline{\hat{\eta}_i}\right) + w_i\right| \le 2\rho_{i1}\left\|\tilde{\theta}_i\right\| + \rho_{i2}\left\|\underline{\tilde{e}_i}\right\| + \rho_{i4} \tag{4.20}$$

$$\left|-\tilde{\theta}_i^T \psi_i\left(\underline{\hat{\eta}_i}\right) + K_{ci}^T \underline{\tilde{e}_i} + w_i\right| \le 2\rho_{i1}\left\|\tilde{\theta}_i\right\| + 2\rho_{i2}\left\|\underline{\tilde{e}_i}\right\| + \rho_{i4} \tag{4.21}$$

Proof of lemma 4.2:

a) The proof of the relation (4.18) is as following:

$$\begin{aligned} \left|\alpha_i(\underline{x}, u_i) - \alpha_i(\underline{x}, u_i^*)\right| &= \left|f_i(\underline{x}) + (g_i(\underline{x}) - b_i)u_i - f_i(\underline{x}) - (g_i(\underline{x}) - b_i)u_i^*\right| \\ &= \left|(g_i(\underline{x}) - b_i)u_i - (g_i(\underline{x}) - b_i)u_i^*\right| \\ &\le \left|g_i(\underline{x}) - b_i\right|\left|u_i - u_i^*\right| \\ &\le b_i\left|u_i - u_i^*\right| \\ &= \left|-\hat{u}_{ai} + \hat{v}_i + u_{ai} - v_i\right| = \left|-\theta_i^T \psi_i\left(\underline{\hat{\eta}_i}\right) - K_{ci}^T \underline{\hat{e}_i} + K_{ci}^T \underline{e_i} + \theta_i^{*T} \psi_i\left(\underline{\eta_i}\right) + \varepsilon_i\left(\underline{\eta}_i\right)\right| \\ &= \left|-\theta_i^T \psi_i\left(\underline{\hat{\eta}}_i\right) + \theta_i^{*T} \psi_i\left(\underline{\eta}_i\right) + \varepsilon_i\left(\underline{\eta}_i\right) + K_{ci}^T \underline{\tilde{e}}_i\right| \\ &= \left|-\tilde{\theta}_i^T \psi_i\left(\underline{\hat{\eta}}_i\right) + \theta_i^{*T}\left[\psi_i\left(\underline{\eta}_i\right) - \psi_i\left(\underline{\hat{\eta}}_i\right)\right] + \varepsilon_i\left(\underline{\eta}_i\right) + K_{ci}^T \underline{\tilde{e}}_i\right| \\ &\le \rho_{i1}\|\tilde{\theta}_i\| + \rho_{i2}\left\|\underline{\tilde{e}}_i\right\| + \rho_{i3} \end{aligned} \tag{4.22}$$

with $\rho_{i1} = sup_t \left\|\psi_i\left(\underline{\hat{\eta}}_i\right)\right\|$, $\rho_{i2} = \|K_{ci}\|$, $\rho_{i3} = M_{\theta_i} sup_t \left\|\psi_i\left(\underline{\hat{\eta}}_i\right) - \psi_i\left(\underline{\eta}_i\right)\right\| + \bar{\varepsilon}_i$, $M_{\theta_i} = \|\theta_i^*\|$.

b) The expression (4.19) is obtained as follows:

$$|w_i| \le \left|\left(\alpha_i(\underline{x}, u_i) - \alpha_i(\underline{x}, u_i^*)\right) + d_i(t, \underline{x}) + \varepsilon_i\left(\underline{\eta}_i\right) + \theta_i^{*T}\left[\psi_i\left(\underline{\eta}_i\right) - \psi_i\left(\underline{\hat{\eta}}_i\right)\right]\right|$$

$$\le |\alpha_i(\underline{x}, u_i) - \alpha_i(\underline{x}, u_i^*)| + |d_i(t, \underline{x})| + \left|\varepsilon_i\left(\underline{\eta}_i\right) + \theta_i^{*T}\left[\psi_i\left(\underline{\eta}_i\right) - \psi_i\left(\underline{\hat{\eta}}_i\right)\right]\right|$$

$$\le \rho_{i1}\|\tilde{\theta}_i\| + \rho_{i2}\|\underline{\tilde{e}}_i\| + \rho_{i3} + d_i^* + \rho_{i3}$$

$$= \rho_{i1}\|\tilde{\theta}_i\| + \rho_{i2}\|\underline{\tilde{e}}_i\| + \rho_{i4} \tag{4.23}$$

where $\rho_{i4} = 2\rho_{i3} + d_i^*$.

c) The term $-\tilde{\theta}_i^T\psi_i\left(\underline{\hat{\eta}}_i\right) + w_i$ can be bounded as:

$$\left|-\tilde{\theta}_i^T\psi_i\left(\underline{\hat{\eta}}_i\right) + w_i\right| \le \left|-\tilde{\theta}_i^T\psi_i\left(\underline{\hat{\eta}}_i\right)\right| + |w_i|$$

$$\le \left|-\tilde{\theta}_i^T\psi_i\left(\underline{\hat{\eta}}_i\right)\right| + \rho_{i1}\|\tilde{\theta}_i\| + \rho_{i2}\|\underline{\tilde{e}}_i\| + \rho_{i4}$$

$$\le 2\rho_{i1}\|\tilde{\theta}_i\| + \rho_{i2}\|\underline{\tilde{e}}_i\| + \rho_{i4}. \tag{4.24}$$

d)

$$\left|-\tilde{\theta}_i^T\psi_i\left(\underline{\hat{\eta}}_i\right) + K_{ci}^T\underline{\tilde{e}}_i + w_i\right| \le \left|\tilde{\theta}_i^T\psi_i\left(\underline{\hat{\eta}}_i\right)\right| + \left|K_{ci}^T\underline{\tilde{e}}_i\right| + |w_i|$$

$$\le \left\|\psi_i\left(\underline{\hat{\eta}}_i\right)\right\| \|\tilde{\theta}_i\| + \|K_{ci}\|\|\underline{\tilde{e}}_i\| + \rho_{i1}\|\tilde{\theta}_i\| + \rho_{i2}\|\underline{\tilde{e}}_i\| + \rho_{i4}$$

$$\le 2\rho_{i1}\|\tilde{\theta}_i\| + 2\rho_{i2}\|\underline{\tilde{e}}_i\| + \rho_{i4}. \tag{4.25}$$

This ends the proof of this Lemma.

