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par

Hesna Delloul

Thème

Validation d'un nouveau protocole de correction d'erreur quantique

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Devant le Jury :

Président :	A. Bounames	Prof.	Univ.Jijel
Rapporteur :	T. Boudjedaa	Prof.	Univ.Jijel
Examinateur :	Kh. Khalfaoui	MCA	Univ.Jijel
Invité :	N. Ferkous	MCA	Univ.Jijel

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Hesna

Chapitre 1

Intoduction générale

Le phénomène de superposition et la mesure, et récemment l'intrication quantique occupe une place particulière et même des fois singulière dans la nouvelle branche de l'information quantique qui les exploite à outrance par leurs effets subtils et étranges dans les domaines tels que l'informatique quantique et la cryptographie quantique. L'informatique quantique est une généralisation de l'informatique classique basée sur les principes de la mécanique quantique où le bit classique est remplacé par ce qu'on appelle un bit quantique ou qubit régi par ces principes. De ce fait, convertir et chercher des nouveaux algorithmes (quantiques) pour la nouvelle technologie qui est l'ordinateur quantique semble primondiale et importante comme première étape de ce développement. Un calcul quantique est un traitement en cascade (ordre en "temps") d'opérations quantiques, en général élémentaires, dites aussi portes quantiques par analogie aux portes logiques. Tout comme pour les circuits de l'informatique classique, un circuit quantique est une combinaison de ces portes élémentaires établies dans un ordre bien défini. En information quantique, l'interaction du système quantique avec son environnement est inévitable et par conséquent l'introduction d'erreur, dans ce cas est un bruit de type (X,Y,Z), sur le qubit dite Erreur quantique. Il est donc naturel et indispensable de corriger ce type d'erreurs. La correction d'erreurs quantiques est une tâche ardue et non évidente méme pour, par exemple, le cas d'un seul qubit. La correction d'erreurs quantiques est un élément essentiel du calcul de tolérance aux pannes qui doit gérer non seulement les erreurs touchant les informations stockées, mais aussi celles provoquées par l'application des portes quantiques, la préparation de nouveaux états ainsi que le processus de mesure. Dans ce qui suit, nous allons refaire tous les calculs d'un nouveau protocole de correction d'Erreur quantique proposé par la plate-forme de

M. Kh. Khalfaoui et nous établissons toutes les propriétés connues des stabilisateurs de Pauli et par la même occasion nous étudions quelques propriétés d'intrication du circuits d'encodage donné par la plate-forme.

Ce mémoire se compose de cinq chapitres :

• Chapitre 1 : Ce chapitre est une introduction sur l'information quantique et les corrections d'erreurs.

• Chapitre 2 : Dans ce chapitre, nous présentons les codes stabilisateurs (un code à 9 qubits équivalent à celui de Shor, un autre à 7 qubits et enfin un dernier à 5 qubits).

• Chapitre 3 : Dans ce chapitre, nous présentons la solution proposée. Nous détaillons le processus d'encodage ainsi que la procédure dedétection d'erreurs.

• Chapitre 4 : Dans ce chapitre, nous présentons une technique géommétrique pour étudier le phénomène d'intrication et nous l'avons appliquée au cas de notre circuit d'encodage.

Enfin, nous terminons par une conclusion générale qui résume notre travail.

Chapitre 2

Codes stabilisateurs

Ces dernières années, des évolutions importantes ont eu lieu et qui semblent prometteuses au niveau du codage de l'information quantique notamment dans le domaine de la correction d'erreurs. Un grand travail théorique a donné naissance à des nouvelles familles de codes. Généralement, les solutions proposées utilisent la redondance d'informations comme outil de détection. C'est une adaptation des correcteurs classiques basée sur l'intrication dans l'étape d'encodage.

Ce chapitre est dédié à la présentation des codes stabilisateurs. A l'heure actuelle, trois solutions ont été proposées et la différence réside dans le nombre de qubits supplémentaire ajoutés dans l'étape d'encodage. Dans la première variante c'est un codage à 9 qubits, la seconde 7 qubits et enfin la plus optimale à 5 qubits.

2.1 L'idée de base

Afin d'introduire les codes stabilisateurs, supposons que les erreurs possibles sont de type X. Soit $|\Psi>$ un qubit quelconque défini par :

$$|\Psi >= \alpha|0 > +\beta|1 > \tag{2.1}$$

Pour protéger ce qubit, on commence d'abord par une étape d'encodage basée sur l'intrication de deux qubits supplémentaires. Les états correspondants à ce niveau sont les suivants :

▶L'état initial :

$$\alpha|000 > +\beta|100 >$$

▶L'application du CNot (1,2) donne l'état :

$$\alpha|000> + \beta|110>$$

▶L'application du CNot (1,3) donne l'état :

$$|\Psi_c>= \alpha|000> + \beta|111>$$

En cours d'un traitement quantique et en cas ou ce qubit codé n'est pas modifié volontairement, il peut se trouver dans l'un des états suivant :

$$\begin{aligned}\alpha|000 &> +\beta|111> \\ X_1[\alpha|000 &> +\beta|111>] = \alpha|100> + \beta|011> \\ X_2[\alpha|000 &> +\beta|111>] = \alpha|010> + \beta|101> \\ X_3[\alpha|000 &> +\beta|111>] = \alpha|001> + \beta|110>\end{aligned}$$

En calcul quantique, il n'existe pas de moyens directs permettant de détecter l'état courant. Pour différencier ces différents cas, la technique basée sur les codes stabilisateurs procède de la manière suivante :

- •On définit des opérateurs de syndrome qui ont la particularité d'avoir ces quatre états comme états propres.
- •Tous ces opérateurs doivent commuter.
- •Pour chaque cas, les valeurs propres associées à ces opérateurs doivent signer l'état courant sans aucune ambiguïté. Cela permet de corriger n'importe quelle erreur si nécessaire.

Pour notre exemple, les deux opérateurs utilisés sont : Z_1Z_2 et Z_2Z_3. La signature de chaque cas est donnée par le tableau ci-dessous

$$\begin{aligned}|\Psi_0 &> = |\Psi>= \alpha|000> + \beta|111> \longrightarrow (+1,+1) \\ |\Psi_1 &> = X_1|\Psi>= \alpha|100> + \beta|011> \longrightarrow (-1,+1) \\ |\Psi_2 &> = X_2|\Psi>= \alpha|010> + \beta|101> \longrightarrow (-1,-1) \\ |\Psi_3 &> = X_3|\Psi>= \alpha|001> + \beta|110> \longrightarrow (+1,-1).\end{aligned}$$

Pour chaque opérateur U, la mesure quantique de la valeur propre associée est mise en œuvre par le circuit suivant :

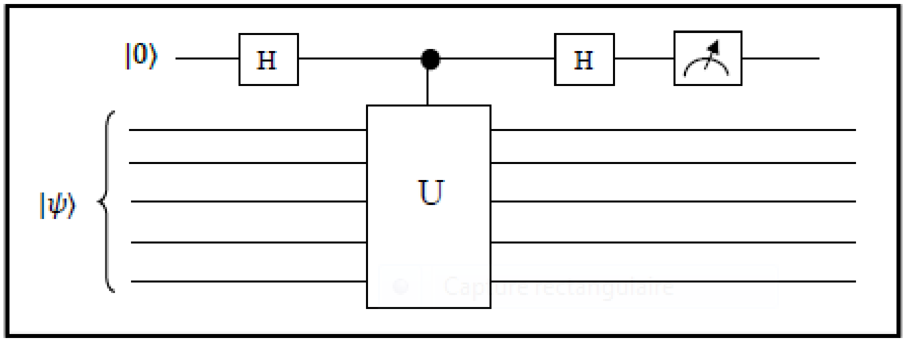

Fig.2.1 :Circuit quantique pour la mesure des syndromes.

*On introduit un qubit supplémentaire initialisé à |0> . Donc, l'état global est :

$$|0> \otimes |\Psi>$$

*On transforme ce qubit par une porte H. Le nouveau état est alors :

$$\frac{|0> + |1>}{\sqrt{2}} \otimes |\Psi>$$

*On applique la transformation U sur |Ψ> mais elle doit être contrôlée par ce qubit auxiliaire, ce qui donne :

$$\frac{1}{\sqrt{2}}(|0> |\Psi> + |1> U|\Psi>)$$

*On applique de nouveau une porte Hadamard sur le qubit auxiliaire. L'état obtenu est le suivant :

$$\frac{1}{\sqrt{2}}(\frac{|0> + |1>}{\sqrt{2}}|\Psi> + \frac{|0> - |1>}{\sqrt{2}}U|\Psi>) = |0> \frac{|\Psi> + U|\Psi>}{2} + |1> \frac{|\Psi> - U|\Psi>}{2}$$

*Finalement, on mesure le qubit auxiliaire. Le dernier résultat montre clairement qu'on obtient :

✓0 si la valeur propre associée est +1.

✓1 si la valeur propre associée est -1.

Remarques :

•Pour chaque cas, les valeurs propres associées aux opérateurs sont appelées syndrome.

•Les deux opérateurs Z_1Z_2 et Z_2Z_3 forment un groupe appelé stabilisateur du code.

•Z_1Z_2 et Z_2Z_3 sont appelés générateurs de ce groupe.

•Les opérations quantiques nécessaires dans l'étape de décodage sont les mêmes utilisées dans le processus d'encodage, mais elles doivent être appliquées dans l'ordre inverse.

•Par linéarité des opérations quantique, l'encodage d'un état : $|\Psi >= \alpha|0> +\beta|1>$

Donne : $|\Psi_c >= \alpha \mid 0\rangle_c + \beta \mid 1\rangle_c$

Cette notation sera utilisée dans le reste de ce mémoire.

Dans le cas général, les erreurs possibles sont de type X, Y et Z. En plus, plusieurs qubits peuvent être altérés. A l'heure actuelle, trois solutions ont été proposées et la différence réside dans le nombre de Qubits supplémentaire ajoutés dans l'étape d'encodage. Dans la première variante c'est un codage à 9 qubits, la seconde 7 qubits et la dernière 5 qubits.

2.2 Codage à neuf qubits

Encodage :

$$\mid \quad 0\rangle_c = \frac{1}{2\sqrt{2}}(\mid 000\rangle + \mid 111\rangle)(\mid 000\rangle + \mid 111\rangle)(\mid 000\rangle + \mid 111\rangle)$$

$$\mid \quad 1\rangle_c = \frac{1}{2\sqrt{2}}(\mid 000\rangle - \mid 111\rangle)(\mid 000\rangle - \mid 111\rangle)(\mid 000\rangle - \mid 111\rangle)$$

Générateurs du groupe stabilisateur :

	8	7	6	5	4	3	2	1	0
M_0	Z	Z	I	I	I	I	I	I	I
M_1	Z	I	Z	I	I	I	I	I	I
M_2	I	I	I	Z	Z	I	I	I	I
M_3	I	I	I	Z	I	Z	I	I	I
M_4	I	I	I	I	I	I	Z	Z	I
M_5	I	I	I	I	I	I	Z	I	Z
M_6	X	X	X	X	X	X	I	I	I
M_7	X	X	X	I	I	I	X	X	X

Fig.2.2 :Les stabilisateurs pour le code 9 qubits.

2.3 Codage à sept qubits

Encodage :

$$|\ \ 0\rangle_c = \frac{1}{\sqrt{8}}\left(1+M_0\right)\left(1+M_1\right)\left(1+M_2\right)|\ 0000000\rangle$$
$$|\ \ 1\rangle_c = \frac{1}{\sqrt{8}}\left(1+M_0\right)\left(1+M_1\right)\left(1+M_2\right)|\ 1111111\rangle$$

Tel que :

$$\begin{aligned} M_0 &= X_0X_4X_5X_6 \\ M_1 &= X_1X_3X_5X_6 \\ M_2 &= X_2X_3X_4X_6 \end{aligned}$$

Syndromes :

Les syndromes d'erreurs pour le code à sept qubits sont résumés dans le tableau qui suit :

	$X_0Y_0Z_0$	$X_1Y_1Z_1$	$X_2Y_2Z_2$	$X_3Y_3Z_3$	$X_4Y_4Z_4$	$X_5Y_5Z_5$	$X_6Y_6Z_6$	1
M_0	$--+$	$+++$	$+++$	$+++$	$--+$	$--+$	$--+$	$+$
M_1	$+++$	$--+$	$+++$	$--+$	$+++$	$--+$	$--+$	$+$
M_2	$+++$	$+++$	$--+$	$--+$	$--+$	$+++$	$--+$	$+$
M_3	$+--$	$+++$	$+++$	$+++$	$+--$	$+--$	$+--$	$+$
M_4	$+++$	$+--$	$+++$	$+--$	$+++$	$+--$	$+--$	$+$
M_5	$+++$	$+++$	$+--$	$+--$	$+--$	$+++$	$+--$	$+$

Fig.2.3 :Syndromes d'erreur pour le code 7 qubits

2.4 Codage à Cinq qubits

2.4.1 Encodage :

Dans ce cas l'encodage se fait par un circuit qui intrique maximalement les qubits en question

$$|\ \ 0\rangle_c = \frac{1}{4}\left(1+M_0\right)\left(1+M_1\right)\left(1+M_2\right)\left(1+M_3\right)|\ 00000\rangle$$
$$|\ \ 1\rangle_c = \frac{1}{4}\left(1+M_0\right)\left(1+M_1\right)\left(1+M_2\right)\left(1+M_3\right)|\ 11111\rangle$$

tel que :

$$\begin{aligned} M_0 &= Z_1X_2X_3Z_4 \\ M_1 &= Z_2X_3X_4Z_0 \\ M_2 &= Z_3X_4X_0Z_1 \\ M_3 &= Z_4X_0X_1Z_2 \end{aligned}$$

Le circuit correspondant est le suivant :

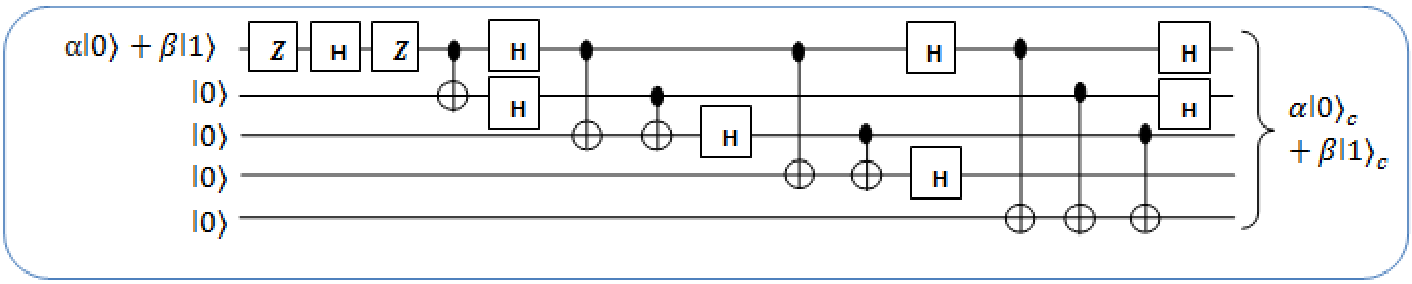

Fig.2.4 :Circuit pour coder un Qubit sur 5 qubits.

2.4.2 Syndromes :

Les valeurs propres associées à chaque cas sont les suivantes :

	$X_0Y_0Z_0$	$X_1Y_1Z_1$	$X_2Y_2Z_2$	$X_3Y_3Z_3$	$X_4Y_4Z_4$	1
$M_0 = Z_1X_2X_3Z_4$	$+++$	$--+$	$+--$	$+--$	$--+$	$+$
$M_1 = Z_2X_3X_4Z_0$	$--+$	$+++$	$--+$	$+--$	$+--$	$+$
$M_2 = Z_3X_4X_0Z_1$	$+--$	$--+$	$+++$	$--+$	$+--$	$+$
$M_3 = Z_4X_0X_1Z_2$	$+--$	$+--$	$--+$	$+++$	$--+$	$+$

Tab2.5 :Syndromes d'erreur pour le code 5 qubits.

Le circuit quantique permettant de réaliser ces calculs est le suivant :

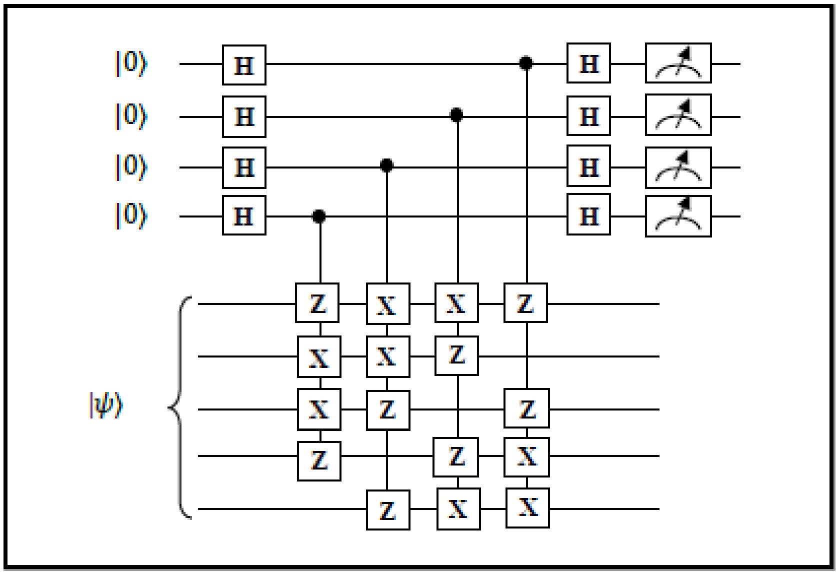

Fig.2.6 :Circuit quantique pour la dtection des syndromes d'erreur.