Assume that P_i is a positive definite solution of the following matrix equation:

$$A_{ci}^T P_i + P_i A_{ci} = -Q_i \tag{4.26}$$

where $A_{ci} = A_i - B_i K_{ci}^T$ and Q_i is an arbitrary positive definite symmetric matrix.
To update the fuzzy parameters, the following adaptive PI law is used:

$$\dot{\theta}_i + \gamma_{2i}\dot{\delta}_{fi} = -\sigma_i\gamma_{1i}\theta_i + \gamma_{1i}\underline{\hat{e}}_i^T P_i B_i \psi_i\left(\underline{\hat{\eta}}_i\right) \tag{4.27}$$

$$\dot{\delta}_{fi} = -\gamma_{2i}\delta_{fi} + \gamma_{2i}\,\delta_i \tag{4.28}$$

with $\delta_i = \sigma_i \theta_i - \underline{\hat{e}_i}^T P_i B_i \psi_i\left(\underline{\hat{\eta}_i}\right)$, where $\gamma_{1i}, \gamma_{2i}, \sigma_i > 0$ are the design constants.

4.4.2 Unified state observer design

In order to solve the problem of the unavailable states, one proposes a unified error observer in this subsection.

Before giving the observer, one introduces the following interesting notations:

1. Let Δ_{λ_i} be a diagonal matrix defined by

$$\Delta_{\lambda_i} = diag\left[1, \frac{1}{\lambda_i}, \dots, \frac{1}{\lambda_i^{m_i-1}}\right]. \tag{4.29}$$

where $\lambda_i > 1$ is a design parameter.

2. Let S_i be the unique solution of the following algebraic Lyapunov equation [FAR04]

$$S_i + A_i^T S_i + S_i A_i = C_i^T C_i. \tag{4.30}$$

One can show that the solution of (4.30) is symmetric and positive definite.

3. Set $\bar{\xi}_i = \Delta_{\lambda_i}\underline{\xi_i}$, $\forall \underline{\xi_i} = \left[\xi_{i1}, \xi_{i2}, \dots, \xi_{im_i}\right]^T \in R^{m_i}$ and $K_i\left(\underline{\xi_i}\right) = [k_i(\xi_{i1}), 0, \dots, 0]^T \in R^{m_i}$ being vectors of smooth or non-smooth functions satisfying

$$\forall \underline{\xi_i} \in R^{m_i}: \bar{\xi}_i^T K_i\left(\underline{\xi_i}\right) \geq \frac{1}{2} \bar{\xi}_i^T C_i^T C_i \bar{\xi}_i \ . \tag{4.31}$$

To estimate the state variables of the system (4.1), the following unified state observer is designed:

$$\begin{cases} \dot{\underline{\hat{e}_i}} = (A_i - B_i K_{ci}^T)\underline{\hat{e}_i} + \lambda_i \Delta_{\lambda_i}^{-1} S_i^{-1} K_i(\tilde{e}_{i1}) \\ \hat{e}_{i1} = C_i \underline{\hat{e}_i} \end{cases} \tag{4.32}$$

where $\underline{\hat{e}_i}$ is the estimate of the tracking error vector $\underline{e_i}$, $\hat{e}_{i1}$is the estimate of the output tracking error e_{i1}, $\tilde{e}_{i1} = y_i - \hat{y}_i = e_{i1} - \hat{e}_{i1}$ is the output observation error. It should be pointed out that the observer (4.32) is composed of a copy of the dynamics (4.17) together with a correction term $\lambda_i \Delta_{\lambda_i}^{-1} S_i^{-1} K_i(\tilde{e}_{i1})$, where $K_i(\tilde{e}_{i1})$ is selected so that condition (4.31) holds. Depending on the choice of $K_i(\tilde{e}_{i1})$, one can get a high-gain observer or a sliding mode observer.

Let us define the observation error vector as $\underline{\tilde{e}_i} = \underline{e_i} - \underline{\hat{e}_i}$, we obtain

$$\begin{cases} \dot{\underline{\tilde{e}}}_i = A_i\underline{\tilde{e}}_i + B_i\left[-\tilde{\theta}_i^T\psi_i\left(\underline{\hat{\eta}}_i\right) + K_{ci}^T\underline{\tilde{e}}_i + w_i\right] - \lambda_i\Delta_{\lambda_i}^{-1}S_i^{-1}K_i(\tilde{e}_{i1}) \\ \tilde{e}_{i1} = C_i\underline{\tilde{e}}_i \end{cases} \tag{4.33}$$

To simplify the stability analysis, let us define a scaling transformation as follows

$$\underline{z}_i = \Delta_{\lambda_i}\underline{\tilde{e}}_i \tag{4.34}$$

One can easily show that $\underline{z}_i$ has the following special properties

(i) $\|\underline{z}_i\| \leq \|\underline{\tilde{e}}_i\| \leq {\lambda_i}^{m_i-1}\|\underline{z}_i\|$ (4.35)

(ii) $C_i\underline{z}_i = z_{i1} = C_i\underline{\tilde{e}}_i = \tilde{e}_{i1}$ (4.36)

Since $\Delta_{\lambda_i}A_i\Delta_{\lambda_i}^{-1}= \lambda_iA_i$ and taking into account the fact that $K_i(\tilde{e}_{i1}) = K_i(z_{i1})$, the system (4.33) can be rewritten in term of $\underline{z}_i$ as follows

$$\begin{cases} \dot{\underline{z}}_i = \lambda_iA_i\underline{z}_i - \lambda_iS_i^{-1}K_i(z_{i1}) + \Delta_{\lambda_i}B_i\left[-\tilde{\theta}_i^T\psi_i\left(\underline{\hat{\eta}}_i\right) + K_{ci}^T\underline{\tilde{e}}_i + w_i\right] \\ z_{i1} = C_i\underline{z}_i \end{cases} \tag{4.37}$$

The following theorem concludes about the properties and stability of the closed-loop system defined by (4.37) and (4.17).

Theorem 4.1: Consider the system (4.1) with its unified state observer (4.32) and its controller defined by (4.15), (4.27) and (4.28). Suppose that Assumptions 1-5 are valid. Then, one has the following nice properties:

- All involved signals in the closed-loop system are Semi-globally Uniformly Ultimately Bounded (UUB).
- The output tracking error e_{i1} and all other related errors remain in a small but adjustable neighborhood of zero.

Proof of Theorem 4.1: Construct the Lyapunov function candidate as follows

$$V_i = V_{i1} + \beta_i(V_{i2} + V_{i3}) \tag{4.38}$$

with $\beta_i = 1/{\lambda_i}^{2m_i-2}$, and

$$V_{i1} = \underline{z}_i^TS_i\underline{z}_i, \tag{4.39}$$

$$V_{i2} = \frac{1}{2}\underline{e}_i^TP_i\underline{e}_i, \tag{4.40}$$

and

$$V_{i3} = \frac{1}{2\gamma_{i1}}\left(\tilde{\theta}_i + \gamma_{i2}\delta_{fi}\right)^T\left(\tilde{\theta}_i + \gamma_{i2}\delta_{fi}\right) + \frac{1}{2}\delta_{fi}^T\delta_{fi} \tag{4.41}$$

Invoking (4.37) and differentiating V_{i1} give:

$$\dot{V}_{i1} = \underline{z}_i^T S_i \underline{\dot{z}}_i + \underline{\dot{z}}_i^T S_i \underline{z}_i$$

$$= -\lambda_i \underline{z}_i^T S_i \underline{z}_i - 2\lambda_i\left[\underline{z}_i^T K_i(z_{i1}) - 0.5\underline{z}_i^T C_i^T C_i \underline{z}_i\right] + 2\underline{z}_i^T S_i \Delta_{\lambda_i} B_i\left[-\tilde{\theta}_i^T \psi_i\left(\underline{\hat{\eta}}_i\right) + K_{ci}^T \underline{\tilde{e}}_i + w_i\right] \tag{4.42}$$

Due to the special forms of the matrix Δ_{λ_i} and the vector B_i, it can be easily shown that $S_i\Delta_{\lambda_i}B_i = \sqrt{\beta_i}S_iB_i$. If the vector function $K_i(z_{i1})$ is designed so that the condition (4.31) is satisfied, the previous relation becomes:

$$\dot{V}_{i1} = -\lambda_i \underline{z}_i^T S_i \underline{z}_i + 2\sqrt{\beta_i}\underline{z}_i^T S_i B_i\left[-\tilde{\theta}_i^T \psi_i\left(\underline{\hat{\eta}}_i\right) + K_{ci}^T \underline{\tilde{e}}_i + w_i\right] \tag{4.43}$$

Using Lemma 4.2 yields

$$\dot{V}_{i1} \le -\lambda_i \underline{z}_i^T S_i \underline{z}_i + 2\sqrt{\beta_i}\|S_iB_i\|\|\underline{z}_i\|\left|-\tilde{\theta}_i^T \psi_i\left(\underline{\hat{\eta}}_i\right) + K_{ci}^T \underline{\tilde{e}}_i + w_i\right|$$

$$\le -\lambda_i\lambda_{imin}(S_i)\|\underline{z}_i\|^2 + 4\sqrt{\beta_i}\rho_{i1}\|S_iB_i\|\|\underline{z}_i\|\|\tilde{\theta}_i\| + 4\sqrt{\beta_i}\rho_{i2}\|S_iB_i\|\|\underline{z}_i\|\|\underline{\tilde{e}}_i\| + 2\sqrt{\beta_i}\rho_{i4}\|S_iB_i\|\|\underline{z}_i\|$$

$$\le -\lambda_i\lambda_{imin}(S_i)\|\underline{z}_i\|^2 + 2\sqrt{\beta_i}\rho_{i5}\|\underline{z}_i\|\|\tilde{\theta}_i\| + 2\sqrt{\beta_i}\rho_{i6}\|\underline{z}_i\|\|\underline{\tilde{e}}_i\| + 2\sqrt{\beta_i}\rho_{i7}\|\underline{z}_i\| \tag{4.44}$$

where $\rho_{i5} = 2\rho_{i1}\|S_iB_i\|$, $\rho_{i6} = 2\rho_{i2}\|S_iB_i\|$, $\rho_{i7} = \rho_{i4}\|S_iB_i\|$, and $\lambda_{imin}(S_i)$ is the smallest eigenvalue of the matrix S_i.

Differentiating V_{i2} with respect to time along of the solution of (4.17) and using Lemma 4.2 yields

$$\dot{V}_{i2} = \frac{1}{2}\underline{e}_i^T P_i \underline{\dot{e}}_i + \frac{1}{2}\underline{\dot{e}}_i^T P_i \underline{e}_i$$

$$= -\frac{1}{2}\underline{e}_i^T Q_i \underline{e}_i - \underline{\hat{e}}_i^T P_i B_i \tilde{\theta}_i^T \psi_i\left(\underline{\hat{\eta}}_i\right) - \underline{\tilde{e}}_i^T P_i B_i \tilde{\theta}_i^T \psi_i\left(\underline{\hat{\eta}}_i\right) + \underline{e}_i^T P_i B_i\left(K_{ci}^T \underline{\tilde{e}}_i + w_i\right)$$

$$\le -\frac{1}{2}\underline{e}_i^T Q_i \underline{e}_i - \underline{\hat{e}}_i^T P_i B_i \tilde{\theta}_i^T \psi_i\left(\underline{\hat{\eta}}_i\right) + \left|\underline{\tilde{e}}_i^T P_i B_i\right|\left|\tilde{\theta}_i^T \psi_i\left(\underline{\hat{\eta}}_i\right)\right| + \left|\underline{e}_i^T P_i B_i\right|\left|K_{ci}^T \underline{\tilde{e}}_i + w_i\right|$$

$$\leq -\frac{1}{2}\lambda_{imin}(Q_i)\|\underline{e}_i\|^2 - \underline{\hat{e}}_i^T P_i B_i \tilde{\theta}_i^T \psi_i\left(\underline{\hat{\eta}}_i\right) + \rho_{i1}\|P_i B_i\| \left\|\underline{\tilde{e}}_i\right\| \|\tilde{\theta}_i\|$$
$$+ \|P_i B_i\|\|\underline{e}_i\|\left(\rho_{i1}\|\tilde{\theta}_i\| + 2\rho_{i2}\|\underline{\tilde{e}}_i\| + \rho_{i4}\right)$$

$$\leq -\frac{1}{2}\lambda_{imin}(Q_i)\|\underline{e}_i\|^2 - \underline{\hat{e}}_i^T P_i B_i \tilde{\theta}_i^T \psi_i\left(\underline{\hat{\eta}}_i\right) + \rho_{i8}\|\underline{\tilde{e}}_i\|\|\tilde{\theta}_i\| + \rho_{i8}\left\|\underline{e}_i\right\| \|\tilde{\theta}_i\| +$$
$$2\rho_{i9}\|\underline{e}_i\|\|\underline{\tilde{e}}_i\| + \rho_{i10}\|\underline{e}_i\| \tag{4.45}$$

where $\rho_{i8} = \rho_{i1}\|P_i B_i\|$, $\rho_{i9} = \rho_{i2}\|P_i B_i\|$, $\rho_{i10} = \rho_{i4}\|P_i B_i\|$.