2.4.3 Décodage :

Le circuit de décodage est le suivant :

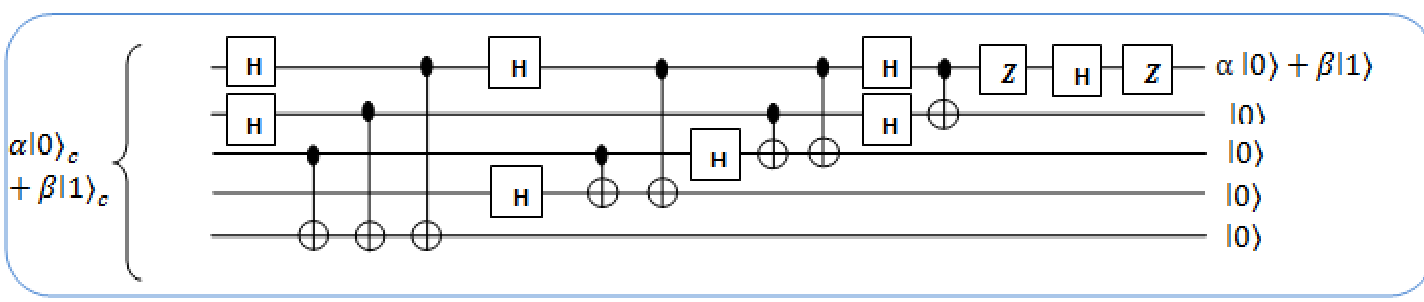

Fig.2.7 :Circuit quantique pour le dcodage de 5 qubits.

2.4.4 Exemple :

Dans ce qui suit, dans le but d'illustrer les calculs de l'alghorithme nous présntons une Erreur X sur le premier qubit ; tous les autres erreurs sont de la même manière :

$$| \Psi_1\rangle = \frac{1}{4}(\alpha | 10000\rangle - \alpha | 11001 \rangle + \alpha | 00001 \rangle + \alpha | 10011 \rangle - \alpha | 01011\rangle + \alpha | 01000 \rangle$$
$$-\alpha | 11010 \rangle - \alpha | 00010 \rangle + \alpha | 11100\rangle - \alpha | 00100 \rangle + \alpha | 10110 \rangle - \alpha | 01110 \rangle - \alpha | 10101 \rangle$$
$$-\alpha | 01101 \rangle - \alpha | 11111 \rangle - \alpha | 00111 \rangle - \beta | 11000 \rangle - \beta | 00000 \rangle - \beta | 10010 \rangle - \beta | 01010 \rangle$$
$$-\beta | 10001\rangle + \beta | 01001 \rangle - \beta | 11011 \rangle + \beta | 00011 \rangle - \beta | 11101 \rangle - \beta | 00101 \rangle + \beta | 10111 \rangle$$
$$+\beta | 01111 \rangle - \beta | 10100 \rangle + \beta | 01100 \rangle + \beta | 11110 \rangle - \beta | 00110 \rangle)$$

Mesure du syndrome : à ce stade la détection des erreurs est effectuée en ajoutant un nouveau circuit dit circuit des syndromes (voir schéma du circuit **Fig.2.6**)

Ajout des quatre qubits du syndrome :

$$| \Psi_1\rangle = \tfrac{1}{4}(\alpha | 100000000\rangle - \alpha | 110010000 \rangle + \alpha | 000010000 \rangle + \alpha | 100110000 \rangle$$
$$-\alpha | 010110000\rangle + \alpha | 010000000 \rangle - \alpha | 110100000 \rangle - \alpha | 000100000 \rangle$$
$$+ \alpha | 111000000\rangle - \alpha | 001000000 \rangle + \alpha | 101100000 \rangle - \alpha | 011100000 \rangle$$
$$-\alpha | 101010000 \rangle - \alpha | 011010000 \rangle - \alpha | 111110000 \rangle - \alpha | 001110000 \rangle$$
$$-\beta | 110000000 \rangle - \beta | 000000000 \rangle - \beta | 100100000 \rangle - \beta | 010100000 \rangle$$
$$-\beta | 100010000\rangle + \beta | 010010000 \rangle - \beta | 110110000 \rangle + \beta | 000110000 \rangle$$
$$-\beta | 111010000 \rangle - \beta | 001010000 \rangle + \beta | 101110000 \rangle + \beta | 011110000 \rangle$$
$$-\beta | 101000000 \rangle + \beta | 011000000 \rangle + \beta | 111100000 \rangle - \beta | 001100000 \rangle)$$

∘ L'application de la Porte H(6):

$$| \Psi_1\rangle = \tfrac{1}{4\sqrt{2}}(\alpha | 100000000\rangle + \alpha | 100001000\rangle - \alpha | 110010000 \rangle - \alpha | 110011000 \rangle$$
$$+\alpha | 000010000\rangle + \alpha | 000011000\rangle + \alpha | 100110000 \rangle + \alpha | 100111000 \rangle$$
$$-\alpha | 010110000\rangle - \alpha | 010111000\rangle + \alpha | 010000000 \rangle + \alpha | 010001000 \rangle$$
$$-\alpha | 110100000\rangle - \alpha | 110101000\rangle - \alpha | 000100000 \rangle - \alpha | 000101000 \rangle$$
$$+ \alpha | 111000000\rangle + \alpha | 111001000\rangle - \alpha | 001000000 \rangle - \alpha | 001001000 \rangle$$
$$+\alpha | 101100000\rangle + \alpha | 101101000\rangle - \alpha | 011100000 \rangle - \alpha | 011101000 \rangle$$
$$-\alpha | 101010000\rangle - \alpha | 101011000\rangle - \alpha | 011010000 \rangle - \alpha | 011011000 \rangle$$
$$-\alpha | 111110000\rangle - \alpha | 111111000\rangle - \alpha | 001110000 \rangle - \alpha | 001111000 \rangle$$
$$-\beta | 110000000\rangle - \beta | 110001000\rangle - \beta | 000000000 \rangle - \beta | 000001000 \rangle$$
$$-\beta | 100100000\rangle - \beta | 100101000\rangle - \beta | 010100000 \rangle - \beta | 010101000 \rangle$$

$-\beta \mid 100010000\rangle - \beta \mid 100011000\rangle + \beta \mid 010010000 \rangle + \beta \mid 010011000 \rangle$

$-\beta \mid 110110000\rangle - \beta \mid 110111000\rangle + \beta \mid 000110000 \rangle + \beta \mid 000111000 \rangle$

-$\beta \mid 111010000\rangle - \beta \mid 111011000\rangle - \beta \mid 001010000 \rangle - \beta \mid 001011000 \rangle$

$+\beta \mid 101110000\rangle + \beta \mid 101111000\rangle + \beta \mid 011110000 \rangle + \beta \mid 011111000 \rangle$

-$\beta \mid 101000000\rangle - \beta \mid 101001000\rangle + \beta \mid 011000000 \rangle + \beta \mid 011001000 \rangle$

$+\beta \mid 111100000\rangle + \beta \mid 111101000 \rangle - \beta \mid 001100000 \rangle - \beta \mid 001101000 \rangle)$

∘ L'application de la Porte H(7):

$\mid \Psi_1\rangle = \frac{1}{8}(\alpha \mid 100000000\rangle + \alpha \mid 100000100\rangle + \alpha \mid 100001000\rangle + \alpha \mid 100001100\rangle$

$-\alpha \mid 110010000\rangle - \alpha \mid 110010100\rangle - \alpha \mid 110011000\rangle - \alpha \mid 110011100\rangle$

$+\alpha \mid 000010000\rangle + \alpha \mid 000010100\rangle + \alpha \mid 000011000\rangle + \alpha \mid 000011100\rangle$

$+\alpha \mid 100110000\rangle + \alpha \mid 100110100\rangle + \alpha \mid 100111000 \rangle + \alpha \mid 100111100 \rangle$

$-\alpha \mid 010110000\rangle - \alpha \mid 010110100\rangle - \alpha \mid 010111000\rangle - \alpha \mid 010111100\rangle$

$+\alpha \mid 010000000\rangle + \alpha \mid 010000100\rangle + \alpha \mid 010001000\rangle + \alpha \mid 010001100\rangle$

$-\alpha \mid 110100000\rangle - \alpha \mid 110100100\rangle - \alpha \mid 110101000\rangle - \alpha \mid 110101100\rangle$

$-\alpha \mid 000100000\rangle - \alpha \mid 000100100\rangle - \alpha \mid 000101000\rangle - \alpha \mid 000101100\rangle$

$+\alpha \mid 111000000\rangle + \alpha \mid 111000100\rangle + \alpha \mid 111001000\rangle + \alpha \mid 111001100\rangle$

$-\alpha \mid 001000000\rangle - \alpha \mid 001000100\rangle - \alpha \mid 001001000\rangle - \alpha \mid 001001100 \rangle$

$+\alpha \mid 101100000\rangle + \alpha \mid 101100100\rangle + \alpha \mid 101101000\rangle + \alpha \mid 101101100\rangle$

$-\alpha \mid 011100000\rangle - \alpha \mid 011100100\rangle - \alpha \mid 011101000\rangle - \alpha \mid 011101100\rangle$

-$\alpha \mid 101010000\rangle - \alpha \mid 101010100\rangle - \alpha \mid 101011000\rangle - \alpha \mid 101011100\rangle$

$-\alpha \mid 011010000\rangle - \alpha \mid 011010100\rangle - \alpha \mid 011011000\rangle - \alpha \mid 011011100\rangle$

$-\alpha \mid 111110000\rangle - \alpha \mid 111110100\rangle - \alpha \mid 111111000\rangle - \alpha \mid 111111100\rangle$

$-\alpha \mid 001110000\rangle - \alpha \mid 001110100\rangle - \alpha \mid 001111000\rangle - \alpha \mid 001111100\rangle$

$-\beta \mid 110000000\rangle - \beta \mid 110000100\rangle - \beta \mid 110001000\rangle - \beta \mid 110001100\rangle$

$-\beta \mid 000000000\rangle - \beta \mid 000000100\rangle - \beta \mid 000001000\rangle - \beta \mid 000001100\rangle$

$-\beta \mid 100100000\rangle - \beta \mid 100100100\rangle - \beta \mid 100101000\rangle - \beta \mid 100101100\rangle$

$-\beta \mid 010100000\rangle - \beta \mid 010100100\rangle - \beta \mid 010101000\rangle - \beta \mid 010101100\rangle$

$-\beta \mid 100010000\rangle - \beta \mid 100010100\rangle - \beta \mid 100011000\rangle - \beta \mid 100011100\rangle$

$+\beta \mid 010010000\rangle + \beta \mid 010010100\rangle + \beta \mid 010011000\rangle + \beta \mid 010011100\rangle$

$-\beta \mid 110110000\rangle - \beta \mid 110110100\rangle - \beta \mid 110111000\rangle - \beta \mid 110111100\rangle$

$+\beta \mid 000110000\rangle + \beta \mid 000110100\rangle + \beta \mid 000111000\rangle + \beta \mid 000111100\rangle$

-$\beta \mid 111010000\rangle - \beta \mid 111010100\rangle - \beta \mid 111011000\rangle - \beta \mid 111011100\rangle$

$-\beta \mid 001010000\rangle - \beta \mid 001010100\rangle - \beta \mid 001011000\rangle - \beta \mid 001011100\rangle$

$+\beta \mid 101110000\rangle + \beta \mid 101110100\rangle + \beta \mid 101111000\rangle + \beta \mid 101111100\rangle$

$+\beta \mid 011110000\rangle + \beta \mid 011110100\rangle + \beta \mid 011111000\rangle + \beta \mid 011111100\rangle$

-$\beta \mid 101000000\rangle - \beta \mid 101000100\rangle - \beta \mid 101001000\rangle - \beta \mid 101001100\rangle$

$+\beta \mid 011000000\rangle + \beta \mid 011000100\rangle + \beta \mid 011001000\rangle + \beta \mid 011001100\rangle$

$+\beta \mid 111100000\rangle + \beta \mid 111100100\rangle + \beta \mid 111101000\rangle + \beta \mid 111101100\rangle$

$-\beta \mid 001100000\rangle - \beta \mid 001100100\rangle - \beta \mid 001101000\rangle - \beta \mid 001101100\rangle)$

∘ <u>L'application de la Porte H(8)</u>:

$\mid \Psi_1\rangle = \frac{1}{8}(\alpha \mid 100000000\rangle + \alpha \mid 100000010\rangle + \alpha \mid 100000100\rangle + \alpha \mid 100000110\rangle$

$+\alpha \mid 100001000\rangle + \alpha \mid 100001010\rangle + \alpha \mid 100001100\rangle + \alpha \mid 100001110\rangle$

$-\alpha \mid 110010000\rangle - \alpha \mid 110010010\rangle - \alpha \mid 110010100\rangle - \alpha \mid 110010110\rangle$

$-\alpha \mid 110011000\rangle - \alpha \mid 110011010\rangle - \alpha \mid 110011100\rangle - \alpha \mid 110011110\rangle$

$+\alpha \mid 000010000\rangle + \alpha \mid 000010010\rangle + \alpha \mid 000010100\rangle + \alpha \mid 000010110\rangle$

$+\alpha \mid 000011000\rangle + \alpha \mid 000011010\rangle + \alpha \mid 000011100\rangle + \alpha \mid 000011110\rangle$

$+\alpha \mid 100110000\rangle + \alpha \mid 100110010\rangle + \alpha \mid 100110100\rangle + \alpha \mid 100110110\rangle$

$+\alpha \mid 100111000\rangle + \alpha \mid 100111010\rangle + \alpha \mid 100111100\rangle + \alpha \mid 100111110\rangle$

$-\alpha \mid 010110000\rangle - \alpha \mid 010110010\rangle - \alpha \mid 010110100\rangle - \alpha \mid 010110110\rangle$

$-\alpha \mid 010111000\rangle - \alpha \mid 010111010\rangle - \alpha \mid 010111100\rangle - \alpha \mid 010111110\rangle$

$+\alpha \mid 010000000\rangle + \alpha \mid 010000010\rangle + \alpha \mid 010000100\rangle + \alpha \mid 010000110\rangle$

$+\alpha \mid 010001000\rangle + \alpha \mid 010001010\rangle + \alpha \mid 010001100\rangle + \alpha \mid 010001110\rangle$

$-\alpha \mid 110100000\rangle - \alpha \mid 110100010\rangle - \alpha \mid 110100100\rangle - \alpha \mid 110100110\rangle$

$-\alpha \mid 110101000\rangle - \alpha \mid 110101010\rangle - \alpha \mid 110101100\rangle - \alpha \mid 110101110\rangle$

$-\alpha \mid 000100000\rangle - \alpha \mid 000100010\rangle - \alpha \mid 000100100\rangle - \alpha \mid 000100110\rangle$

$-\alpha \mid 000101000\rangle - \alpha \mid 000101010\rangle - \alpha \mid 000101100\rangle - \alpha \mid 000101110\rangle$

$+\alpha \mid 111000000\rangle + \alpha \mid 111000010\rangle + \alpha \mid 111000100\rangle + \alpha \mid 111000110\rangle$

$+\alpha \mid 111001000\rangle + \alpha \mid 111001010\rangle + \alpha \mid 111001100\rangle + \alpha \mid 111001110\rangle$

$-\alpha \mid 001000000\rangle - \alpha \mid 001000010\rangle - \alpha \mid 001000100\rangle - \alpha \mid 001000110\rangle$

$-\alpha \mid 001001000\rangle - \alpha \mid 001001010\rangle - \alpha \mid 001001100\rangle - \alpha \mid 001001110\rangle$

$+\alpha \mid 101100000\rangle + \alpha \mid 101100010\rangle + \alpha \mid 101100100\rangle + \alpha \mid 101100110\rangle$

$+\alpha \mid 101101000\rangle + \alpha \mid 101101010\rangle + \alpha \mid 101101100\rangle + \alpha \mid 101101110\rangle$

$-\alpha \mid 011100000\rangle - \alpha \mid 011100010\rangle - \alpha \mid 011100100\rangle - \alpha \mid 011100110\rangle$

$-\alpha \mid 011101000\rangle - \alpha \mid 011101010\rangle - \alpha \mid 011101100\rangle - \alpha \mid 011101110\rangle$

-$\alpha \mid 101010000\rangle - \alpha \mid 101010010\rangle - \alpha \mid 101010100\rangle - \alpha \mid 101010110\rangle$

$-\alpha \mid 101011000\rangle - \alpha \mid 101011010\rangle - \alpha \mid 101011100\rangle - \alpha \mid 101011110\rangle$

$-\alpha \mid 011010000\rangle - \alpha \mid 011010010\rangle - \alpha \mid 011010100\rangle - \alpha \mid 011010110\rangle$

$-\alpha \mid 011011000\rangle - \alpha \mid 011011010\rangle - \alpha \mid 011011100\rangle - \alpha \mid 011011110\rangle$

$-\alpha \mid 111110000\rangle - \alpha \mid 111110010\rangle - \alpha \mid 111110100\rangle - \alpha \mid 111110110\rangle$

$-\alpha \mid 111111000\rangle - \alpha \mid 111111010\rangle - \alpha \mid 111111100\rangle - \alpha \mid 111111110\rangle$

$-\alpha \mid 001110000\rangle - \alpha \mid 001110010\rangle - \alpha \mid 001110100\rangle - \alpha \mid 001110110\rangle$

$-\alpha \mid 001111000\rangle - \alpha \mid 001111010\rangle - \alpha \mid 001111100\rangle - \alpha \mid 001111110\rangle$

$-\beta \mid 110000000\rangle - \beta \mid 110000010\rangle - \beta \mid 110000100\rangle - \beta \mid 110000110\rangle$

$-\beta \mid 110001000\rangle - \beta \mid 110001010\rangle - \beta \mid 110001100\rangle - \beta \mid 110001110\rangle$