Differentiating V_{i3} along of the solutions of (4.27) and (4.28) gives:

$$\dot{V}_{i3} = \frac{1}{\gamma_{i1}}\left(\tilde{\theta}_i + \gamma_{i2}\delta_{fi}\right)^T\left(\dot{\tilde{\theta}}_i + \gamma_{i2}\dot{\delta}_{fi}\right) + \delta_{fi}^T\dot{\delta}_{fi}$$
$$= \left(\tilde{\theta}_i + \gamma_{i2}\delta_{fi}\right)^T(-\delta_i) + \delta_{fi}^T\left(-\gamma_{2i}\delta_{fi} + \gamma_{2i}\,\delta_i\right)$$
$$= -\sigma_i\tilde{\theta}_i^T\theta_i + \underline{\hat{e}}_i^T P_i B_i \tilde{\theta}_i^T \psi_i\left(\underline{\hat{\eta}}_i\right) - \gamma_{i2}\left\|\delta_{fi}\right\|^2 \tag{4.46}$$

Using the following inequality

$$-\sigma_i\tilde{\theta}_i^T\theta_i \leq -\frac{\sigma_i}{2}\left\|\tilde{\theta}_i\right\|^2 + \frac{\sigma_i}{2}\|\theta_i^*\|^2,$$

yields

$$\dot{V}_{i3} \leq -\frac{\sigma_i}{2}\left\|\tilde{\theta}_i\right\|^2 - \gamma_{i2}\left\|\delta_{fi}\right\|^2 + \frac{\sigma_i}{2}\|\theta_i^*\|^2 + \underline{\hat{e}}_i^T P_i B_i \tilde{\theta}_i^T \psi_i\left(\underline{\hat{\eta}}_i\right) \tag{4.47}$$

Then, using the property $\sqrt{\beta_i}\|\underline{\tilde{e}}_i\| \leq \|\underline{z}_i\|$ and from (4.47), (4.45) and (4.44), the time derivative of V_i can be bounded as follows:

$$\dot{V}_i \leq -\lambda_i\lambda_{imin}(S_i)\|\underline{z}_i\|^2 + 2\sqrt{\beta_i}\rho_{i5}\|\underline{z}_i\|\|\tilde{\theta}_i\| + 2\sqrt{\beta_i}\rho_{i6}\|\underline{z}_i\|\|\underline{\tilde{e}}_i\| + 2\sqrt{\beta_i}\rho_{i7}\|\underline{z}_i\| -$$
$$\frac{1}{2}\lambda_{imin}\left(Q_i\right)\beta_i\left\|\underline{e}_i\right\|^2 + \rho_{i8}\beta_i\left\|\underline{\tilde{e}}_i\right\|\|\tilde{\theta}_i\| + \rho_{i8}\beta_i\|\underline{e}_i\|\|\tilde{\theta}_i\| + 2\rho_{i9}\beta_i\|\underline{e}_i\|\|\underline{\tilde{e}}_i\| + \rho_{i10}\beta_i\|\underline{e}_i\| -$$
$$\frac{\beta_i\sigma_i}{2}\left\|\tilde{\theta}_i\right\|^2 - \beta_i\gamma_{i2}\left\|\delta_{fi}\right\|^2 + \frac{\beta_i\sigma_i}{2}\|\theta_i^*\|^2 \tag{4.48}$$

Using the following inequalities:

$$2\sqrt{\beta_i}\rho_{i6}\|\underline{z}_i\|\|\underline{\tilde{e}}_i\| \leq 2\rho_{i6}\|\underline{z}_i\|^2,$$

$$2\sqrt{\beta_i}\rho_{i5}\|\underline{z_i}\|\|\tilde{\theta}_i\| \le \frac{\rho_{i5}^2}{\alpha_{i1}}\|\underline{z_i}\|^2 + \beta_i\alpha_{i1}\|\tilde{\theta}_i\|^2,$$

$$\rho_{i8}\sqrt{\beta_i}\|\underline{z_i}\|\|\tilde{\theta}_i\| \le \frac{\rho_{i8}^2}{4\alpha_{i2}}\|\underline{z_i}\|^2 + \beta_i\alpha_{i2}\|\tilde{\theta}_i\|^2,$$

$$\beta_i\rho_{i8}\|\underline{e_i}\|\|\tilde{\theta}_i\| \le \frac{\beta_i\rho_{i8}^2}{4\alpha_{i3}}\|\underline{e_i}\|^2 + \beta_i\alpha_{i3}\|\tilde{\theta}_i\|^2,$$

$$2\rho_{i9}\sqrt{\beta_i}\|\underline{z_i}\|\|\underline{e_i}\| \le \frac{\rho_{i9}^2}{\alpha_{i4}}\|\underline{z_i}\|^2 + \beta_i\alpha_{i4}\|\underline{e_i}\|^2,$$

$$2\sqrt{\beta_i}\rho_{i7}\|\underline{z_i}\| \le \frac{\rho_{i7}^2}{\alpha_{i5}}\|\underline{z_i}\|^2 + \beta_i\alpha_{i5},$$

$$\beta_i\rho_{i10}\|\underline{e_i}\| \le \frac{\beta_i\rho_{i10}^2}{4\alpha_{i6}}\|\underline{e_i}\|^2 + \beta_i\alpha_{i6},$$

where $\alpha_{ij} > 0$, for $j = 1, \dots, 6$, one gets finally:

$$\dot{V}_i \le -\left(\lambda_i\lambda_{imin}(S_i) - \bar{\lambda}_i\right)\|\underline{z_i}\|^2 - 0.5\beta_i(\sigma_i - \bar{\sigma}_i)\|\tilde{\theta}_i\|^2 - \beta_i\gamma_{i2}\|\delta_{fi}\|^2 - 0.5\beta_i\left(\lambda_{imin}(Q_i) - \bar{\lambda}_{Q_i}\right)\|\underline{e_i}\|^2 + \beta_i\bar{\mu}_i \quad (4.49)$$

Or equivalently

$$\dot{V}_i \le -\beta_i\left(\lambda_i\lambda_{imin}(S_i) - \bar{\lambda}_i\right)\|\underline{\tilde{e}_i}\|^2 - 0.5\beta_i(\sigma_i - \bar{\sigma}_i)\|\tilde{\theta}_i\|^2 - \beta_i\gamma_{i2}\|\delta_{fi}\|^2 - 0.5\beta_i\left(\lambda_{imin}(Q_i) - \bar{\lambda}_{Q_i}\right)\|\underline{e_i}\|^2 + \beta_i\bar{\mu}_i \quad (4.50)$$

where $\bar{\lambda}_i = 2\rho_{i6} + \frac{\rho_{i5}^2}{\alpha_{i1}} + \frac{\rho_{i8}^2}{4\alpha_{i2}} + \frac{\rho_{i9}^2}{\alpha_{i4}} + \frac{\rho_{i7}^2}{\alpha_{i5}}$, $\bar{\lambda}_{Q_i} = \frac{\rho_{i8}^2}{2\alpha_{i3}} + \frac{\rho_{i10}^2}{2\alpha_{i6}} + 2\alpha_{i4}$, $\bar{\sigma}_i = 2(\alpha_{i1} + \alpha_{i2} + \alpha_{i3})$ and $\bar{\mu}_i = \frac{\sigma_i}{2}\|\theta_i^*\|^2 + \alpha_{i5} + \alpha_{i6}$.

By selecting $\lambda_i > \bar{\lambda}_i/\lambda_{imin}(S_i)$, $\lambda_{imin}(Q_i) > \bar{\lambda}_{Q_i}$ and $\sigma_i > \bar{\sigma}_i$, we can guarantee that $\dot{V}_i$ is negative as long as $\underline{\tilde{e}_i}$ is outside the compact set:

$$\Omega_{\underline{\tilde{e}_i}} = \left\{\underline{\tilde{e}_i} \middle| \|\underline{\tilde{e}_i}\| \le \sqrt{\frac{\bar{\mu}_i}{\lambda_i\lambda_{imin}(S_i) - \bar{\lambda}_i}}\right\} \quad (4.51)$$

According to a standard Lyapunov theorem [IOA96], one can conclude that $\underline{\tilde{e}}_i$ is bounded and will converge to $\Omega_{\underline{\tilde{e}}_i}$. Moreover, the radius of this compact set can be made arbitrary small if λ_i is selected to be sufficiently large.

Similarly, the signal $\underline{e}_i$ is bounded and will converge to a set $\Omega_{\underline{e}_i}$ defined by

$$\Omega_{\underline{e}_i} = \left\{ \underline{e}_i \middle| \|\underline{e}_i\| \leq \sqrt{\frac{2\bar{\mu}_i}{\lambda_{imin}(Q_i) - \bar{\lambda}_Q}} \right\} \tag{4.52}$$

whose radius can be made also as small as possible, if $\lambda_{imin}(Q_i)$ is taken sufficiently large. The parameter estimation error vector $\tilde{\theta}_i$ is also bounded and converges to a set $\Omega_{\tilde{\theta}_i}$ which is defined as

$$\Omega_{\tilde{\theta}_i} = \left\{ \tilde{\theta}_i \middle| \|\tilde{\theta}_i\| \leq \sqrt{\frac{2\bar{\mu}_i}{\sigma_i - \bar{\sigma}_i}} \right\} \tag{4.53}$$

The filtered signals δ_{fi} also remain to be bounded and will converge to a set $\Omega_{\delta_{fi}}$ defined by:

$$\Omega_{\delta_{fi}} = \left\{ \delta_{fi} \middle| \|\delta_{fi}\| \leq \sqrt{\frac{\bar{\mu}_i}{\gamma_{i2}}} \right\} \tag{4.54}$$

It is clear that the boundedness of the other signals (as e.g. $\underline{\hat{e}}_i, u_i$) can be established from that of δ_{fi}, $\tilde{\theta}_i$, $\underline{\tilde{e}}_i$ and $\underline{e}_i$. This ends the proof of the theorem.

Remark 4.1: *The choice of the expression of $K_i(\tilde{e}_{i1})$ which satisfies condition (4.31), can result in different types of observers such as:*

1. *A high-gain observer: $K_{iHG} = C_i^T\left(C_i\underline{\tilde{e}}_i\right)$ [SES00, TON02].*
2. *A smooth sliding-mode observer: $K_{iSM} = l_iC_i^T\left(C_iTanh\left(k_{i0}\underline{\tilde{e}}_i\right)\right)$, $K_{iSM} = C_i^T\left(C_i\underline{\tilde{e}}_i\right) + l_iC_i^T\left(C_iTanh\left(k_{i0}\underline{\tilde{e}}_i\right)\right)$, $K_{iSM} = C_i^T\left(C_i\underline{\tilde{e}}_i\right) + l_iC_i^T\left(C_iSat\left(k_{i0}\underline{\tilde{e}}_i\right)\right)$, $K_{iSM} = l_iC_i^T\left(C_i\underline{\tilde{e}}_i/\left(\varepsilon_{i0} + \left|C_i\underline{\tilde{e}}_i\right|\right)\right)$ [FIL03].*
3. *A non-smooth sliding-mode observer: $K_{iSM} = C_i^T\left(C_i\underline{\tilde{e}}_i\right) + l_i\left(\left|C_i\underline{\tilde{e}}_i\right|\right)^{p_i}C_i^TC_iSign(\underline{\tilde{e}}_i)$, $K_{iSM} = l_i\left(\left|C_i\underline{\tilde{e}}_i\right|\right)^{0.5}C_i^TC_iSign(\underline{\tilde{e}}_i)$ [JIA02, CAO00].*

where k_{i0}, ε_{i0} and p_i are positive real constants.

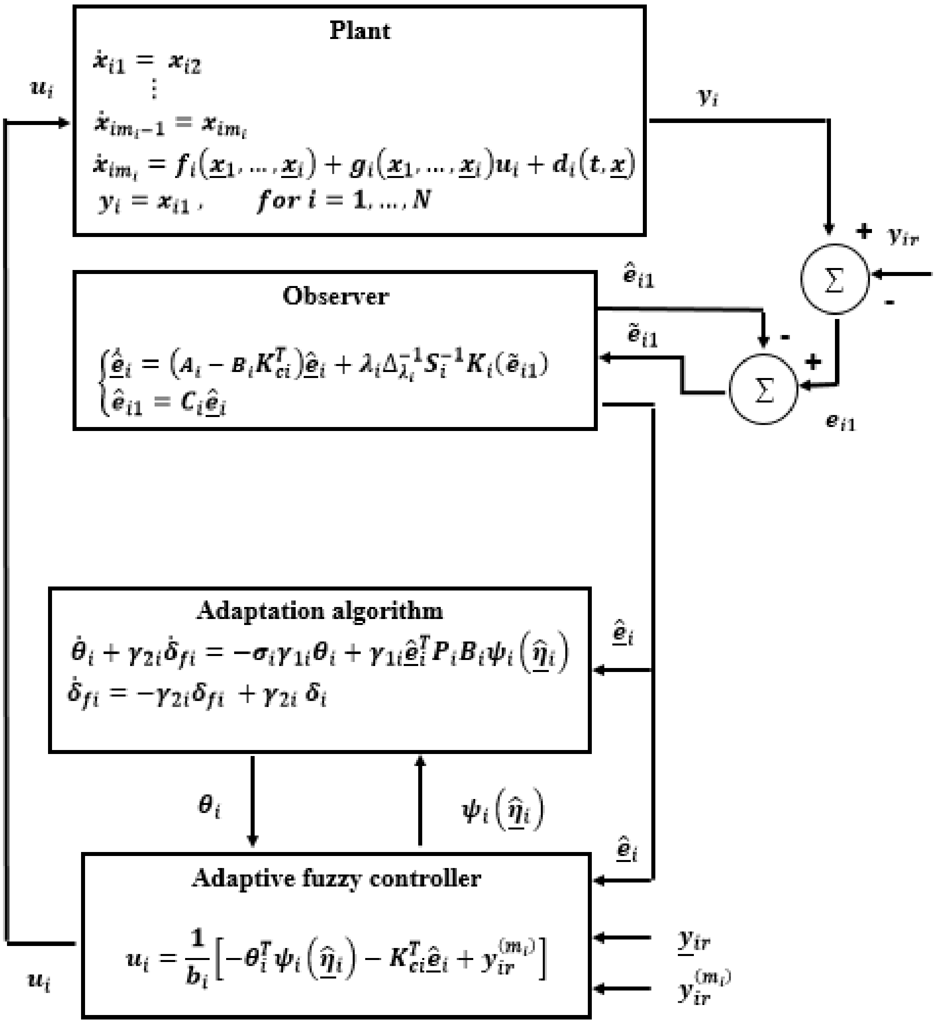

Figure 4.2: *Unified Observer-based Adaptive Fuzzy Control*

4.5 Simulation results

To demonstrate the efficiency of the proposed control scheme, two numerical simulation examples are performed using two nonlinear multivariable systems.