$-\beta \mid 000000000\rangle - \beta \mid 000000010\rangle - \beta \mid 000000100\rangle - \beta \mid 000000110\rangle$

$-\beta \mid 000001000\rangle - \beta \mid 000001010\rangle - \beta \mid 000001100\rangle - \beta \mid 000001110\rangle$

$-\beta \mid 100100000\rangle - \beta \mid 100100010\rangle - \beta \mid 100100100\rangle - \beta \mid 100100110\rangle$

$-\beta \mid 100101000\rangle - \beta \mid 100101010\rangle - \beta \mid 100101100\rangle - \beta \mid 100101110\rangle$

$-\beta \mid 010100000\rangle - \beta \mid 010100010\rangle - \beta \mid 010100100\rangle - \beta \mid 010100110\rangle$

$-\beta \mid 010101000\rangle - \beta \mid 010101010\rangle - \beta \mid 010101100\rangle - \beta \mid 010101110\rangle$

$-\beta \mid 100010000\rangle - \beta \mid 100010010\rangle - \beta \mid 100010100\rangle - \beta \mid 100010110\rangle$

$-\beta \mid 100011000\rangle - \beta \mid 100011010\rangle - \beta \mid 100011100\rangle - \beta \mid 100011110\rangle$

$+\beta \mid 010010000\rangle + \beta \mid 010010010\rangle + \beta \mid 010010100\rangle + \beta \mid 010010110\rangle$

$+\beta \mid 010011000\rangle + \beta \mid 010011010\rangle + \beta \mid 010011100\rangle + \beta \mid 010011110\rangle$

$-\beta \mid 110110000\rangle - \beta \mid 110110010\rangle - \beta \mid 110110100\rangle - \beta \mid 110110110\rangle$

$-\beta \mid 110111000\rangle - \beta \mid 110111010\rangle - \beta \mid 110111100\rangle - \beta \mid 110111110\rangle$

$+\beta \mid 000110000\rangle + \beta \mid 000110010\rangle + \beta \mid 000110100\rangle + \beta \mid 000110110\rangle$

$+\beta \mid 000111000\rangle + \beta \mid 000111010\rangle + \beta \mid 000111100\rangle + \beta \mid 000111110\rangle$

-$\beta \mid 111010000\rangle - \beta \mid 111010010\rangle - \beta \mid 111010100\rangle - \beta \mid 111010110\rangle$

$-\beta \mid 111011000\rangle - \beta \mid 111011010\rangle - \beta \mid 111011100\rangle - \beta \mid 111011110\rangle$

$-\beta \mid 001010000\rangle - \beta \mid 001010010\rangle - \beta \mid 001010100\rangle - \beta \mid 001010110\rangle$

$-\beta \mid 001011000\rangle - \beta \mid 001011010\rangle - \beta \mid 001011100\rangle - \beta \mid 001011110\rangle$

$+\beta \mid 101110000\rangle + \beta \mid 101110010\rangle + \beta \mid 101110100\rangle + \beta \mid 101110110\rangle$

$+\beta \mid 101111000\rangle + \beta \mid 101111010\rangle + \beta \mid 101111100\rangle + \beta \mid 101111110\rangle$

$+\beta \mid 011110000\rangle + \beta \mid 011110010\rangle + \beta \mid 011110100\rangle + \beta \mid 011110110\rangle$

$+\beta \mid 011111000\rangle + \beta \mid 011111010\rangle + \beta \mid 011111100\rangle + \beta \mid 011111110\rangle$

$-\beta \mid 101000000\rangle - \beta \mid 101000010\rangle - \beta \mid 101000100\rangle - \beta \mid 101000110\rangle$

$-\beta \mid 101001000\rangle - \beta \mid 101001010\rangle - \beta \mid 101001100\rangle - \beta \mid 101001110\rangle$

$+\beta \mid 011000000\rangle + \beta \mid 011000010\rangle + \beta \mid 011000100\rangle + \beta \mid 011000110\rangle$

$+\beta \mid 011001000\rangle + \beta \mid 011001010\rangle + \beta \mid 011001100\rangle + \beta \mid 011001110\rangle$

$+\beta \mid 111100000\rangle + \beta \mid 111100010\rangle + \beta \mid 111100100\rangle + \beta \mid 111100110\rangle$

$+\beta \mid 111101000\rangle + \beta \mid 111101010\rangle + \beta \mid 111101100\rangle + \beta \mid 111101110\rangle$

$-\beta \mid 001100000\rangle - \beta \mid 001100010\rangle - \beta \mid 001100100\rangle - \beta \mid 001100110\rangle$

$-\beta \mid 001101000\rangle - \beta \mid 001101010\rangle - \beta \mid 001101100\rangle) - \beta \mid 001101110\rangle)$

○ L'application de la Porte H(9):

$\mid \Psi_1\rangle = \frac{1}{16}(\alpha \mid 100000000\rangle + \alpha \mid 100000001\rangle + \alpha \mid 100000010\rangle + \alpha \mid 100000011\rangle$

$+\alpha \mid 100000100\rangle + \alpha \mid 100000101\rangle + \alpha \mid 100000110\rangle + \alpha \mid 100000111\rangle$

$+\alpha \mid 100001000\rangle + \alpha \mid 100001001\rangle + \alpha \mid 100001010\rangle + \alpha \mid 100001011\rangle$

$+\alpha \mid 100001100\rangle + \alpha \mid 100001101\rangle + \alpha \mid 100001110\rangle + \alpha \mid 100001111\rangle$

$-\alpha \mid 110010000\rangle - \alpha \mid 110010001\rangle - \alpha \mid 110010010\rangle - \alpha \mid 110010011\rangle$

$-\alpha \mid 110010100\rangle - \alpha \mid 110010101\rangle - \alpha \mid 110010110\rangle - \alpha \mid 110010111\rangle$

$-\alpha \mid 110011000\rangle - \alpha \mid 110011001\rangle - \alpha \mid 110011010\rangle - \alpha \mid 110011011\rangle$

$-\alpha \mid 110011100\rangle - \alpha \mid 110011101\rangle - \alpha \mid 110011110\rangle - \alpha \mid 110011111\rangle$

$+\alpha \mid 000010000\rangle + \alpha \mid 000010001\rangle + \alpha \mid 000010010\rangle + \alpha \mid 000010011\rangle$

$+\alpha \mid 000010100\rangle + \alpha \mid 000010101\rangle + \alpha \mid 000010110\rangle + \alpha \mid 000010111\rangle$

$+\alpha \mid 000011000\rangle + \alpha \mid 000011001\rangle + \alpha \mid 000011010\rangle + \alpha \mid 000011011\rangle$

$+\alpha \mid 000011100\rangle + \alpha \mid 000011101\rangle + \alpha \mid 000011110\rangle + \alpha \mid 000011111\rangle$

$+\alpha \mid 100110000\rangle + \alpha \mid 100110001\rangle + \alpha \mid 100110010\rangle + \alpha \mid 100110011\rangle$

$+\alpha \mid 100110100\rangle + \alpha \mid 100110101\rangle + \alpha \mid 100110110\rangle + \alpha \mid 100110111\rangle$

$+\alpha \mid 100111000\rangle + \alpha \mid 100111001\rangle + \alpha \mid 100111010\rangle + \alpha \mid 100111011\rangle$

$+\alpha \mid 100111100\rangle + \alpha \mid 100111101\rangle + \alpha \mid 100111110\rangle + \alpha \mid 100111111\rangle$

$-\alpha \mid 010110000\rangle - \alpha \mid 010110001\rangle - \alpha \mid 010110010\rangle - \alpha \mid 010110011\rangle$

$-\alpha \mid 010110100\rangle - \alpha \mid 010110101\rangle - \alpha \mid 010110110\rangle - \alpha \mid 010110111\rangle$

$-\alpha \mid 010111000\rangle - \alpha \mid 010111001\rangle - \alpha \mid 010111010\rangle - \alpha \mid 010111011\rangle$

$-\alpha \mid 010111100\rangle - \alpha \mid 010111101\rangle - \alpha \mid 010111110\rangle - \alpha \mid 010111111\rangle$

$+\alpha \mid 010000000\rangle + \alpha \mid 010000001\rangle + \alpha \mid 010000010\rangle + \alpha \mid 010000011\rangle$

$+\alpha \mid 010000100\rangle + \alpha \mid 010000101\rangle + \alpha \mid 010000110\rangle + \alpha \mid 010000111\rangle$

$+\alpha \mid 010001000\rangle + \alpha \mid 010001001\rangle + \alpha \mid 01\ 0001010\rangle + \alpha \mid 01\ 0001011\rangle$

$+\alpha \mid 010001100\rangle + \alpha \mid 010001101\rangle + \alpha \mid 010001110\rangle \ + \alpha \mid 010001111\rangle$

$-\alpha \mid 110100000\rangle - \alpha \mid 110100001\rangle \ - \alpha \mid 110100010\rangle - \alpha \mid 110100011\rangle$

$-\alpha \mid 110100100\rangle - \alpha \mid 110100101\rangle - \alpha \mid 110100110\rangle - \alpha \mid 110100111\rangle$

$-\alpha \mid 110101000\rangle - \alpha \mid 110101001\rangle - \alpha \mid 110101010\rangle - \alpha \mid 110101011\rangle$

$-\alpha \mid 110101100\rangle \ - \alpha \mid 110101101\rangle - \alpha \mid 110101110\rangle - \alpha \mid 110101111\rangle$

$-\alpha \mid 000100000\rangle - \alpha \mid 000100001\rangle - \alpha \mid 000100010\rangle - \alpha \mid 000100011\rangle$

$-\alpha \mid 000100100\rangle - \alpha \mid 000100101\rangle - \alpha \mid 000100110\rangle - \alpha \mid 000100111\rangle$

$-\alpha \mid 000101000\rangle - \alpha \mid 000101001\rangle - \alpha \mid 000101010\rangle - \alpha \mid 000101011\rangle$

$-\alpha \mid 000101100\rangle - \alpha \mid 000101101\rangle - \alpha \mid 000101110\rangle - \alpha \mid 000101111\rangle$

$+\alpha \mid 111000000\rangle + \alpha \mid 111000001\rangle + \alpha \mid 111000010\rangle + \alpha \mid 111000011\rangle$

$+\alpha \mid 111000100\rangle + \alpha \mid 111000101\rangle + \alpha \mid 111000110\rangle \ + \alpha \mid 111000111\rangle$

$+\alpha \mid 111001000\rangle + \alpha \mid 111001001\rangle + \alpha \mid 111001010\rangle + \alpha \mid 111001011\rangle$

$+\alpha \mid 111001100\rangle + \alpha \mid 111001101\rangle + \alpha \mid 111001110\rangle + \alpha \mid 111001111\rangle$

$-\alpha \mid 001000000\rangle - \alpha \mid 001000001\rangle - \alpha \mid 001000010\rangle - \alpha \mid 001000011\rangle$

$-\alpha \mid 001000100\rangle - \alpha \mid 001000101\rangle - \alpha \mid 001000110\rangle - \alpha \mid 001000111\rangle$

$-\alpha \mid 001001000\rangle - \alpha \mid 001001001\rangle - \alpha \mid 001001010\rangle - \alpha \mid 001001011\rangle$

$-\alpha \mid 001001100\rangle - \alpha \mid 001001101\rangle - \alpha \mid 001001110 \ \rangle - \alpha \mid 001001111 \ \rangle$

$+\alpha \mid 101100000\rangle + \alpha \mid 101100001\rangle + \alpha \mid 101100010\rangle + \alpha \mid 101100011\rangle$

$+\alpha \mid 101100100\rangle + \alpha \mid 101100101\rangle \ + \alpha \mid 101100110\rangle + \alpha \mid 101100111\rangle$

$+\alpha \mid 101101000\rangle + \alpha \mid 101101001\rangle + \alpha \mid 101101010\rangle + \alpha \mid 101101011\rangle$

$+\alpha \mid 101101100\rangle + \alpha \mid 101101101\rangle + \alpha \mid 101101110\rangle + \alpha \mid 101101111\rangle$

$-\alpha \mid 011100000\rangle - \alpha \mid 011100001\rangle - \alpha \mid 011100010\rangle - \alpha \mid 011100011\rangle$

$-\alpha \mid 011100100\rangle - \alpha \mid 011100101\rangle - \alpha \mid 011100110\rangle - \alpha \mid 011100111\rangle$

$-\alpha \mid 011101000\rangle - \alpha \mid 011101001\rangle - \alpha \mid 011101010\rangle - \alpha \mid 011101011\rangle$

$-\alpha \mid 011101100\rangle - \alpha \mid 011101101\rangle - \alpha \mid 011101110\rangle - \alpha \mid 011101111\rangle$

-$\alpha \mid 101010000\rangle - \alpha \mid 101010001\rangle - \alpha \mid 101010010\rangle - \alpha \mid 101010011\rangle$

$-\alpha \mid 101010100\rangle - \alpha \mid 101010101\rangle - \alpha \mid 101010110\rangle - \alpha \mid 101010111\rangle$

$-\alpha \mid 101011000\rangle - \alpha \mid 101011001\rangle - \alpha \mid 101011010\rangle - \alpha \mid 101011011\rangle$

$-\alpha \mid 101011100\rangle - \alpha \mid 101011101\rangle - \alpha \mid 101011110\rangle - \alpha \mid 101011111\rangle$

$-\alpha \mid 011010000\rangle - \alpha \mid 011010001\rangle - \alpha \mid 011010010\rangle - \alpha \mid 011010011\rangle$

$-\alpha \mid 011010100\rangle - \alpha \mid 011010101\rangle - \alpha \mid 011010110\rangle - \alpha \mid 011010111\rangle$

$-\alpha \mid 011011000\rangle - \alpha \mid 011011001\rangle - \alpha \mid 011011010\rangle - \alpha \mid 011011011\rangle$

$-\alpha \mid 011011100\rangle - \alpha \mid 011011101\rangle - \alpha \mid 011011110\rangle - \alpha \mid 011011111\rangle$

$-\alpha \mid 111110000\rangle - \alpha \mid 111110001\rangle - \alpha \mid 111110010\rangle - \alpha \mid 111110011\rangle$

$-\alpha \mid 111110100\rangle - \alpha \mid 111110101\rangle - \alpha \mid 111110110\rangle - \alpha \mid 111110111\rangle$

$-\alpha \mid 111111000\rangle - \alpha \mid 111111001\rangle - \alpha \mid 111111010\rangle - \alpha \mid 111111011\rangle$

$-\alpha \mid 111111100\rangle - \alpha \mid 111111101\rangle - \alpha \mid 111111110\rangle - \alpha \mid 111111111\rangle$

$-\alpha \mid 001110000\rangle - \alpha \mid 001110001\rangle - \alpha \mid 001110010\rangle - \alpha \mid 001110011\rangle$

$-\alpha \mid 001110100\rangle - \alpha \mid 001110101\rangle - \alpha \mid 001110110\rangle - \alpha \mid 001110111\rangle$

$-\alpha \mid 001111000\rangle - \alpha \mid 001111001\rangle - \alpha \mid 001111010\rangle - \alpha \mid 001111011\rangle$

$-\alpha \mid 001111100\rangle - \alpha \mid 001111101\rangle - \alpha \mid 001111110\rangle - \alpha \mid 001111111\rangle$

$-\beta \mid 110000000\rangle - \beta \mid 110000001\rangle - \beta \mid 110000010\rangle - \beta \mid 110000011\rangle$

$-\beta \mid 110000100\rangle - \beta \mid 110000101\rangle - \beta \mid 110000110\rangle - \beta \mid 110000111\rangle$

$-\beta \mid 110001000\rangle - \beta \mid 110001001\rangle - \beta \mid 110001010\rangle - \beta \mid 110001011\rangle$

$-\beta \mid 110001100\rangle - \beta \mid 110001101\rangle - \beta \mid 110001110\rangle - \beta \mid 110001111\rangle$

$-\beta \mid 000000000\rangle - \beta \mid 000000001\rangle - \beta \mid 000000010\rangle - \beta \mid 000000011\rangle$

$-\beta \mid 000000100\rangle - \beta \mid 000000101\rangle - \beta \mid 000000110\rangle - \beta \mid 000000111\rangle$

$-\beta \mid 000001000\rangle - \beta \mid 000001001\rangle - \beta \mid 000001010\rangle - \beta \mid 000001011\rangle$

$-\beta \mid 000001100\rangle - \beta \mid 000001101\rangle - \beta \mid 000001110\rangle - \beta \mid 000001111\rangle$

$-\beta \mid 100100000\rangle - \beta \mid 100100001\rangle - \beta \mid 100100010\rangle - \beta \mid 100100011\rangle$

$-\beta \mid 100100100\rangle - \beta \mid 100100101\rangle - \beta \mid 100100110\rangle - \beta \mid 100100111\rangle$

$-\beta \mid 100101000\rangle - \beta \mid 100101001\rangle - \beta \mid 100101010\rangle - \beta \mid 100101011\rangle$

$-\beta \mid 100101100\rangle - \beta \mid 100101101\rangle - \beta \mid 100101110\rangle - \beta \mid 100101111\rangle$

$-\beta \mid 010100000\rangle - \beta \mid 010100001\rangle - \beta \mid 010100010\rangle - \beta \mid 010100011\rangle$

$-\beta \mid 010100100\rangle - \beta \mid 010100101\rangle - \beta \mid 010100110\rangle - \beta \mid 010100111\rangle$

$-\beta \mid 010101000\rangle - \beta \mid 010101001\rangle - \beta \mid 010101010\rangle - \beta \mid 010101011\rangle$

$-\beta \mid 010101100\rangle - \beta \mid 010101101\rangle - \beta \mid 010101110\rangle - \beta \mid 010101111\rangle$

$-\beta \mid 100010000\rangle - \beta \mid 100010001\rangle - \beta \mid 100010010\rangle - \beta \mid 100010011\rangle$

$-\beta \mid 100010100\rangle - \beta \mid 100010101\rangle - \beta \mid 100010110\rangle - \beta \mid 100010111\rangle$

$-\beta \mid 100011000\rangle - \beta \mid 100011001\rangle - \beta \mid 100011010\rangle - \beta \mid 100011011\rangle$

$-\beta \mid 100011100\rangle - \beta \mid 100011101\rangle - \beta \mid 100011110\rangle - \beta \mid 100011111\rangle$

$+\beta \mid 010010000\rangle + \beta \mid 010010001\rangle + \beta \mid 010010010\rangle + \beta \mid 010010011\rangle$

$+\beta \mid 010010100\rangle + \beta \mid 010010101\rangle + \beta \mid 010010110\rangle + \beta \mid 010010111\rangle$

$+\beta \mid 010011000\rangle + \beta \mid 010011001\rangle + \beta \mid 010011010\rangle + \beta \mid 010011011\rangle$

$+\beta \mid 010011100\rangle + \beta \mid 010011101\rangle + \beta \mid 010011110\rangle + \beta \mid 010011111\rangle$

$-\beta \mid 110110000\rangle - \beta \mid 110110001\rangle - \beta \mid 110110010\rangle - \beta \mid 110110011\rangle$

$-\beta \mid 110110100\rangle - \beta \mid 110110101\rangle - \beta \mid 110110110\rangle - \beta \mid 110110111\rangle$

$-\beta \mid 110111000\rangle - \beta \mid 110111001\rangle - \beta \mid 110111010\rangle - \beta \mid 110111011\rangle$

$-\beta \mid 110111100\rangle - \beta \mid 110111101\rangle - \beta \mid 110111110\rangle - \beta \mid 110111111\rangle$

$+\beta \mid 000110000\rangle + \beta \mid 000110001\rangle + \beta \mid 000110010\rangle + \beta \mid 000110011\rangle$

$+\beta \mid 000110100\rangle + \beta \mid 000110101\rangle + \beta \mid 000110110\rangle + \beta \mid 000110111\rangle$

$+\beta \mid 000111000\rangle + \beta \mid 000111001\rangle + \beta \mid 000111010\rangle + \beta \mid 000111011\rangle$

$+\beta \mid 000111100\rangle + \beta \mid 000111101\rangle + \beta \mid 000111110\rangle + \beta \mid 000111111\rangle$

-$\beta \mid 111010000\rangle - \beta \mid 111010001\rangle - \beta \mid 111010010\rangle - \beta \mid 111010011\rangle$

$-\beta \mid 111010100\rangle - \beta \mid 111010101\rangle - \beta \mid 111010110\rangle - \beta \mid 111010111\rangle$

$-\beta \mid 111011000\rangle - \beta \mid 111011001\rangle - \beta \mid 111011010\rangle - \beta \mid 111011011\rangle$

$-\beta \mid 111011100\rangle - \beta \mid 111011101\rangle - \beta \mid 111011110\rangle - \beta \mid 111011111\rangle$

$-\beta \mid 001010000\rangle - \beta \mid 001010001\rangle - \beta \mid 001010010\rangle - \beta \mid 001010011\rangle$

$-\beta \mid 001010100\rangle - \beta \mid 001010101\rangle - \beta \mid 001010110\rangle - \beta \mid 001010111\rangle$

$-\beta \mid 001011000\rangle - \beta \mid 001011001\rangle - \beta \mid 001011010\rangle - \beta \mid 001011011\rangle$

$-\beta \mid 001011100\rangle - \beta \mid 001011101\rangle - \beta \mid 001011110\rangle - \beta \mid 001011111\rangle$

$+\beta \mid 101110000\rangle + \beta \mid 101110001\rangle + \beta \mid 101110010\rangle + \beta \mid 101110011\rangle$

$+\beta \mid 101110100\rangle + \beta \mid 101110101\rangle + \beta \mid 101110110\rangle + \beta \mid 101110111\rangle$

$+\beta \mid 101111000\rangle + \beta \mid 101111001\rangle + \beta \mid 101111010\rangle + \beta \mid 101111011\rangle$

$+\beta \mid 101111100\rangle + \beta \mid 101111101\rangle + \beta \mid 101111110\rangle + \beta \mid 101111111\rangle$

$+\beta \mid 011110000\rangle + \beta \mid 011110001\rangle + \beta \mid 011110010\rangle + \beta \mid 011110011\rangle$

$+\beta \mid 011110100\rangle + \beta \mid 011110101\rangle + \beta \mid 011110110\rangle + \beta \mid 011110111\rangle$

$+\beta \mid 011111000\rangle + \beta \mid 011111001\rangle + \beta \mid 011111010\rangle + \beta \mid 011111011\rangle$

$+\beta \mid 011111100\rangle + \beta \mid 011111101\rangle + \beta \mid 011111110\rangle + \beta \mid 011111111\rangle$

-$\beta \mid 101000000\rangle - \beta \mid 101000001\rangle - \beta \mid 101000010\rangle - \beta \mid 101000011\rangle$

$-\beta \mid 101000100\rangle - \beta \mid 101000101\rangle - \beta \mid 101000110\rangle - \beta \mid 101000111\rangle$

$-\beta \mid 101001000\rangle - \beta \mid 101001001\rangle - \beta \mid 101001010\rangle - \beta \mid 101001011\rangle$

$-\beta \mid 101001100\rangle - \beta \mid 101001101\rangle - \beta \mid 101001110\rangle - \beta \mid 101001111\rangle$

$+\beta \mid 011000000\rangle + \beta \mid 011000001\rangle + \beta \mid 011000010\rangle + \beta \mid 011000011\rangle$

$+\beta \mid 011000100\rangle + \beta \mid 011000101\rangle + \beta \mid 011000110\rangle + \beta \mid 011000111\rangle$

$+\beta \mid 011001000\rangle + \beta \mid 011001001\rangle + \beta \mid 011001010\rangle + \beta \mid 011001011\rangle$

$+\beta \mid 011001100\rangle + \beta \mid 011001101\rangle + \beta \mid 011001110\rangle + \beta \mid 011001111\rangle$

$+\beta \mid 111100000\rangle + \beta \mid 111100001\rangle + \beta \mid 111100010\rangle + \beta \mid 111100011\rangle$

$+\beta \mid 111100100\rangle + \beta \mid 111100101\rangle + \beta \mid 111100110\rangle + \beta \mid 111100111\rangle$

$+\beta \mid 111101000\rangle + \beta \mid 111101001\rangle + \beta \mid 111101010\rangle + \beta \mid 111101011\rangle$

$+\beta \mid 111101100\rangle + \beta \mid 111101101\rangle + \beta \mid 111101110\rangle + \beta \mid 111101111\rangle$

$-\beta \mid 001100000\rangle - \beta \mid 001100001\rangle - \beta \mid 001100010\rangle - \beta \mid 001100011\rangle$

$-\beta \mid 001100100\rangle - \beta \mid 001100101\rangle - \beta \mid 001100110\rangle - \beta \mid 001100111\rangle$

$-\beta \mid 001101000\rangle - \beta \mid 001101001\rangle - \beta \mid 001101010\rangle - \beta \mid 001101011\rangle$

$-\beta \mid 001101100\rangle - \beta \mid 001101101\rangle - \beta \mid 001101110\rangle - \beta \mid 001101111\rangle)$

○ **Application controlée des stabilisateurs** :

✓ Pour $M_0 = Z(1)\,X(2)\,X(3)\,Z(4)\,I(5)$

$\mid \Psi_1\rangle = \frac{1}{16}(\alpha \mid 100000000\rangle + \alpha \mid 100000001\rangle + \alpha \mid 100000010\rangle + \alpha \mid 100000011\rangle$

$+\alpha \mid 100000100\rangle + \alpha \mid 100000101\rangle + \alpha \mid 100000110\rangle + \alpha \mid 100000111\rangle$

$-\alpha \mid 111001000\rangle - \alpha \mid 111001001\rangle - \alpha \mid 111001010\rangle - \alpha \mid 111001011\rangle$

$-\alpha \mid 111001100\rangle - \alpha \mid 111001101\rangle - \alpha \mid 111001110\rangle - \alpha \mid 111001111\rangle$

$-\alpha \mid 110010000\rangle - \alpha \mid 110010001\rangle - \alpha \mid 110010010\rangle - \alpha \mid 110010011\rangle$

$-\alpha \mid 110010100\rangle - \alpha \mid 110010101\rangle - \alpha \mid 110010110\rangle - \alpha \mid 110010111\rangle$

$+\alpha \mid 101011000\rangle + \alpha \mid 101011001\rangle + \alpha \mid 101011010\rangle + \alpha \mid 101011011\rangle$

$+\alpha \mid 101011100\rangle + \alpha \mid 101011101\rangle + \alpha \mid 101011110\rangle + \alpha \mid 101011111\rangle$

$+\alpha \mid 000010000\rangle + \alpha \mid 000010001\rangle + \alpha \mid 000010010\rangle + \alpha \mid 000010011\rangle$

$+\alpha \mid 000010100\rangle + \alpha \mid 000010101\rangle + \alpha \mid 000010110\rangle + \alpha \mid 000010111\rangle$

$+\alpha \mid 011011000\rangle + \alpha \mid 011011001\rangle + \alpha \mid 011011010\rangle + \alpha \mid 011011011\rangle$

$+\alpha \mid 011011100\rangle + \alpha \mid 011011101\rangle + \alpha \mid 011011110\rangle + \alpha \mid 011011111\rangle$

$+\alpha \mid 100110000\rangle + \alpha \mid 100110001\rangle + \alpha \mid 100110010\rangle + \alpha \mid 100110011\rangle$

$+\alpha \mid 100110100\rangle + \alpha \mid 100110101\rangle + \alpha \mid 100110110\rangle + \alpha \mid 100110111\rangle$

$+\alpha \mid 111111000\rangle + \alpha \mid 111111001\rangle + \alpha \mid 111111010\rangle + \alpha \mid 111111011\rangle$

$+\alpha \mid 111111100\rangle + \alpha \mid 111111101\rangle + \alpha \mid 111111110\rangle + \alpha \mid 111111111\rangle$

$-\alpha \mid 010110000\rangle - \alpha \mid 010110001\rangle - \alpha \mid 010110010\rangle - \alpha \mid 010110011\rangle$

$-\alpha \mid 010110100\rangle - \alpha \mid 010110101\rangle - \alpha \mid 010110110\rangle - \alpha \mid 010110111\rangle$

$+\alpha \mid 001111000\rangle + \alpha \mid 001111001\rangle + \alpha \mid 001111010\rangle + \alpha \mid 001111011\rangle$

$+\alpha \mid 001111100\rangle + \alpha \mid 001111101\rangle + \alpha \mid 001111110\rangle + \alpha \mid 001111111\rangle$

$+\alpha \mid 010000000\rangle + \alpha \mid 010000001\rangle + \alpha \mid 010000010\rangle + \alpha \mid 010000011\rangle$

$+\alpha \mid 010000100\rangle + \alpha \mid 010000101\rangle + \alpha \mid 010000110\rangle + \alpha \mid 010000111\rangle$

$+\alpha \mid 001001000\rangle + \alpha \mid 001001001\rangle + \alpha \mid 001001010\rangle + \alpha \mid 001001011\rangle$

$+\alpha \mid 001001100\rangle + \alpha \mid 001001101\rangle + \alpha \mid 001001110\rangle + \alpha \mid 001001111\rangle$

$-\alpha \mid 110100000\rangle - \alpha \mid 110100001\rangle - \alpha \mid 110100010\rangle - \alpha \mid 110100011\rangle$

$-\alpha \mid 110100100\rangle - \alpha \mid 110100101\rangle - \alpha \mid 110100110\rangle - \alpha \mid 110100111\rangle$

$-\alpha \mid 101101000\rangle - \alpha \mid 101101001\rangle - \alpha \mid 101101010\rangle - \alpha \mid 101101011\rangle$

$-\alpha \mid 101101100\rangle - \alpha \mid 101101101\rangle - \alpha \mid 101101110\rangle - \alpha \mid 101101111\rangle$

$-\alpha \mid 000100000\rangle - \alpha \mid 000100001\rangle - \alpha \mid 000100010\rangle - \alpha \mid 000100011\rangle$

$-\alpha \mid 000100100\rangle - \alpha \mid 000100101\rangle - \alpha \mid 000100110\rangle - \alpha \mid 000100111\rangle$

$+\alpha \mid 011101000\rangle + \alpha \mid 011101001\rangle + \alpha \mid 011101010\rangle + \alpha \mid 011101011\rangle$

$+\alpha \mid 011101100\rangle + \alpha \mid 011101101\rangle + \alpha \mid 011101110\rangle + \alpha \mid 011101111\rangle$

$+\alpha \mid 111000000\rangle + \alpha \mid 111000001\rangle + \alpha \mid 111000010\rangle + \alpha \mid 111000011\rangle$

$+\alpha \mid 111000100\rangle + \alpha \mid 111000101\rangle + \alpha \mid 111000110\rangle + \alpha \mid 111000111\rangle$

$-\alpha \mid 100001000\rangle - \alpha \mid 100001001\rangle - \alpha \mid 100001010\rangle - \alpha \mid 100001011\rangle$

$-\alpha \mid 100001100\rangle - \alpha \mid 100001101\rangle - \alpha \mid 100001110\rangle - \alpha \mid 100001111\rangle$

$-\alpha \mid 001000000\rangle - \alpha \mid 001000001\rangle - \alpha \mid 001000010\rangle - \alpha \mid 001000011\rangle$

$-\alpha \mid 001000100\rangle - \alpha \mid 001000101\rangle - \alpha \mid 001000110\rangle - \alpha \mid 001000111\rangle$

$-\alpha \mid 010001000\rangle - \alpha \mid 010001001\rangle - \alpha \mid 010001010\rangle - \alpha \mid 010001011\rangle$

$-\alpha \mid 010001100\rangle - \alpha \mid 010001101\rangle - \alpha \mid 010001110\rangle - \alpha \mid 010001111\rangle$

$+\alpha \mid 101100000\rangle + \alpha \mid 101100001\rangle + \alpha \mid 101100010\rangle + \alpha \mid 101100011\rangle$

$+\alpha \mid 101100100\rangle + \alpha \mid 101100101\rangle + \alpha \mid 101100110\rangle + \alpha \mid 101100111\rangle$

$+\alpha \mid 110101000\rangle + \alpha \mid 110101001\rangle + \alpha \mid 110101010\rangle + \alpha \mid 110101011\rangle$

$+\alpha \mid 110101100\rangle + \alpha \mid 110101101\rangle + \alpha \mid 110101110\rangle + \alpha \mid 110101111\rangle$

$-\alpha \mid 011100000\rangle - \alpha \mid 011100001\rangle - \alpha \mid 011100010\rangle - \alpha \mid 011100011\rangle$

$-\alpha \mid 011100100\rangle - \alpha \mid 011100101\rangle - \alpha \mid 011100110\rangle - \alpha \mid 011100111\rangle$

$+\alpha \mid 000101000\rangle + \alpha \mid 000101001\rangle + \alpha \mid 000101010\rangle + \alpha \mid 000101011\rangle$

$+\alpha \mid 000101100\rangle + \alpha \mid 000101101\rangle + \alpha \mid 000101110\rangle + \alpha \mid 000101111\rangle$

-$\alpha \mid 101010000\rangle - \alpha \mid 101010001\rangle - \alpha \mid 101010010\rangle - \alpha \mid 101010011\rangle$

$-\alpha \mid 101010100\rangle - \alpha \mid 101010101\rangle - \alpha \mid 101010110\rangle - \alpha \mid 101010111\rangle$

$+\alpha \mid 110011000\rangle + \alpha \mid 110011001\rangle + \alpha \mid 110011010\rangle + \alpha \mid 110011011\rangle$

$+\alpha \mid 110011100\rangle + \alpha \mid 110011101\rangle + \alpha \mid 110011110\rangle + \alpha \mid 110011111\rangle$