4.5.1 Example 1

Consider again the mass-spring-damper system presented in section 3.4.2 (see Figure 3.3). By choosing: $x_{11} = y_1, x_{12} = \dot{y}_1, x_{21} = y_2, x_{22} = \dot{y}_2,$ we can rewrite the dynamical equation of this MIMO system as follows:

$$\begin{cases}\dot{x}_{11} = x_{12} \\ \dot{x}_{12} = \frac{1}{M_1}u_1 + \frac{1}{M_1}[-f_{K1}(x) - f_{B1}(x) + f_{K2}(x) + f_{B2}(x) - f_{C1}(x) + f_{C2}(x) + d_1 \\ y_1 = x_{11} \\ \dot{x}_{21} = x_{22} \\ \dot{x}_{12} = \frac{1}{M_2}u_2 + \frac{1}{M_2}[-f_{K2}(x) - f_{B1}(x) - f_{C2}(x) + d_2 \\ y_2 = x_{21}\end{cases} \tag{4.55}$$

All parameters and functions of this model are defined as in section 3.4.2.

With loss of generality, it is assumed that the external disturbances $d_i(t)$ is a square wave having an amplitude ∓ 1 with a period of 2π (s). The control objective is to force the system to track the references as $y_{r1} = \sin(t)$ and $y_{r2} = \sin(t)$, under condition that only the system outputs y_i are measurable.

From Remark 4.1, for the proposed fuzzy adaptive controller (4.15), one designs the following sliding mode observer to estimate the tracking errors' vector.

$$\begin{cases}\dot{\underline{\hat{e}}}_i = (A_i - B_i K_{ci}^T)\underline{\hat{e}_i} + \lambda_i \Delta_{\lambda_i}^{-1} S_i^{-1} K_{iSM}(\tilde{e}_{i1}) \\ \hat{e}_{i1} = C_i \underline{\hat{e}_i}\end{cases} \tag{4.56}$$

where $K_{iSM} = l_i C_i^T \left(C_i Tanh\left(k_{i0}\underline{\tilde{e}_i}\right)\right)$.

To approximate the uncertain nonlinear functions, the presented controller (4.15) uses two adaptive fuzzy systems, namely $\theta_1{}^T\psi_1\left(\underline{\hat{\eta}}_1\right)$ and $\theta_2{}^T\psi_2\left(\underline{\hat{\eta}}_2\right)$, whose respective inputs are $\underline{\hat{\eta}}_1 = \left[\underline{\hat{x}}_1^T, \hat{v}_1\right]^T$ and $\underline{\hat{\eta}}_2 = \left[\underline{\hat{x}}_1^T, \hat{v}_2\right]^T$. Three fuzzy sets are defined for each input of these fuzzy systems over the following intervals: [-2, 2] for $\underline{\hat{x}}_i$, and [-10, 10] for $\hat{v}_i$. The corresponding membership functions are selected as in [BOU08d].

The design parameters are set as: $\gamma_{11} = \gamma_{21} = 50$, $\gamma_{12} = \gamma_{22} = 20$, $\sigma_1 = \sigma_2 = 2, b_1 = b_2 = 1$, , $K_{ci}^T = [100,20]$, $\lambda_1 = \lambda_2 = 10$, $k_{01} = k_{02} = 50$, $l_1 = l_2 = 0.1$. The initial conditions are selected as: $\underline{x}_i(0) = [0.5,\ 0, 0.5,\ 0]^T$, $\underline{\hat{e}}_i(0) = [-0.3, -1, -0.8, -1]^T$, $\theta_{1i}(0) = \theta_{2i}(0) = \delta_{f1}(0) = \delta_{f2}(0) = 0$.

Figure 4.3 displays the simulation results obtained by using the proposed fuzzy adaptive controller. It is clear from Figures 4.3(a)-(d) that the system states track well theirs corresponding reference signals and the sliding-mode observer adequately constructs the system states. Figure 4.3(e) illustrates well the boundedness of the norm of the fuzzy adaptive parameters. Figure 4.3(f) shows that the applied control signals are bounded, smooth and admissible.

4.5.2 Example 2

Consider again the 2 DOF polar manipulator robot presented in section 2.4.1 (see Figure 2.1).

With loss of generality, it is assumed that the external disturbances $d(t)$ is a square wave having an amplitude ∓ 1 with a period of 2π (s). The control objective is to force the system to track the reference signals as $y_{r1} = 0.5 + 0.5\sin(2t)$ and $y_{r2} = 1 + 0.2\sin(4t)$, under condition