$-\alpha \mid 011010000\rangle - \alpha \mid 011010001\rangle - \alpha \mid 011010010\rangle - \alpha \mid 011010011\rangle$

$$-\alpha \mid 011010100\rangle - \alpha \mid 011010101\rangle - \alpha \mid 011010110\rangle - \alpha \mid 011010111\rangle$$
$$-\alpha \mid 000011000\rangle - \alpha \mid 000011001\rangle - \alpha \mid 000011010\rangle - \alpha \mid 000011011\rangle$$
$$-\alpha \mid 000011100\rangle - \alpha \mid 000011101\rangle - \alpha \mid 000011110\rangle - \alpha \mid 000011111\rangle$$
$$-\alpha \mid 111110000\rangle - \alpha \mid 111110001\rangle - \alpha \mid 111110010\rangle - \alpha \mid 111110011\rangle$$
$$-\alpha \mid 111110100\rangle - \alpha \mid 111110101\rangle - \alpha \mid 111110110\rangle - \alpha \mid 111110111\rangle$$
$$-\alpha \mid 100111000\rangle - \alpha \mid 100111001\rangle - \alpha \mid 100111010\rangle - \alpha \mid 100111011\rangle$$
$$-\alpha \mid 100111100\rangle - \alpha \mid 100111101\rangle - \alpha \mid 100111110\rangle - \alpha \mid 100111111\rangle$$
$$-\alpha \mid 001110000\rangle - \alpha \mid 001110001\rangle - \alpha \mid 001110010\rangle - \alpha \mid 001110011\rangle$$
$$-\alpha \mid 001110100\rangle - \alpha \mid 001110101\rangle - \alpha \mid 001110110\rangle - \alpha \mid 001110111\rangle$$
$$+\alpha \mid 010111000\rangle + \alpha \mid 010111001\rangle + \alpha \mid 010111010\rangle + \alpha \mid 010111011\rangle$$
$$+\alpha \mid 010111100\rangle + \alpha \mid 010111101\rangle + \alpha \mid 010111110\rangle + \alpha \mid 010111111\rangle$$
$$-\beta \mid 110000000\rangle - \beta \mid 110000001\rangle - \beta \mid 110000010\rangle - \beta \mid 110000011\rangle$$
$$-\beta \mid 110000100\rangle - \beta \mid 110000101\rangle - \beta \mid 110000110\rangle - \beta \mid 110000111\rangle$$
$$+\beta \mid 101001000\rangle + \beta \mid 101001001\rangle + \beta \mid 101001010\rangle + \beta \mid 101001011\rangle$$
$$+\beta \mid 101001100\rangle + \beta \mid 101001101\rangle + \beta \mid 101001110\rangle + \beta \mid 101001111\rangle$$
$$-\beta \mid 000000000\rangle - \beta \mid 000000001\rangle - \beta \mid 000000010\rangle - \beta \mid 000000011\rangle$$
$$-\beta \mid 000000100\rangle - \beta \mid 000000101\rangle - \beta \mid 000000110\rangle - \beta \mid 000000111\rangle$$
$$-\beta \mid 011001000\rangle - \beta \mid 011001001\rangle - \beta \mid 011001010\rangle - \beta \mid 011001011\rangle$$
$$-\beta \mid 011001100\rangle - \beta \mid 011001101\rangle - \beta \mid 011001110\rangle - \beta \mid 011001111\rangle$$
$$-\beta \mid 100100000\rangle - \beta \mid 100100001\rangle - \beta \mid 100100010\rangle - \beta \mid 100100011\rangle$$
$$-\beta \mid 100100100\rangle - \beta \mid 100100101\rangle - \beta \mid 100100110\rangle - \beta \mid 100100111\rangle$$
$$-\beta \mid 111101000\rangle - \beta \mid 111101001\rangle - \beta \mid 111101010\rangle - \beta \mid 111101011\rangle$$
$$-\beta \mid 111101100\rangle - \beta \mid 111101101\rangle - \beta \mid 111101110\rangle - \beta \mid 111101111\rangle$$
$$-\beta \mid 010100000\rangle - \beta \mid 010100001\rangle - \beta \mid 010100010\rangle - \beta \mid 010100011\rangle$$
$$-\beta \mid 010100100\rangle - \beta \mid 010100101\rangle - \beta \mid 010100110\rangle - \beta \mid 010100111\rangle$$
$$+\beta \mid 001101000\rangle + \beta \mid 001101001\rangle + \beta \mid 001101010\rangle + \beta \mid 001101011\rangle$$
$$+\beta \mid 001101100\rangle + \beta \mid 001101101\rangle + \beta \mid 001101110\rangle + \beta \mid 001101111\rangle$$
$$-\beta \mid 100010000\rangle - \beta \mid 100010001\rangle - \beta \mid 100010010\rangle - \beta \mid 100010011\rangle$$
$$-\beta \mid 100010100\rangle - \beta \mid 100010101\rangle - \beta \mid 100010110\rangle - \beta \mid 100010111\rangle$$
$$+\beta \mid 111011000\rangle + \beta \mid 111011001\rangle + \beta \mid 111011010\rangle + \beta \mid 111011011\rangle$$
$$+\beta \mid 111011100\rangle + \beta \mid 111011101\rangle + \beta \mid 111011110\rangle + \beta \mid 111011111\rangle$$
$$+\beta \mid 010010000\rangle + \beta \mid 010010001\rangle + \beta \mid 010010010\rangle + \beta \mid 010010011\rangle$$

$+\beta \mid 010010100\rangle + \beta \mid 010010101\rangle + \beta \mid 010010110\rangle + \beta \mid 010010111\rangle$

$+\beta \mid 001011000\rangle + \beta \mid 001011001\rangle + \beta \mid 001011010\rangle + \beta \mid 001011011\rangle$

$+\beta \mid 001011100\rangle + \beta \mid 001011101\rangle + \beta \mid 001011110\rangle + \beta \mid 001011111\rangle$

$-\beta \mid 110110000\rangle - \beta \mid 110110001\rangle - \beta \mid 110110010\rangle - \beta \mid 110110011\rangle$

$-\beta \mid 110110100\rangle - \beta \mid 110110101\rangle - \beta \mid 110110110\rangle - \beta \mid 110110111\rangle$

$-\beta \mid 101111000\rangle - \beta \mid 101111001\rangle - \beta \mid 101111010\rangle - \beta \mid 101111011\rangle$

$-\beta \mid 101111100\rangle - \beta \mid 101111101\rangle - \beta \mid 101111110\rangle - \beta \mid 101111111\rangle$

$+\beta \mid 000110000\rangle + \beta \mid 000110001\rangle + \beta \mid 000110010\rangle + \beta \mid 000110011\rangle$

$+\beta \mid 000110100\rangle + \beta \mid 000110101\rangle + \beta \mid 000110110\rangle + \beta \mid 000110111\rangle$

$-\beta \mid 011111000\rangle - \beta \mid 011111001\rangle - \beta \mid 011111010\rangle - \beta \mid 011111011\rangle$

$-\beta \mid 011111100\rangle - \beta \mid 011111101\rangle - \beta \mid 011111110\rangle - \beta \mid 011111111\rangle$

-$\beta \mid 111010000\rangle - \beta \mid 111010001\rangle - \beta \mid 111010010\rangle - \beta \mid 111010011\rangle$

$-\beta \mid 111010100\rangle - \beta \mid 111010101\rangle - \beta \mid 111010110\rangle - \beta \mid 111010111\rangle$

$+\beta \mid 100011000\rangle + \beta \mid 100011001\rangle + \beta \mid 100011010\rangle + \beta \mid 100011011\rangle$

$+\beta \mid 100011100\rangle + \beta \mid 100011101\rangle + \beta \mid 100011110\rangle + \beta \mid 100011111\rangle$

$-\beta \mid 001010000\rangle - \beta \mid 001010001\rangle - \beta \mid 001010010\rangle - \beta \mid 001010011\rangle$

$-\beta \mid 001010100\rangle - \beta \mid 001010101\rangle - \beta \mid 001010110\rangle - \beta \mid 001010111\rangle$

$-\beta \mid 010011000\rangle - \beta \mid 010011001\rangle - \beta \mid 010011010\rangle - \beta \mid 010011011\rangle$

$-\beta \mid 010011100\rangle - \beta \mid 010011101\rangle - \beta \mid 010011110\rangle - \beta \mid 010011111\rangle$

$+\beta \mid 101110000\rangle + \beta \mid 101110001\rangle + \beta \mid 101110010\rangle + \beta \mid 101110011\rangle$

$+\beta \mid 101110100\rangle + \beta \mid 101110101\rangle + \beta \mid 101110110\rangle + \beta \mid 101110111\rangle$

$+\beta \mid 110111000\rangle + \beta \mid 110111001\rangle + \beta \mid 110111010\rangle + \beta \mid 110111011\rangle$

$+\beta \mid 110111100\rangle + \beta \mid 110111101\rangle + \beta \mid 110111110\rangle + \beta \mid 110111111\rangle$

$+\beta \mid 011110000\rangle + \beta \mid 011110001\rangle + \beta \mid 011110010\rangle + \beta \mid 011110011\rangle$

$+\beta \mid 011110100\rangle + \beta \mid 011110101\rangle + \beta \mid 011110110\rangle + \beta \mid 011110111\rangle$

$-\beta \mid 000111000\rangle - \beta \mid 000111001\rangle - \beta \mid 000111010\rangle - \beta \mid 000111011\rangle$

$-\beta \mid 000111100\rangle - \beta \mid 000111101\rangle - \beta \mid 000111110\rangle - \beta \mid 000111111\rangle$

-$\beta \mid 101000000\rangle - \beta \mid 101000001\rangle - \beta \mid 101000010\rangle - \beta \mid 101000011\rangle$

$-\beta \mid 101000100\rangle - \beta \mid 101000101\rangle - \beta \mid 101000110\rangle - \beta \mid 101000111\rangle$

$+\beta \mid 110001000\rangle + \beta \mid 110001001\rangle + \beta \mid 110001010\rangle + \beta \mid 110001011\rangle$

$+\beta \mid 110001100\rangle + \beta \mid 110001101\rangle + \beta \mid 110001110\rangle + \beta \mid 110001111\rangle$

$+\beta \mid 011000000\rangle + \beta \mid 011000001\rangle + \beta \mid 011000010\rangle + \beta \mid 011000011\rangle$

$+\beta \mid 011000100\rangle + \beta \mid 011000101\rangle + \beta \mid 011000110\rangle + \beta \mid 011000111\rangle$

$+\beta \mid 000001000\rangle + \beta \mid 000001001\rangle + \beta \mid 000001010\rangle + \beta \mid 000001011\rangle$

$+\beta \mid 000001100\rangle + \beta \mid 000001101\rangle + \beta \mid 000001110\rangle + \beta \mid 000001111\rangle$

$+\beta \mid 111100000\rangle + \beta \mid 111100001\rangle + \beta \mid 111100010\rangle + \beta \mid 111100011\rangle$

$+\beta \mid 111100100\rangle + \beta \mid 111100101\rangle + \beta \mid 111100110\rangle + \beta \mid 111100111\rangle$

$+\beta \mid 100101000\rangle + \beta \mid 100101001\rangle + \beta \mid 100101010\rangle + \beta \mid 100101011\rangle$

$+\beta \mid 100101100\rangle + \beta \mid 100101101\rangle + \beta \mid 100101110\rangle + \beta \mid 100101111\rangle$

$-\beta \mid 001100000\rangle - \beta \mid 001100001\rangle - \beta \mid 001100010\rangle - \beta \mid 001100011\rangle$

$-\beta \mid 001100100\rangle - \beta \mid 001100101\rangle - \beta \mid 001100110\rangle - \beta \mid 001100111\rangle$

$+\beta \mid 010101000\rangle + \beta \mid 010101001\rangle + \beta \mid 010101010\rangle + \beta \mid 010101011\rangle$

$+\beta \mid 010101100\rangle + \beta \mid 010101101\rangle + \beta \mid 010101110\rangle + \beta \mid 010101111\rangle)$

✓ Pour $M_1 = X(1)\,X(2)\,Z(3)\,I(4)\,Z(5)$

$\mid \Psi_1\rangle = \frac{1}{16}(\alpha \mid 100000000\rangle + \alpha \mid 100000001\rangle + \alpha \mid 100000010\rangle + \alpha \mid 100000011\rangle$

$+\alpha \mid 010000100\rangle + \alpha \mid 010000101\rangle + \alpha \mid 010000110\rangle + \alpha \mid 010000111\rangle$

$-\alpha \mid 111001000\rangle - \alpha \mid 111001001\rangle - \alpha \mid 111001010\rangle - \alpha \mid 111001011\rangle$

$+\alpha \mid 001001100\rangle + \alpha \mid 001001101\rangle + \alpha \mid 001001110\rangle + \alpha \mid 001001111\rangle$

$-\alpha \mid 110010000\rangle - \alpha \mid 110010001\rangle - \alpha \mid 110010010\rangle - \alpha \mid 110010011\rangle$

$+\alpha \mid 000010100\rangle + \alpha \mid 000010101\rangle + \alpha \mid 000010110\rangle + \alpha \mid 000010111\rangle$

$+\alpha \mid 101011000\rangle + \alpha \mid 101011001\rangle + \alpha \mid 101011010\rangle + \alpha \mid 101011011\rangle$

$+\alpha \mid 011011100\rangle + \alpha \mid 011011101\rangle + \alpha \mid 011011110\rangle + \alpha \mid 011011111\rangle$

$+\alpha \mid 000010000\rangle + \alpha \mid 000010001\rangle + \alpha \mid 000010010\rangle + \alpha \mid 000010011\rangle$

$-\alpha \mid 110010100\rangle - \alpha \mid 110010101\rangle - \alpha \mid 110010110\rangle - \alpha \mid 110010111\rangle$

$+\alpha \mid 011011000\rangle + \alpha \mid 011011001\rangle + \alpha \mid 011011010\rangle + \alpha \mid 011011011\rangle$

$+\alpha \mid 101011100\rangle + \alpha \mid 101011101\rangle + \alpha \mid 101011110\rangle + \alpha \mid 101011111\rangle$

$+\alpha \mid 100110000\rangle + \alpha \mid 100110001\rangle + \alpha \mid 100110010\rangle + \alpha \mid 100110011\rangle$

$-\alpha \mid 010110100\rangle - \alpha \mid 010110101\rangle - \alpha \mid 010110110\rangle - \alpha \mid 010110111\rangle$

$+\alpha \mid 111111000\rangle + \alpha \mid 111111001\rangle + \alpha \mid 111111010\rangle + \alpha \mid 111111011\rangle$

$+\alpha \mid 001111100\rangle + \alpha \mid 001111101\rangle + \alpha \mid 001111110\rangle + \alpha \mid 001111111\rangle$

$-\alpha \mid 010110000\rangle - \alpha \mid 010110001\rangle - \alpha \mid 010110010\rangle - \alpha \mid 010110011\rangle$

$+\alpha \mid 100110100\rangle + \alpha \mid 100110101\rangle + \alpha \mid 100110110\rangle + \alpha \mid 100110111\rangle$

$+\alpha \mid 001111000\rangle + \alpha \mid 001111001\rangle + \alpha \mid 001111010\rangle + \alpha \mid 001111011\rangle$

$+\alpha \mid 111111100\rangle + \alpha \mid 111111101\rangle + \alpha \mid 111111110\rangle + \alpha \mid 111111111\rangle$

$+\alpha \mid 010000000\rangle + \alpha \mid 010000001\rangle + \alpha \mid 010000010\rangle + \alpha \mid 010000011\rangle$

$+\alpha \mid 100000100\rangle + \alpha \mid 100000101\rangle + \alpha \mid 100000110\rangle + \alpha \mid 100000111\rangle$

$+\alpha \mid 001001000\rangle + \alpha \mid 001001001\rangle + \alpha \mid 001001010\rangle + \alpha \mid 001001011\rangle$

$-\alpha \mid 111001100\rangle - \alpha \mid 111001101\rangle - \alpha \mid 111001110\rangle - \alpha \mid 111001111\rangle$

$-\alpha \mid 110100000\rangle - \alpha \mid 110100001\rangle - \alpha \mid 110100010\rangle - \alpha \mid 110100011\rangle$

$-\alpha \mid 000100100\rangle - \alpha \mid 000100101\rangle - \alpha \mid 000100110\rangle - \alpha \mid 000100111\rangle$

$-\alpha \mid 101101000\rangle - \alpha \mid 101101001\rangle - \alpha \mid 101101010\rangle - \alpha \mid 101101011\rangle$

$+\alpha \mid 011101100\rangle + \alpha \mid 011101101\rangle + \alpha \mid 011101110\rangle + \alpha \mid 011101111\rangle$

$-\alpha \mid 000100000\rangle - \alpha \mid 000100001\rangle - \alpha \mid 000100010\rangle - \alpha \mid 000100011\rangle$

$-\alpha \mid 110100100\rangle - \alpha \mid 110100101\rangle - \alpha \mid 110100110\rangle - \alpha \mid 110100111\rangle$

$+\alpha \mid 011101000\rangle + \alpha \mid 011101001\rangle + \alpha \mid 011101010\rangle + \alpha \mid 011101011\rangle$

$-\alpha \mid 101101100\rangle - \alpha \mid 101101101\rangle - \alpha \mid 101101110\rangle - \alpha \mid 101101111\rangle$

$+\alpha \mid 111000000\rangle + \alpha \mid 111000001\rangle + \alpha \mid 111000010\rangle + \alpha \mid 111000011\rangle$

$-\alpha \mid 001000100\rangle - \alpha \mid 001000101\rangle - \alpha \mid 001000110\rangle - \alpha \mid 001000111\rangle$

$-\alpha \mid 100001000\rangle - \alpha \mid 100001001\rangle - \alpha \mid 100001010\rangle - \alpha \mid 100001011\rangle$

$-\alpha \mid 010001100\rangle - \alpha \mid 010001101\rangle - \alpha \mid 010001110\rangle - \alpha \mid 010001111\rangle$

$-\alpha \mid 001000000\rangle - \alpha \mid 001000001\rangle - \alpha \mid 001000010\rangle - \alpha \mid 001000011\rangle$

$+\alpha \mid 111000100\rangle + \alpha \mid 111000101\rangle + \alpha \mid 111000110\rangle + \alpha \mid 111000111\rangle$

$-\alpha \mid 010001000\rangle - \alpha \mid 010001001\rangle - \alpha \mid 010001010\rangle - \alpha \mid 010001011\rangle$

$-\alpha \mid 100001100\rangle - \alpha \mid 100001101\rangle - \alpha \mid 100001110\rangle - \alpha \mid 100001111\rangle$

$+\alpha \mid 101100000\rangle + \alpha \mid 101100001\rangle + \alpha \mid 101100010\rangle + \alpha \mid 101100011\rangle$

$-\alpha \mid 011100100\rangle - \alpha \mid 011100101\rangle - \alpha \mid 011100110\rangle - \alpha \mid 011100111\rangle$

$+\alpha \mid 110101000\rangle + \alpha \mid 110101001\rangle + \alpha \mid 110101010\rangle + \alpha \mid 110101011\rangle$

$+\alpha \mid 000101100\rangle + \alpha \mid 000101101\rangle + \alpha \mid 000101110\rangle + \alpha \mid 000101111\rangle$

$-\alpha \mid 011100000\rangle - \alpha \mid 011100001\rangle - \alpha \mid 011100010\rangle - \alpha \mid 011100011\rangle$