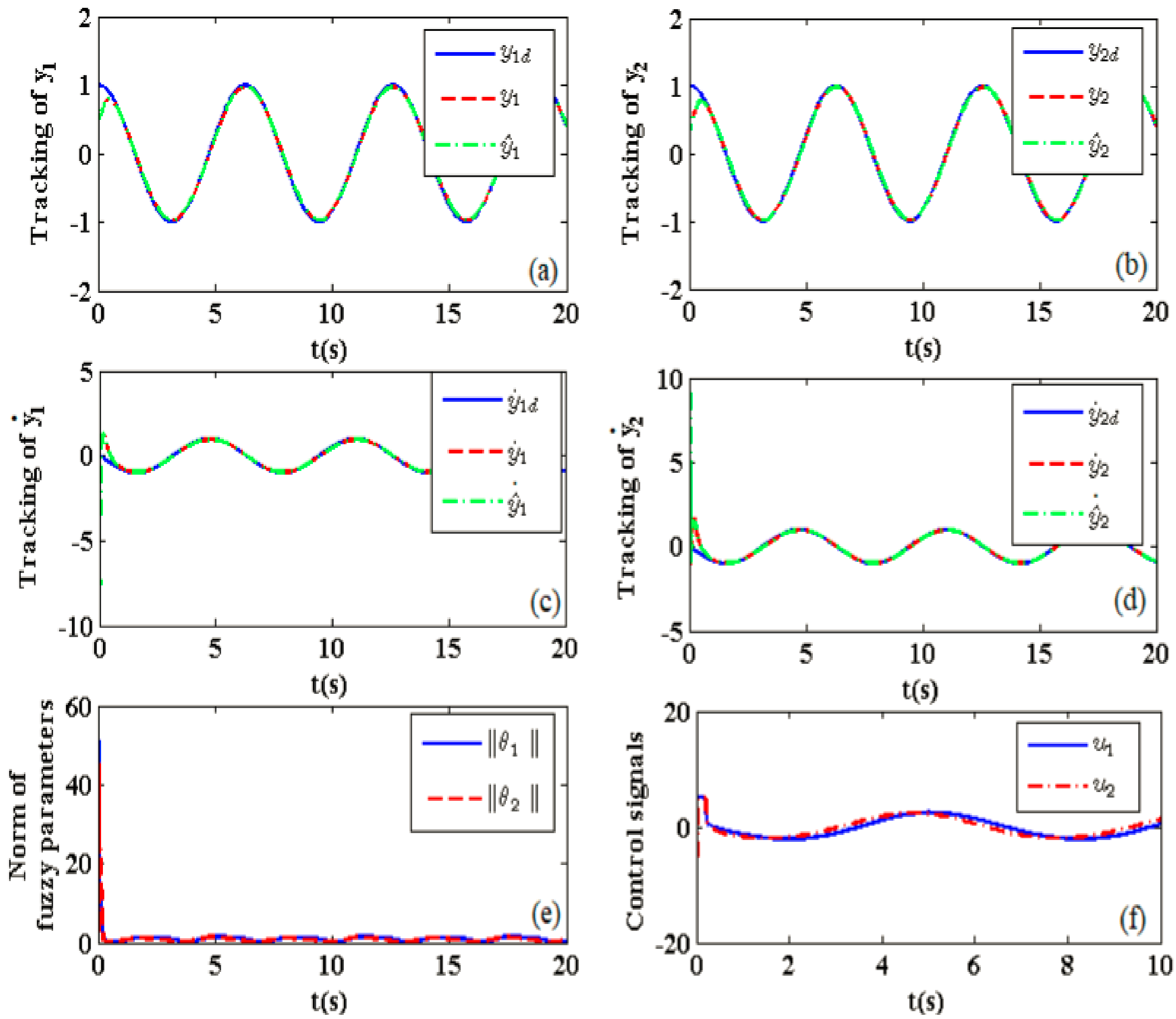

Figure 4.3: *Simulation results for a mass–spring–damper system (example 1): (a) Trajectory of* y_{d1}, y_1 *and* $\hat{y}_1$. *(b) Trajectory of* y_{d2}, y_2 *and* $\hat{y}_2$ *(c) Trajectory of* $\dot{y}_{d1}$, $\dot{y}_1$ *and* $\dot{\hat{y}}_1$. *(d) Trajectory of* $\dot{y}_{d2}$, $\dot{y}_2$ *and* $\dot{\hat{y}}_2$, *(e) Norm of fuzzy parameters:* $\|\theta_1\|$ *and* $\|\theta_2\|$,*(f) Control signals:* u_1 *and* u_2.

that only the system outputs, y_1 and y_2, are measurable. Again, one designs the sliding mode observer (4.56) to estimate the tracking errors' vector.

To approximate the uncertain nonlinear functions, the presented controller (4.15) uses two adaptive fuzzy systems, namely $\theta_1{}^T\psi_1\left(\underline{\hat{\eta}}_1\right)$ and $\theta_2{}^T\psi_2\left(\underline{\hat{\eta}}_2\right)$, whose respective inputs are $\underline{\hat{\eta}}_1 = \left[\underline{\hat{x}}_1^T, \hat{v}_1\right]^T$ and $\underline{\hat{\eta}}_2 = \left[\underline{\hat{x}}_1^T, \hat{v}_2\right]^T$. Three fuzzy sets are defined for each input of these fuzzy systems over the following intervals: [-2, 2] for $\underline{\hat{x}}_i$, and [-10, 10] for $\hat{v}_i$. The corresponding membership functions are selected as in [BOU08d].

The design parameters are set as: $\gamma_{11} = \gamma_{21} = 50,\ \gamma_{12} = \gamma_{22} = 20,\quad \sigma_1 = \sigma_2 = 2, b_1 = b_2 = 1, K_{ci}^T = [100,20],\quad \lambda_1 = \lambda_2 = 10,\quad k_{01} = k_{02} = 50$. The initial conditions are selected as: $\underline{x}_i(0) = [0.5,\ 0, 0.5,\ 0]^T,\quad \underline{\hat{e}}_i(0) = [-0.3, -1, -0.8, -1]^T,\quad \theta_{1i}(0) = \quad \theta_{2i}(0) = \delta_{f1}(0) = \delta_{f2}(0) = 0$.

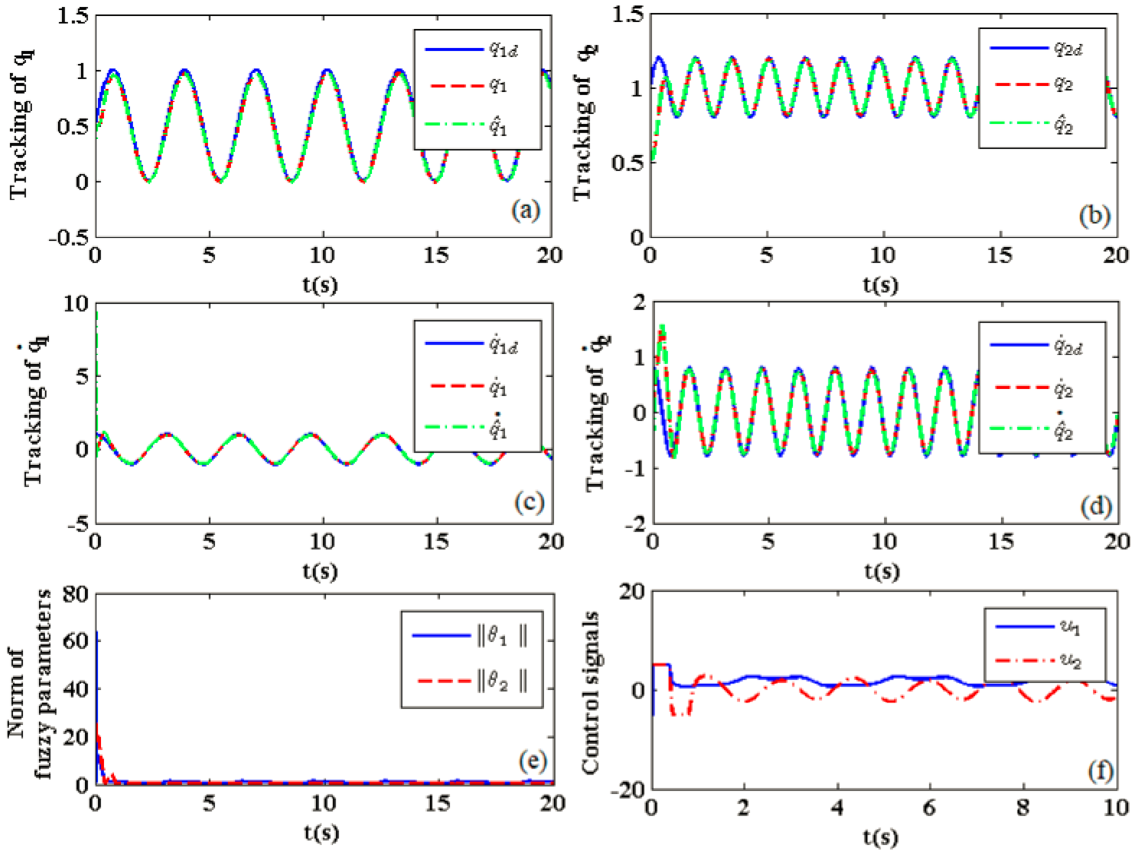

Figure 4.4: *Simulation results for a 2 DOF polar manipulator robot (example 2): (a) Trajectory of $q_{d1}, q_1$and $\hat{q}_1$. (b) Trajectory of $q_{d2}, q_2$and $\hat{q}_2$ (c) Trajectory of $\dot{q}_{d1}, \dot{q}_1$ and t $\dot{\hat{q}}_1$. (d) Trajectory of $\dot{q}_{d2}, \dot{q}_2$ and t $\dot{\hat{q}}_2$, (e) Norm of fuzzy parameters: $\|\theta_1\|$ and $\|\theta_2\|$,(f) Control signals: u_1 and u_2.*

Simulation results are presented in Figure 4.4. From Figures 4.4(a)-(d), it is can be seen that the system states and these of the observer track the imposed reference trajectories effectively

and with precision even in the presence of disturbances and unknown dynamics. Figure 4.4(e) illustates the boundedness of the norm of the fuzzy parameters. And finally, the Figure 4.4(f) confirms the theoretical analysis concerning the boundedness of the control signals generated.

4.4 Conclusion

In this chapter, a unified approach for designing fuzzy adaptive output-feedback controller for a class of uncertain multivariable nonlinear systems has been presented. In this adaptive control scheme, neither differentiation of the system output nor exact knowledge of nonlinearities are required. Indeed, a unified observer has been designed to indirectly construct the state vector and adaptive fuzzy systems have been used to online model the unknown dynamics. On the basis of the tracking error estimate, a PI adaptation law equipped by the so-called $\sigma - modification$ has been proposed for estimating the unknown fuzzy parameters. A Lyapunov's approach has been used to make conclusions about the practical stability of the closed-loop system. The performances of this proposed control scheme have been numerically demonstrated in a realistic simulation framework involving two nonlinear system control problems.

General conclusion

The work of this thesis focuses on two major themes:

- ✓ Design of a fuzzy adaptive state-feedback control scheme for uncertain MIMO nonlinear systems. The used fuzzy systems have been updated using a novel PI law, which provides fast parameters update and hence fast convergence of the tracking errors.
- ✓ Development of two output-feedback adaptive fuzzy controllers for a class of uncertain multivariable nonlinear systems. In these control schemes, high-gain observers and Unified observers have been constructed to estimate the immeasurable state variables.

In the first chapter, we have exhibited the basic notions of fuzzy systems, adaptive control and high-gain observers. Also, we have presented some mathematical background necessary for the design of the control schemes developed in this thesis.

In the second chapter, we have considered the tracking control problem for a class of multivariable nonlinear systems. This class has been assumed already in (or to be transformable to) Brunovsky canonical form with available states but with uncertain dynamics. The adaptive fuzzy systems have been employed to online learn unknown dynamics. The adaptive parameters' convergence as well as the tracking performances have been enhanced via the designing of an adaptation PI law. To facilitate the control design procedure and stability analysis, a matrix factorization lemma has been exploited. The later consists to decompose the control gain matrix into a symmetric positive-definite matrix, a diagonal matrix with diagonal entries +1 or -1 and a unity upper triangular matrix. By using this useful matrix factorization, one does not need that the control gain matrix is necessarily symmetrical or with a definite sign. To prove the stability of the overall closed-loop system and to design the update laws, a Lyapunov approach has been used. The viability and the efficiency of the obtained fundamental results are clearly illustrated through a numerical simulation involving the usual benchmark examples of the adaptive control community. Finally, a comparative study has been also conducted to study the effect or the contribution of each term added in the main control law or in the proposed adaptation law.

In the third chapter, we have suggested a novel observer-based adaptive control methodology for a class of uncertain MIMO nonlinear mechanical systems. Just like the previous chapter, these systems under-consideration have been supposed to be with a control gain matrix having non-zero leading principal minors as well as a non-symmetric structure. But, unlike the previous class, the proposed controller has been designed under the constraint that only system outputs are available for measurement. Hence, it is needed to design a state observer to construct the unmeasured states. Adaptive fuzzy systems have been incorporated in the controller to tackle unknown nonlinear functions. By using a Lyapunov analysis method, it is proven that all the signals in the closed-loop system are UUB and the systems outputs practically track their corresponding imposed reference signals. Extensive simulation results have revealed that the proposed adaptive control strategy is promising and it provides stable and high control performance for nonlinear processes.

In the last chapter, a unified observer-based fuzzy adaptive output-feedback controller for a special class of uncertain multivariable nonlinear systems has been presented. On the basis of the tracking errors' estimates, and without resorting to the so-called strictly-Positive Real (SPR) condition, a PI law augmented by a $\sigma - modification$ for updating the fuzzy adjustable parameters has been proposed. Then, a unified observer has been employed to estimate the tracking error vectors. Indeed, the observer corrector term involves a well-defined design function which has been shown to be verified by the commonly used observers, namely HG observer, (smooth or non-smooth) sliding mode observers,…. A Lyapunov analysis approach has been used to guarantee an UUB property of the observation and tracking errors, as well as of all other signals involved in the closed-loop system.

Although the proposed fundamental results answer to multiple problems, they remain perfectible and open the way to new axes of research. Among the directions considered promising, we mainly retain:

- Design of an adaptive fuzzy control based on a unified observer for a class of MIMO nonlinear fractional-order systems

- Development of output-feedback adaptive fuzzy controllers for MIMO nonlinear systems with non-smooth actuator nonlinearities and/or state constraints.

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II. International Conferences with Acts and Reading Committee:

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