$+\alpha \mid 101100100\rangle + \alpha \mid 101100101\rangle + \alpha \mid 101100110\rangle + \alpha \mid 101100111\rangle$

$+\alpha \mid 000101000\rangle + \alpha \mid 000101001\rangle + \alpha \mid 000101010\rangle + \alpha \mid 000101011\rangle$

$+\alpha \mid 110101100\rangle + \alpha \mid 110101101\rangle + \alpha \mid 110101110\rangle + \alpha \mid 110101111\rangle$

-$\alpha \mid 101010000\rangle - \alpha \mid 101010001\rangle - \alpha \mid 101010010\rangle - \alpha \mid 101010011\rangle$

$-\alpha \mid 011010100\rangle - \alpha \mid 011010101\rangle - \alpha \mid 011010110\rangle - \alpha \mid 011010111\rangle$

$+\alpha \mid 110011000\rangle + \alpha \mid 110011001\rangle + \alpha \mid 110011010\rangle + \alpha \mid 110011011\rangle$

$-\alpha \mid 000011100\rangle - \alpha \mid 000011101\rangle - \alpha \mid 000011110\rangle - \alpha \mid 000011111\rangle$

$-\alpha \mid 011010000\rangle - \alpha \mid 011010001\rangle - \alpha \mid 011010010\rangle - \alpha \mid 011010011\rangle$

$-\alpha \mid 101010100\rangle - \alpha \mid 101010101\rangle - \alpha \mid 101010110\rangle - \alpha \mid 101010111\rangle$

$-\alpha \mid 000011000\rangle - \alpha \mid 000011001\rangle - \alpha \mid 000011010\rangle - \alpha \mid 000011011\rangle$

$+\alpha \mid 110011100\rangle + \alpha \mid 110011101\rangle + \alpha \mid 110011110\rangle + \alpha \mid 110011111\rangle$

$-\alpha \mid 111110000\rangle - \alpha \mid 111110001\rangle - \alpha \mid 111110010\rangle - \alpha \mid 111110011\rangle$

$-\alpha \mid 001110100\rangle - \alpha \mid 001110101\rangle - \alpha \mid 001110110\rangle - \alpha \mid 001110111\rangle$

$-\alpha \mid 100111000\rangle - \alpha \mid 100111001\rangle - \alpha \mid 100111010\rangle - \alpha \mid 100111011\rangle$

$+\alpha \mid 010111100\rangle + \alpha \mid 010111101\rangle + \alpha \mid 010111110\rangle + \alpha \mid 010111111\rangle$

$-\alpha \mid 001110000\rangle - \alpha \mid 001110001\rangle - \alpha \mid 001110010\rangle - \alpha \mid 001110011\rangle$

$-\alpha \mid 111110100\rangle - \alpha \mid 111110101\rangle - \alpha \mid 111110110\rangle - \alpha \mid 111110111\rangle$

$+\alpha \mid 010111000\rangle + \alpha \mid 010111001\rangle + \alpha \mid 010111010\rangle + \alpha \mid 010111011\rangle$

$-\alpha \mid 100111100\rangle - \alpha \mid 100111101\rangle - \alpha \mid 100111110\rangle - \alpha \mid 100111111\rangle$

$-\beta \mid 110000000\rangle - \beta \mid 110000001\rangle - \beta \mid 110000010\rangle - \beta \mid 110000011\rangle$

$-\beta \mid 000000100\rangle - \beta \mid 000000101\rangle - \beta \mid 000000110\rangle - \beta \mid 000000111\rangle$

$+\beta \mid 101001000\rangle + \beta \mid 101001001\rangle + \beta \mid 101001010\rangle + \beta \mid 101001011\rangle$

$-\beta \mid 011001100\rangle - \beta \mid 011001101\rangle - \beta \mid 011001110\rangle - \beta \mid 011001111\rangle$

$-\beta \mid 000000000\rangle - \beta \mid 000000001\rangle - \beta \mid 000000010\rangle - \beta \mid 000000011\rangle$

$-\beta \mid 110000100\rangle - \beta \mid 110000101\rangle - \beta \mid 110000110\rangle - \beta \mid 110000111\rangle$

$-\beta \mid 011001000\rangle - \beta \mid 011001001\rangle - \beta \mid 011001010\rangle - \beta \mid 011001011\rangle$

$+\beta \mid 101001100\rangle + \beta \mid 101001101\rangle + \beta \mid 101001110\rangle + \beta \mid 101001111\rangle$

$-\beta \mid 100100000\rangle - \beta \mid 100100001\rangle - \beta \mid 100100010\rangle - \beta \mid 100100011\rangle$

$-\beta \mid 010100100\rangle - \beta \mid 010100101\rangle - \beta \mid 010100110\rangle - \beta \mid 010100111\rangle$

$-\beta \mid 111101000\rangle - \beta \mid 111101001\rangle - \beta \mid 111101010\rangle - \beta \mid 111101011\rangle$

$+\beta \mid 001101100\rangle + \beta \mid 001101101\rangle + \beta \mid 001101110\rangle + \beta \mid 001101111\rangle$

$-\beta \mid 010100000\rangle - \beta \mid 010100001\rangle - \beta \mid 010100010\rangle - \beta \mid 010100011\rangle$

$-\beta \mid 100100100\rangle - \beta \mid 100100101\rangle - \beta \mid 100100110\rangle - \beta \mid 100100111\rangle$

$+\beta \mid 001101000\rangle + \beta \mid 001101001\rangle + \beta \mid 001101010\rangle + \beta \mid 001101011\rangle$

$-\beta \mid 111101100\rangle - \beta \mid 111101101\rangle - \beta \mid 111101110\rangle - \beta \mid 111101111\rangle$

$-\beta \mid 100010000\rangle - \beta \mid 100010001\rangle - \beta \mid 100010010\rangle - \beta \mid 100010011\rangle$

$+\beta \mid 010010100\rangle + \beta \mid 010010101\rangle + \beta \mid 010010110\rangle + \beta \mid 010010111\rangle$

$+\beta \mid 111011000\rangle + \beta \mid 111011001\rangle + \beta \mid 111011010\rangle + \beta \mid 111011011\rangle$

$+\beta \mid 001011100\rangle + \beta \mid 001011101\rangle + \beta \mid 001011110\rangle + \beta \mid 001011111\rangle$

$+\beta \mid 010010000\rangle + \beta \mid 010010001\rangle + \beta \mid 010010010\rangle + \beta \mid 010010011\rangle$

$-\beta \mid 100010100\rangle - \beta \mid 100010101\rangle - \beta \mid 100010110\rangle - \beta \mid 100010111\rangle$

$+\beta \mid 001011000\rangle + \beta \mid 001011001\rangle + \beta \mid 001011010\rangle + \beta \mid 001011011\rangle$

$+\beta \mid 111011100\rangle + \beta \mid 111011101\rangle + \beta \mid 111011110\rangle + \beta \mid 111011111\rangle$

$-\beta \mid 110110000\rangle - \beta \mid 110110001\rangle - \beta \mid 110110010\rangle - \beta \mid 110110011\rangle$

$+\beta \mid 000110100\rangle + \beta \mid 000110101\rangle + \beta \mid 000110110\rangle + \beta \mid 000110111\rangle$

$-\beta \mid 101111000\rangle - \beta \mid 101111001\rangle - \beta \mid 101111010\rangle - \beta \mid 101111011\rangle$

$-\beta \mid 011111100\rangle - \beta \mid 011111101\rangle - \beta \mid 011111110\rangle - \beta \mid 011111111\rangle$

$+\beta \mid 000110000\rangle + \beta \mid 000110001\rangle + \beta \mid 000110010\rangle + \beta \mid 000110011\rangle$

$-\beta \mid 110110100\rangle - \beta \mid 110110101\rangle - \beta \mid 110110110\rangle - \beta \mid 110110111\rangle$

$-\beta \mid 011111000\rangle - \beta \mid 011111001\rangle - \beta \mid 011111010\rangle - \beta \mid 011111011\rangle$

$-\beta \mid 101111100\rangle - \beta \mid 101111101\rangle - \beta \mid 101111110\rangle - \beta \mid 101111111\rangle$

-$\beta \mid 111010000\rangle - \beta \mid 111010001\rangle - \beta \mid 111010010\rangle - \beta \mid 111010011\rangle$

$-\beta \mid 001010100\rangle - \beta \mid 001010101\rangle - \beta \mid 001010110\rangle - \beta \mid 001010111\rangle$

$+\beta \mid 100011000\rangle + \beta \mid 100011001\rangle + \beta \mid 100011010\rangle + \beta \mid 100011011\rangle$

$-\beta \mid 010011100\rangle - \beta \mid 010011101\rangle - \beta \mid 010011110\rangle - \beta \mid 010011111\rangle$

$-\beta \mid 001010000\rangle - \beta \mid 001010001\rangle - \beta \mid 001010010\rangle - \beta \mid 001010011\rangle$

$-\beta \mid 111010100\rangle - \beta \mid 111010101\rangle - \beta \mid 111010110\rangle - \beta \mid 111010111\rangle$

$-\beta \mid 010011000\rangle - \beta \mid 010011001\rangle - \beta \mid 010011010\rangle - \beta \mid 010011011\rangle$

$+\beta \mid 100011100\rangle + \beta \mid 100011101\rangle + \beta \mid 100011110\rangle + \beta \mid 100011111\rangle$

$+\beta \mid 101110000\rangle + \beta \mid 101110001\rangle + \beta \mid 101110010\rangle + \beta \mid 101110011\rangle$

$+\beta \mid 011110100\rangle + \beta \mid 011110101\rangle + \beta \mid 011110110\rangle + \beta \mid 011110111\rangle$

$+\beta \mid 110111000\rangle + \beta \mid 110111001\rangle + \beta \mid 110111010\rangle + \beta \mid 110111011\rangle$

$-\beta \mid 000111100\rangle - \beta \mid 000111101\rangle - \beta \mid 000111110\rangle - \beta \mid 000111111\rangle$

$+\beta \mid 011110000\rangle + \beta \mid 011110001\rangle + \beta \mid 011110010\rangle + \beta \mid 011110011\rangle$

$+\beta \mid 101110100\rangle + \beta \mid 101110101\rangle + \beta \mid 101110110\rangle + \beta \mid 101110111\rangle$

$-\beta \mid 000111000\rangle - \beta \mid 000111001\rangle - \beta \mid 000111010\rangle - \beta \mid 000111011\rangle$

$+\beta \mid 110111100\rangle + \beta \mid 110111101\rangle + \beta \mid 110111110\rangle + \beta \mid 110111111\rangle$

-$\beta \mid 101000000\rangle - \beta \mid 101000001\rangle - \beta \mid 101000010\rangle - \beta \mid 101000011\rangle$

$+\beta \mid 011000100\rangle + \beta \mid 011000101\rangle + \beta \mid 011000110\rangle + \beta \mid 011000111\rangle$

$+\beta \mid 110001000\rangle + \beta \mid 110001001\rangle + \beta \mid 110001010\rangle + \beta \mid 110001011\rangle$

$-\beta \mid 000001100\rangle - \beta \mid 000001101\rangle - \beta \mid 000001110\rangle - \beta \mid 000001111\rangle$

$+\beta \mid 011000000\rangle + \beta \mid 011000001\rangle + \beta \mid 011000010\rangle + \beta \mid 011000011\rangle$

$-\beta \mid 101000100\rangle - \beta \mid 101000101\rangle - \beta \mid 101000110\rangle - \beta \mid 101000111\rangle$

$+\beta \mid 000001000\rangle + \beta \mid 000001001\rangle + \beta \mid 000001010\rangle + \beta \mid 000001011\rangle$

$+\beta \mid 110001100\rangle + \beta \mid 110001101\rangle + \beta \mid 110001110\rangle + \beta \mid 110001111\rangle$

$+\beta \mid 111100000\rangle + \beta \mid 111100001\rangle + \beta \mid 111100010\rangle + \beta \mid 111100011\rangle$

$-\beta \mid 001100100\rangle - \beta \mid 001100101\rangle - \beta \mid 001100110\rangle - \beta \mid 001100111\rangle$

$+\beta \mid 100101000\rangle + \beta \mid 100101001\rangle + \beta \mid 100101010\rangle + \beta \mid 100101011\rangle$

$+\beta \mid 010101100\rangle + \beta \mid 010101101\rangle + \beta \mid 010101110\rangle + \beta \mid 010101111\rangle$

$-\beta \mid 001100000\rangle - \beta \mid 001100001\rangle - \beta \mid 001100010\rangle - \beta \mid 001100011\rangle$

$+\beta \mid 111100100\rangle + \beta \mid 111100101\rangle + \beta \mid 111100110\rangle + \beta \mid 111100111\rangle$

$+\beta \mid 010101000\rangle + \beta \mid 010101001\rangle + \beta \mid 010101010\rangle + \beta \mid 010101011\rangle$

$+\beta \mid 010101100\rangle + \beta \mid 010101101\rangle + \beta \mid 010101110\rangle + \beta \mid 010101111\rangle)$

✓ <u>Pour $M_2 = X(1)\, Z(2)\, I(3)\, Z(4)\, X(5)$</u>

$\mid \Psi_1\rangle = \dfrac{1}{16}(\alpha \mid 100000000\rangle + \alpha \mid 100000001\rangle + \alpha \mid 000010010\rangle + \alpha \mid 000010011\rangle$

$+\alpha \mid 010000100\rangle + \alpha \mid 010000101\rangle - \alpha \mid 110010110\rangle - \alpha \mid 110010111\rangle$

$-\alpha \mid 111001000\rangle - \alpha \mid 111001001\rangle + \alpha \mid 011011010\rangle + \alpha \mid 011011011\rangle$

$+\alpha \mid 001001100\rangle + \alpha \mid 001001101\rangle + \alpha \mid 101011110\rangle + \alpha \mid 101011111\rangle$

$-\alpha \mid 110010000\rangle - \alpha \mid 110010001\rangle + \alpha \mid 010000010\rangle + \alpha \mid 010000011\rangle$

$+\alpha \mid 000010100\rangle + \alpha \mid 000010101\rangle + \alpha \mid 100000110\rangle + \alpha \mid 100000111\rangle$

$+\alpha \mid 101011000\rangle + \alpha \mid 101011001\rangle + \alpha \mid 001001010\rangle + \alpha \mid 001001011\rangle$

$+\alpha \mid 011011100\rangle + \alpha \mid 011011101\rangle - \alpha \mid 111001110\rangle - \alpha \mid 111001111\rangle$

$+\alpha \mid 000010000\rangle + \alpha \mid 000010001\rangle + \alpha \mid 100000010\rangle + \alpha \mid 100000011\rangle$

$-\alpha \mid 110010100\rangle - \alpha \mid 110010101\rangle + \alpha \mid 010000110\rangle + \alpha \mid 010000111\rangle$

$+\alpha \mid 011011000\rangle + \alpha \mid 011011001\rangle - \alpha \mid 111001010\rangle - \alpha \mid 111001011\rangle$

$+\alpha \mid 101011100\rangle + \alpha \mid 101011101\rangle + \alpha \mid 001001110\rangle + \alpha \mid 001001111\rangle$

$+\alpha \mid 100110000\rangle + \alpha \mid 100110001\rangle - \alpha \mid 000100010\rangle - \alpha \mid 000100011\rangle$

$-\alpha \mid 010110100\rangle - \alpha \mid 010110101\rangle - \alpha \mid 110100110\rangle - \alpha \mid 110100111\rangle$

$+\alpha \mid 111111000\rangle + \alpha \mid 111111001\rangle + \alpha \mid 011101010\rangle + \alpha \mid 011101011\rangle$

$+\alpha \mid 001111100\rangle + \alpha \mid 001111101\rangle - \alpha \mid 101101110\rangle - \alpha \mid 101101111\rangle$

$-\alpha \mid 010110000\rangle - \alpha \mid 010110001\rangle - \alpha \mid 110100010\rangle - \alpha \mid 110100011\rangle$

$+\alpha \mid 100110100\rangle + \alpha \mid 100110101\rangle - \alpha \mid 000100110\rangle - \alpha \mid 000100111\rangle$

$+\alpha \mid 001111000\rangle + \alpha \mid 001111001\rangle - \alpha \mid 101101010\rangle - \alpha \mid 101101011\rangle$

$+\alpha \mid 111111100\rangle + \alpha \mid 111111101\rangle + \alpha \mid 011101110\rangle + \alpha \mid 011101111\rangle$

$+\alpha \mid 010000000\rangle + \alpha \mid 010000001\rangle - \alpha \mid 110010010\rangle - \alpha \mid 110010011\rangle$

$+\alpha \mid 100000100\rangle + \alpha \mid 100000101\rangle + \alpha \mid 000010110\rangle + \alpha \mid 000010111\rangle$

$+\alpha \mid 001001000\rangle + \alpha \mid 001001001\rangle + \alpha \mid 101011010\rangle + \alpha \mid 101011011\rangle$

$-\alpha \mid 111001100\rangle - \alpha \mid 111001101\rangle + \alpha \mid 011011110\rangle + \alpha \mid 011011111\rangle$

$-\alpha \mid 110100000\rangle - \alpha \mid 110100001\rangle - \alpha \mid 010110010\rangle - \alpha \mid 010110011\rangle$

$-\alpha \mid 000100100\rangle - \alpha \mid 000100101\rangle + \alpha \mid 100110110\rangle + \alpha \mid 100110111\rangle$

$-\alpha \mid 101101000\rangle - \alpha \mid 101101001\rangle + \alpha \mid 001111010\rangle + \alpha \mid 001111011\rangle$

$+\alpha \mid 011101100\rangle + \alpha \mid 011101101\rangle + \alpha \mid 111111110\rangle + \alpha \mid 111111111\rangle$

$-\alpha \mid 000100000\rangle - \alpha \mid 000100001\rangle + \alpha \mid 100110010\rangle + \alpha \mid 100110011\rangle$

$-\alpha \mid 110100100\rangle - \alpha \mid 110100101\rangle - \alpha \mid 010110110\rangle - \alpha \mid 010110111\rangle$

$+\alpha \mid 011101000\rangle + \alpha \mid 011101001\rangle + \alpha \mid 111111010\rangle + \alpha \mid 111111011\rangle$

$-\alpha \mid 101101100\rangle - \alpha \mid 101101101\rangle + \alpha \mid 001111110\rangle + \alpha \mid 001111111\rangle$

$+\alpha \mid 111000000\rangle + \alpha \mid 111000001\rangle - \alpha \mid 011010010\rangle - \alpha \mid 011010011\rangle$

$-\alpha \mid 001000100\rangle - \alpha \mid 001000101\rangle - \alpha \mid 101010110\rangle - \alpha \mid 101010111\rangle$

$-\alpha \mid 100001000\rangle - \alpha \mid 100001001\rangle - \alpha \mid 000011010\rangle - \alpha \mid 000011011\rangle$

$-\alpha \mid 010001100\rangle - \alpha \mid 010001101\rangle + \alpha \mid 110011110\rangle + \alpha \mid 110011111\rangle$

$-\alpha \mid 001000000\rangle - \alpha \mid 001000001\rangle - \alpha \mid 101010010\rangle - \alpha \mid 101010011\rangle$

$+\alpha \mid 111000100\rangle + \alpha \mid 111000101\rangle - \alpha \mid 011010110\rangle - \alpha \mid 011010111\rangle$

$-\alpha \mid 010001000\rangle - \alpha \mid 010001001\rangle + \alpha \mid 110011010\rangle + \alpha \mid 110011011\rangle$

$-\alpha \mid 100001100\rangle - \alpha \mid 100001101\rangle - \alpha \mid 100011110\rangle - \alpha \mid 100011111\rangle$

$+\alpha \mid 101100000\rangle + \alpha \mid 101100001\rangle - \alpha \mid 001110010\rangle - \alpha \mid 001110011\rangle$

$-\alpha \mid 011100100\rangle - \alpha \mid 011100101\rangle - \alpha \mid 111110110\rangle - \alpha \mid 111110111\rangle$

$+\alpha \mid 110101000\rangle + \alpha \mid 110101001\rangle + \alpha \mid 010111010\rangle + \alpha \mid 010111011\rangle$

$+\alpha \mid 000101100\rangle + \alpha \mid 000101101\rangle - \alpha \mid 100111110\rangle - \alpha \mid 100111111\rangle$

$-\alpha \mid 011100000\rangle - \alpha \mid 011100001\rangle - \alpha \mid 111110010\rangle - \alpha \mid 11110011\rangle$

$+\alpha \mid 101100100\rangle + \alpha \mid 101100101\rangle - \alpha \mid 001110110\rangle - \alpha \mid 001110111\rangle$

$+\alpha \mid 000101000\rangle + \alpha \mid 000101001\rangle - \alpha \mid 100111010\rangle - \alpha \mid 100111011\rangle$

$+\alpha \mid 110101100\rangle + \alpha \mid 110101101\rangle + \alpha \mid 010111110\rangle + \alpha \mid 010111111\rangle$

-$\alpha \mid 101010000\rangle - \alpha \mid 101010001\rangle - \alpha \mid 001000010\rangle - \alpha \mid 001000011\rangle$

$-\alpha \mid 011010100\rangle - \alpha \mid 011010101\rangle + \alpha \mid 111000110\rangle + \alpha \mid 111000111\rangle$

$+\alpha \mid 110011000\rangle + \alpha \mid 110011001\rangle - \alpha \mid 010001010\rangle - \alpha \mid 010001011\rangle$

$$-\alpha \mid 000011100\rangle - \alpha \mid 000011101\rangle - \alpha \mid 100001110\rangle - \alpha \mid 100001111\rangle$$
$$-\alpha \mid 011010000\rangle - \alpha \mid 011010001\rangle + \alpha \mid 111000010\rangle + \alpha \mid 111000011\rangle$$
$$-\alpha \mid 101010100\rangle - \alpha \mid 101010101\rangle - \alpha \mid 001000110\rangle - \alpha \mid 001000111\rangle$$
$$-\alpha \mid 000011000\rangle - \alpha \mid 000011001\rangle - \alpha \mid 100001010\rangle - \alpha \mid 100001011\rangle$$
$$+\alpha \mid 110011100\rangle + \alpha \mid 110011101\rangle - \alpha \mid 010001110\rangle - \alpha \mid 010001111\rangle$$
$$-\alpha \mid 111110000\rangle - \alpha \mid 111110001\rangle - \alpha \mid 011100010\rangle - \alpha \mid 011100011\rangle$$
$$-\alpha \mid 001110100\rangle - \alpha \mid 001110101\rangle + \alpha \mid 101100110\rangle + \alpha \mid 1011010111\rangle$$
$$-\alpha \mid 100111000\rangle - \alpha \mid 100111001\rangle + \alpha \mid 000101010\rangle + \alpha \mid 000101011\rangle$$
$$+\alpha \mid 010111100\rangle + \alpha \mid 010111101\rangle + \alpha \mid 110101110\rangle + \alpha \mid 110101111\rangle$$
$$-\alpha \mid 001110000\rangle - \alpha \mid 001110001\rangle + \alpha \mid 101100010\rangle + \alpha \mid 101100011\rangle$$
$$-\alpha \mid 111110100\rangle - \alpha \mid 111110101\rangle - \alpha \mid 011100110\rangle - \alpha \mid 011100111\rangle$$
$$+\alpha \mid 010111000\rangle + \alpha \mid 010111001\rangle + \alpha \mid 110101010\rangle + \alpha \mid 110101011\rangle$$
$$-\alpha \mid 100111100\rangle - \alpha \mid 100111101\rangle + \alpha \mid 000101110\rangle + \alpha \mid 100111111\rangle$$
$$-\beta \mid 110000000\rangle - \beta \mid 110000001\rangle + \beta \mid 010010010\rangle + \beta \mid 010010011\rangle$$
$$-\beta \mid 000000100\rangle - \beta \mid 000000101\rangle - \beta \mid 100010110\rangle - \beta \mid 100010111\rangle$$
$$+\beta \mid 101001000\rangle + \beta \mid 101001001\rangle + \beta \mid 001011010\rangle + \beta \mid 001011011\rangle$$
$$-\beta \mid 011001100\rangle - \beta \mid 011001101\rangle + \beta \mid 111011110\rangle + \beta \mid 111011111\rangle$$
$$-\beta \mid 000000000\rangle - \beta \mid 000000001\rangle - \beta \mid 100010010\rangle - \beta \mid 100010011\rangle$$
$$-\beta \mid 110000100\rangle - \beta \mid 110000101\rangle - \beta \mid 010010110\rangle - \beta \mid 010010111\rangle$$
$$-\beta \mid 011001000\rangle - \beta \mid 011001001\rangle + \beta \mid 111011010\rangle + \beta \mid 111011011\rangle$$
$$+\beta \mid 101001100\rangle + \beta \mid 101001101\rangle + \beta \mid 001011110\rangle + \beta \mid 001011111\rangle$$
$$-\beta \mid 100100000\rangle - \beta \mid 100100001\rangle + \beta \mid 000110010\rangle + \beta \mid 000110011\rangle$$
$$-\beta \mid 010100100\rangle - \beta \mid 010100101\rangle - \beta \mid 110110110\rangle - \beta \mid 110110111\rangle$$
$$-\beta \mid 111101000\rangle - \beta \mid 111101001\rangle - \beta \mid 011111010\rangle - \beta \mid 011111011\rangle$$
$$+\beta \mid 001101100\rangle + \beta \mid 001101101\rangle - \beta \mid 101111110\rangle - \beta \mid 101111111\rangle$$
$$-\beta \mid 010100000\rangle - \beta \mid 010100001\rangle - \beta \mid 110110010\rangle - \beta \mid 110110011\rangle$$
$$-\beta \mid 100100100\rangle - \beta \mid 100100101\rangle + \beta \mid 000110110\rangle + \beta \mid 000110111\rangle$$
$$+\beta \mid 001101000\rangle + \beta \mid 001101001\rangle - \beta \mid 101111010\rangle - \beta \mid 101111011\rangle$$
$$-\beta \mid 111101100\rangle - \beta \mid 111101101\rangle - \beta \mid 011111110\rangle - \beta \mid 011111111\rangle$$
$$-\beta \mid 100010000\rangle - \beta \mid 100010001\rangle - \beta \mid 000000010\rangle - \beta \mid 000000011\rangle$$
$$+\beta \mid 010010100\rangle + \beta \mid 010010101\rangle - \beta \mid 110000110\rangle - \beta \mid 110000111\rangle$$
$$+\beta \mid 111011000\rangle + \beta \mid 111011001\rangle - \beta \mid 011001010\rangle - \beta \mid 011001011\rangle$$

$+\beta \mid 001011100\rangle + \beta \mid 001011101\rangle + \beta \mid 101001110\rangle + \beta \mid 101001111\rangle$

$+\beta \mid 010010000\rangle + \beta \mid 010010001\rangle - \beta \mid 110000010\rangle - \beta \mid 110000011\rangle$

$-\beta \mid 100010100\rangle - \beta \mid 100010101\rangle - \beta \mid 000000110\rangle - \beta \mid 000000111\rangle$

$+\beta \mid 001011000\rangle + \beta \mid 001011001\rangle + \beta \mid 101001010\rangle + \beta \mid 101001011\rangle$

$+\beta \mid 111011100\rangle + \beta \mid 111011101\rangle - \beta \mid 011001110\rangle - \beta \mid 011001111\rangle$

$-\beta \mid 110110000\rangle - \beta \mid 110110001\rangle - \beta \mid 010100010\rangle - \beta \mid 010100011\rangle$

$+\beta \mid 000110100\rangle + \beta \mid 000110101\rangle - \beta \mid 100100110\rangle - \beta \mid 100100111\rangle$

$-\beta \mid 101111000\rangle - \beta \mid 101111001\rangle + \beta \mid 001101010\rangle + \beta \mid 001101011\rangle$

$-\beta \mid 011111100\rangle - \beta \mid 011111101\rangle - \beta \mid 111101110\rangle - \beta \mid 111101111\rangle$

$+\beta \mid 000110000\rangle + \beta \mid 000110001\rangle - \beta \mid 100100010\rangle - \beta \mid 100100011\rangle$

$-\beta \mid 110110100\rangle - \beta \mid 110110101\rangle - \beta \mid 010100110\rangle - \beta \mid 010100111\rangle$

$-\beta \mid 011111000\rangle - \beta \mid 011111001\rangle - \beta \mid 111101010\rangle - \beta \mid 111101011\rangle$

$-\beta \mid 101111100\rangle - \beta \mid 101111101\rangle + \beta \mid 001101110\rangle + \beta \mid 001101111\rangle$

-$\beta \mid 111010000\rangle - \beta \mid 111010001\rangle + \beta \mid 011000010\rangle + \beta \mid 011000011\rangle$

$-\beta \mid 001010100\rangle - \beta \mid 001010101\rangle - \beta \mid 101000110\rangle - \beta \mid 101000111\rangle$

$+\beta \mid 100011000\rangle + \beta \mid 100011001\rangle + \beta \mid 000001010\rangle + \beta \mid 000001011\rangle$

$-\beta \mid 010011100\rangle - \beta \mid 010011101\rangle + \beta \mid 110001110\rangle + \beta \mid 110001111\rangle$

$-\beta \mid 001010000\rangle - \beta \mid 001010001\rangle - \beta \mid 101000010\rangle - \beta \mid 101000011\rangle$

$-\beta \mid 111010100\rangle - \beta \mid 111010101\rangle + \beta \mid 011000110\rangle + \beta \mid 011000111\rangle$

$-\beta \mid 010011000\rangle - \beta \mid 010011001\rangle + \beta \mid 110001010\rangle + \beta \mid 110001011\rangle$

$+\beta \mid 100011100\rangle + \beta \mid 100011101\rangle + \beta \mid 000001110\rangle + \beta \mid 000001111\rangle$

$+\beta \mid 101110000\rangle + \beta \mid 101110001\rangle - \beta \mid 001100010\rangle - \beta \mid 001100011\rangle$

$+\beta \mid 011110100\rangle + \beta \mid 011110101\rangle + \beta \mid 111100110\rangle + \beta \mid 111100111\rangle$

$+\beta \mid 110111000\rangle + \beta \mid 110111001\rangle + \beta \mid 010101010\rangle + \beta \mid 010101011\rangle$

$-\beta \mid 000111100\rangle - \beta \mid 000111101\rangle + \beta \mid 100101110\rangle + \beta \mid 100101111\rangle$

$+\beta \mid 011110000\rangle + \beta \mid 011110001\rangle + \beta \mid 111100010\rangle + \beta \mid 111100011\rangle$

$+\beta \mid 101110100\rangle + \beta \mid 101110101\rangle - \beta \mid 001100110\rangle - \beta \mid 001100111\rangle$

$-\beta \mid 000111000\rangle - \beta \mid 000111001\rangle + \beta \mid 100101010\rangle + \beta \mid 100101011\rangle$

$+\beta \mid 110111100\rangle + \beta \mid 110111101\rangle + \beta \mid 010101110\rangle + \beta \mid 010101111\rangle$

-$\beta \mid 101000000\rangle - \beta \mid 101000001\rangle - \beta \mid 001010010\rangle - \beta \mid 001010011\rangle$

$+\beta \mid 011000100\rangle + \beta \mid 011000101\rangle - \beta \mid 111010110\rangle - \beta \mid 111010111\rangle$

$+\beta \mid 110001000\rangle + \beta \mid 110001001\rangle - \beta \mid 010011010\rangle - \beta \mid 010011011\rangle$

$-\beta\ \mid 000001100\rangle - \beta\ \mid 000001101\rangle - \beta\ \mid 100011110\rangle - \beta\ \mid 100011111\rangle$

$+\beta \mid 011000000\rangle + \beta \mid 011000001\rangle - \beta \mid 111010010\rangle - \beta \mid 111010011\rangle$

$-\beta \mid 101000100\rangle - \beta \mid 101000101\rangle - \beta \mid 001010110\rangle - \beta \mid 001010111\rangle$

$+\beta \mid 000001000\rangle + \beta \mid 000001001\rangle + \beta \mid 100011010\rangle + \beta \mid 100011011\rangle$

$+\beta \mid 110001100\rangle + \beta \mid 110001101\rangle - \beta \mid 010011110\rangle - \beta \mid 010011111\rangle$

$+\beta \mid 111100000\rangle + \beta \mid 111100001\rangle + \beta \mid 011110010\rangle + \beta \mid 011110011\rangle$

$-\beta \mid 001100100\rangle - \beta \mid 001100101\rangle + \beta \mid 101110110\rangle + \beta \mid 101110111\rangle$

$+\beta \mid 100101000\rangle + \beta \mid 100101001\rangle - \beta \mid 000111010\rangle - \beta \mid 000111011\rangle$

$+\beta \mid 010101100\rangle + \beta \mid 010101101\rangle + \beta \mid 110111110\rangle + \beta \mid 110111111\rangle$

$-\beta \mid 001100000\rangle - \beta \mid 001100001\rangle + \beta \mid 101110010\rangle + \beta \mid 101110011\rangle$

$+\beta \mid 111100100\rangle\ + \beta \mid 111100101\rangle + \beta \mid 011110110\rangle + \beta \mid 011110111\rangle$

$+\beta \mid 010101000\rangle + \beta \mid 010101001\rangle + \beta \mid 110111010\rangle\ + \beta \mid 110111011\rangle$

$+\beta \mid\ 010101100\rangle + \beta \mid\ 010101101\rangle + \beta \mid\ 110111110\rangle + \beta \mid\ 110111111\rangle)$

✓ <u>Pour $M_3 = Z(1)\,I(2)\,Z(3)\,X(4)\,X(5)$</u>

$\mid\ \Psi_1\rangle\ = \frac{1}{16}(\alpha \mid 100000000\rangle - \alpha \mid 100110001\rangle + \alpha \mid 000010010\rangle + \alpha \mid\ 000100011\rangle$

$+\alpha \mid 010000100\rangle + \alpha \mid 010110101\rangle - \alpha \mid 110010110\rangle + \alpha \mid\ 110100111\rangle$

$-\alpha \mid 111001000\rangle - \alpha \mid 111111001\rangle + \alpha \mid 011011010\rangle - \alpha \mid\ 011101011\rangle$

$+\alpha \mid 001001100\rangle - \alpha \mid 001111101\rangle + \alpha \mid 101011110\rangle + \alpha \mid\ 101101111\rangle$

$-\alpha \mid 110010000\rangle + \alpha \mid 110100001\rangle + \alpha \mid 010000010\rangle + \alpha \mid\ 010110011\rangle$

$+\alpha \mid 000010100\rangle + \alpha \mid 000100101\rangle + \alpha \mid 100000110\rangle - \alpha \mid\ 100110111\rangle$

$+\alpha \mid 101011000\rangle + \alpha \mid 101101001\rangle + \alpha \mid 001001010\rangle - \alpha \mid\ 001111011\rangle$

$+\alpha \mid 011011100\rangle - \alpha \mid 011101101\rangle - \alpha \mid 111001110\rangle - \alpha \mid\ 111111111\rangle$

$+\alpha \mid 000010000\rangle + \alpha \mid 000100001\rangle + \alpha \mid 100000010\rangle - \alpha \mid\ 100110011\rangle$

$-\alpha \mid 110010100\rangle + \alpha \mid 110100101\rangle + \alpha \mid 010000110\rangle + \alpha \mid\ 010110111\rangle$

$+\alpha \mid 011011000\rangle - \alpha \mid 011101001\rangle - \alpha \mid 111001010\rangle - \alpha \mid\ 111111011\rangle$

$+\alpha \mid 101011100\rangle + \alpha \mid 101101101\rangle + \alpha \mid 001001110\rangle - \alpha \mid\ 001111111\rangle$

$+\alpha \mid 100110000\rangle - \alpha \mid 100000001\rangle - \alpha \mid 000100010\rangle - \alpha \mid\ 000010011\rangle$

$-\alpha \mid 010110100\rangle - \alpha \mid 010000101\rangle - \alpha \mid 110100110\rangle + \alpha \mid\ 110010111\rangle$

$+\alpha \mid 111111000\rangle\ +\alpha \mid 111001001\rangle\ +\alpha \mid 011101010\rangle\ -\alpha \mid\ 011011011\rangle$

$+\alpha \mid 001111100\rangle - \alpha \mid 001001101\ \rangle\ -\alpha \mid 101101110\ \rangle\ -\alpha \mid\ 101011111\ \rangle$

$-\alpha \mid 010110000\rangle - \alpha \mid 010000001\rangle - \alpha \mid 110100010\rangle + \alpha \mid\ 110010011\rangle$

$+\alpha \mid 100110100\rangle - \alpha \mid 100000101\rangle - \alpha \mid 000100110\rangle - \alpha \mid\ 000010111\rangle$

$+\alpha \mid 001111000\rangle - \alpha \mid 001001001\rangle - \alpha \mid 101101010\rangle - \alpha \mid 101011011\rangle$
$+\alpha \mid 111111100\rangle + \alpha \mid 111001101\rangle + \alpha \mid 011101110\rangle - \alpha \mid 011011111\rangle$
$+\alpha \mid 010000000\rangle + \alpha \mid 010110001\rangle - \alpha \mid 110010010\rangle + \alpha \mid 110100011\rangle$
$+\alpha \mid 100000100\rangle - \alpha \mid 100110101\rangle + \alpha \mid 000010110\rangle + \alpha \mid 000100111\rangle$
$+\alpha \mid 001001000\rangle - \alpha \mid 001111001\rangle + \alpha \mid 101011010\rangle + \alpha \mid 101101011\rangle$
$-\alpha \mid 111001100\rangle - \alpha \mid 111111101\rangle + \alpha \mid 011011110\rangle - \alpha \mid 011101111\rangle$
$-\alpha \mid 110100000\rangle + \alpha \mid 110010001\rangle - \alpha \mid 010110010\rangle - \alpha \mid 010000011\rangle$
$-\alpha \mid 000100100\rangle - \alpha \mid 000010101\rangle + \alpha \mid 100110110\rangle - \alpha \mid 100000111\rangle$
$-\alpha \mid 101101000\rangle - \alpha \mid 101011001\rangle + \alpha \mid 001111010\rangle - \alpha \mid 001001011\rangle$
$+\alpha \mid 011101100\rangle - \alpha \mid 011011101\rangle + \alpha \mid 111111110\rangle + \alpha \mid 111001111\rangle$
$-\alpha \mid 000100000\rangle - \alpha \mid 000010001\rangle + \alpha \mid 100110010\rangle - \alpha \mid 100000011\rangle$
$-\alpha \mid 110100100\rangle + \alpha \mid 110010101\rangle - \alpha \mid 010110110\rangle - \alpha \mid 010000111\rangle$
$+\alpha \mid 011101000\rangle - \alpha \mid 011011001\rangle + \alpha \mid 111111010\rangle + \alpha \mid 111001011\rangle$
$-\alpha \mid 101101100\rangle - \alpha \mid 101011101\rangle + \alpha \mid 001111110\rangle - \alpha \mid 001001111\rangle$
$+\alpha \mid 111000000\rangle + \alpha \mid 111110001\rangle - \alpha \mid 011010010\rangle + \alpha \mid 011100011\rangle$
$-\alpha \mid 001000100\rangle + \alpha \mid 001110101\rangle - \alpha \mid 101010110\rangle - \alpha \mid 101100111\rangle$
$-\alpha \mid 100001000\rangle + \alpha \mid 100111001\rangle - \alpha \mid 000011010\rangle - \alpha \mid 000101011\rangle$
$-\alpha \mid 010001100\rangle - \alpha \mid 010111101\rangle + \alpha \mid 110011110\rangle - \alpha \mid 110101111\rangle$
$-\alpha \mid 001000000\rangle + \alpha \mid 001110001\rangle - \alpha \mid 101010010\rangle - \alpha \mid 101100011\rangle$
$+\alpha \mid 111000100\rangle + \alpha \mid 111110101\rangle - \alpha \mid 011010110\rangle + \alpha \mid 011100111\rangle$
$-\alpha \mid 010001000\rangle - \alpha \mid 010111001\rangle + \alpha \mid 110011010\rangle - \alpha \mid 110101011\rangle$
$-\alpha \mid 100001100\rangle + \alpha \mid 100111101\rangle - \alpha \mid 100011110\rangle + \alpha \mid 100101111\rangle$
$+\alpha \mid 101100000\rangle + \alpha \mid 101010001\rangle - \alpha \mid 001110010\rangle + \alpha \mid 001000011\rangle$
$-\alpha \mid 011100100\rangle + \alpha \mid 011010101\rangle - \alpha \mid 111110110\rangle - \alpha \mid 111000111\rangle$
$+\alpha \mid 110101000\rangle - \alpha \mid 110011001\rangle + \alpha \mid 010111010\rangle + \alpha \mid 010001011\rangle$
$+\alpha \mid 000101100\rangle + \alpha \mid 000011101\rangle - \alpha \mid 100111110\rangle + \alpha \mid 100001111\rangle$
$-\alpha \mid 011100000\rangle + \alpha \mid 011010001\rangle - \alpha \mid 111110010\rangle - \alpha \mid 11101011\rangle$
$+\alpha \mid 101100100\rangle + \alpha \mid 101010101\rangle - \alpha \mid 001110110\rangle + \alpha \mid 001000111\rangle$
$+\alpha \mid 000101000\rangle + \alpha \mid 000011001\rangle - \alpha \mid 100111010\rangle + \alpha \mid 100001011\rangle$
$+\alpha \mid 110101100\rangle - \alpha \mid 110011101\rangle + \alpha \mid 010111110\rangle + \alpha \mid 010001111\rangle$
$-\alpha \mid 101010000\rangle - \alpha \mid 101100001\rangle - \alpha \mid 001000010\rangle + \alpha \mid 001110011\rangle$
$-\alpha \mid 011010100\rangle + \alpha \mid 011100101\rangle + \alpha \mid 111000110\rangle + \alpha \mid 111110111\rangle$

$$+\alpha \mid 110011000\rangle - \alpha \mid 110101001\rangle - \alpha \mid 010001010\rangle - \alpha \mid 010111011\rangle$$
$$-\alpha \mid 000011100\rangle - \alpha \mid 000101101\rangle - \alpha \mid 100001110\rangle + \alpha \mid 100111111\rangle$$
$$-\alpha \mid 011010000\rangle + \alpha \mid 011100001\rangle + \alpha \mid 111000010\rangle + \alpha \mid 111110011\rangle$$
$$-\alpha \mid 101010100\rangle - \alpha \mid 101100101\rangle - \alpha \mid 001000110\rangle + \alpha \mid 001110111\rangle$$
$$-\alpha \mid 000011000\rangle - \alpha \mid 000101001\rangle - \alpha \mid 100001010\rangle + \alpha \mid 100111011\rangle$$
$$+\alpha \mid 110011100\rangle - \alpha \mid 110101101\rangle - \alpha \mid 010001110\rangle - \alpha \mid 010111111\rangle$$
$$-\alpha \mid 111110000\rangle - \alpha \mid 111000001\rangle - \alpha \mid 011100010\rangle + \alpha \mid 011010011\rangle$$
$$-\alpha \mid 001110100\rangle + \alpha \mid 001000101\rangle + \alpha \mid 101100110\rangle + \alpha \mid 1010110111\rangle$$
$$-\alpha \mid 100111000\rangle + \alpha \mid 100001001\rangle + \alpha \mid 000101010\rangle + \alpha \mid 000011011\rangle$$
$$+\alpha \mid 010111100\rangle + \alpha \mid 010001101\rangle + \alpha \mid 110101110\rangle - \alpha \mid 110011111\rangle$$
$$-\alpha \mid 001110000\rangle + \alpha \mid 001000001\rangle + \alpha \mid 101100010\rangle + \alpha \mid 101010011\rangle$$
$$-\alpha \mid 111110100\rangle - \alpha \mid 111000101\rangle - \alpha \mid 011100110\rangle + \alpha \mid 011010111\rangle$$
$$+\alpha \mid 010111000\rangle + \alpha \mid 010001001\rangle + \alpha \mid 110101010\rangle - \alpha \mid 110011011\rangle$$
$$-\alpha \mid 100111100\rangle + \alpha \mid 100001101\rangle + \alpha \mid 000101110\rangle - \alpha \mid 100001111\rangle$$
$$-\beta \mid 110000000\rangle + \beta \mid 110110001\rangle + \beta \mid 010010010\rangle + \beta \mid 010100011\rangle$$
$$-\beta \mid 000000100\rangle - \beta \mid 000110101\rangle - \beta \mid 100010110\rangle + \beta \mid 100100111\rangle$$
$$+\beta \mid 101001000\rangle + \beta \mid 101111001\rangle + \beta \mid 001011010\rangle - \beta \mid 001101011\rangle$$
$$-\beta \mid 011001100\rangle + \beta \mid 011111101\rangle + \beta \mid 111011110\rangle + \beta \mid 111101111\rangle$$
$$-\beta \mid 000000000\rangle - \beta \mid 000110001\rangle - \beta \mid 100010010\rangle + \beta \mid 100100011\rangle$$
$$-\beta \mid 110000100\rangle + \beta \mid 110110101\rangle - \beta \mid 010010110\rangle - \beta \mid 010100111\rangle$$
$$-\beta \mid 011001000\rangle + \beta \mid 011111001\rangle + \beta \mid 111011010\rangle + \beta \mid 111101011\rangle$$
$$+\beta \mid 101001100\rangle + \beta \mid 101111101\rangle + \beta \mid 001011110\rangle - \beta \mid 001101111\rangle$$
$$-\beta \mid 100100000\rangle + \beta \mid 100010001\rangle + \beta \mid 000110010\rangle + \beta \mid 000000011\rangle$$
$$-\beta \mid 010100100\rangle - \beta \mid 010010101\rangle - \beta \mid 110110110\rangle + \beta \mid 110000111\rangle$$
$$-\beta \mid 111101000\rangle - \beta \mid 111011001\rangle - \beta \mid 011111010\rangle + \beta \mid 011001011\rangle$$
$$+\beta \mid 001101100\rangle - \beta \mid 001011101\rangle - \beta \mid 101111110\rangle - \beta \mid 101001111\rangle$$
$$-\beta \mid 010100000\rangle - \beta \mid 010010001\rangle - \beta \mid 110110010\rangle + \beta \mid 110000011\rangle$$
$$-\beta \mid 100100100\rangle + \beta \mid 100010101\rangle + \beta \mid 000110110\rangle + \beta \mid 000000111\rangle$$
$$+\beta \mid 001101000\rangle - \beta \mid 001011001\rangle - \beta \mid 101111010\rangle - \beta \mid 101001011\rangle$$
$$-\beta \mid 111101100\rangle - \beta \mid 011011101\rangle - \beta \mid 011111110\rangle + \beta \mid 011001111\rangle$$
$$-\beta \mid 100010000\rangle + \beta \mid 100100001\rangle - \beta \mid 000000010\rangle - \beta \mid 000110011\rangle$$
$$+\beta \mid 010010100\rangle + \beta \mid 010100101\rangle - \beta \mid 110000110\rangle + \beta \mid 110110111\rangle$$

$+\beta \mid 111011000\rangle + \beta \mid 111101001\rangle - \beta \mid 011001010\rangle + \beta \mid 011111011\rangle$

$+\beta \mid 001011100\rangle - \beta \mid 001101101\rangle + \beta \mid 101001110\rangle + \beta \mid 101111111\rangle$

$+\beta \mid 010010000\rangle + \beta \mid 010100001\rangle - \beta \mid 110000010\rangle + \beta \mid 110110011\rangle$

$-\beta \mid 100010100\rangle + \beta \mid 100100101\rangle - \beta \mid 000000110\rangle - \beta \mid 000110111\rangle$

$+\beta \mid 001011000\rangle - \beta \mid 001101001\rangle + \beta \mid 101001010\rangle + \beta \mid 101111011\rangle$

$+\beta \mid 111011100\rangle + \beta \mid 111101101\rangle - \beta \mid 011001110\rangle + \beta \mid 011111111\rangle$

$-\beta \mid 110110000\rangle + \beta \mid 110000001\rangle - \beta \mid 010100010\rangle - \beta \mid 010010011\rangle$

$+\beta \mid 000110100\rangle + \beta \mid 000000101\rangle - \beta \mid 100100110\rangle + \beta \mid 100010111\rangle$

$-\beta \mid 101111000\rangle - \beta \mid 101001001\rangle + \beta \mid 001101010\rangle - \beta \mid 001011011\rangle$

$-\beta \mid 011111100\rangle + \beta \mid 011001101\rangle - \beta \mid 111101110\rangle - \beta \mid 111011111\rangle$

$+\beta \mid 000110000\rangle + \beta \mid 000000001\rangle - \beta \mid 100100010\rangle + \beta \mid 100010011\rangle$

$-\beta \mid 110110100\rangle + \beta \mid 110000101\rangle - \beta \mid 010100110\rangle - \beta \mid 010010111\rangle$

$-\beta \mid 011111000\rangle + \beta \mid 011001001\rangle - \beta \mid 111101010\rangle - \beta \mid 111011011\rangle$

$-\beta \mid 101111100\rangle - \beta \mid 101001101\rangle + \beta \mid 001101110\rangle - \beta \mid 001011111\rangle$

-$\beta \mid 111010000\rangle - \beta \mid 111100001\rangle + \beta \mid 011000010\rangle - \beta \mid 011110011\rangle$

$-\beta \mid 001010100\rangle + \beta \mid 001100101\rangle - \beta \mid 101000110\rangle - \beta \mid 101110111\rangle$

$+\beta \mid 100011000\rangle - \beta \mid 100101001\rangle + \beta \mid 000001010\rangle + \beta \mid 000111011\rangle$

$-\beta \mid 010011100\rangle - \beta \mid 010101101\rangle + \beta \mid 110001110\rangle - \beta \mid 110111111\rangle$

$-\beta \mid 001010000\rangle + \beta \mid 001100001\rangle - \beta \mid 101000010\rangle - \beta \mid 101110011\rangle$

$-\beta \mid 111010100\rangle - \beta \mid 111100101\rangle + \beta \mid 011000110\rangle - \beta \mid 011110111\rangle$

$-\beta \mid 010011000\rangle - \beta \mid 010101001\rangle + \beta \mid 110001010\rangle - \beta \mid 110111011\rangle$

$+\beta \mid 100011100\rangle - \beta \mid 100101101\rangle + \beta \mid 000001110\rangle + \beta \mid 000111111\rangle$

$+\beta \mid 101110000\rangle + \beta \mid 101000001\rangle - \beta \mid 001100010\rangle + \beta \mid 001010011\rangle$

$+\beta \mid 011110100\rangle - \beta \mid 011000101\rangle + \beta \mid 111100110\rangle + \beta \mid 111010111\rangle$

$+\beta \mid 110111000\rangle - \beta \mid 110001001\rangle + \beta \mid 010101010\rangle + \beta \mid 010011011\rangle$

$-\beta \mid 000111100\rangle - \beta \mid 000001101\rangle + \beta \mid 100101110\rangle - \beta \mid 100011111\rangle$

$+\beta \mid 011110000\rangle - \beta \mid 011000001\rangle + \beta \mid 111100010\rangle + \beta \mid 111010011\rangle$

$+\beta \mid 101110100\rangle + \beta \mid 101000101\rangle - \beta \mid 001100110\rangle + \beta \mid 001010111\rangle$

$-\beta \mid 000111000\rangle - \beta \mid 000001001\rangle + \beta \mid 100101010\rangle - \beta \mid 100011011\rangle$

$+\beta \mid 110111100\rangle - \beta \mid 110001101\rangle + \beta \mid 010101110\rangle + \beta \mid 010011111\rangle$

-$\beta \mid 101000000\rangle - \beta \mid 101110001\rangle - \beta \mid 001010010\rangle + \beta \mid 001100011\rangle$

$+\beta \mid 011000100\rangle - \beta \mid 011110101\rangle - \beta \mid 111010110\rangle - \beta \mid 111100111\rangle$

$+\beta \mid 110001000\rangle - \beta \mid 110111001\rangle - \beta \mid 010011010\rangle - \beta \mid 010101011\rangle$
$-\beta \mid 000001100\rangle - \beta \mid 000111101\rangle - \beta \mid 100011110\rangle + \beta \mid 100101111\rangle$
$+\beta \mid 011000000\rangle - \beta \mid 011110001\rangle - \beta \mid 111010010\rangle - \beta \mid 111100011\rangle$
$-\beta \mid 101000100\rangle - \beta \mid 101110101\rangle - \beta \mid 001010110\rangle + \beta \mid 001100111\rangle$
$+\beta \mid 000001000\rangle + \beta \mid 000111001\rangle + \beta \mid 100011010\rangle - \beta \mid 100101011\rangle$
$+\beta \mid 110001100\rangle - \beta \mid 110111101\rangle - \beta \mid 010011110\rangle - \beta \mid 010101111\rangle$
$+\beta \mid 111100000\rangle + \beta \mid 111010001\rangle + \beta \mid 011110010\rangle - \beta \mid 011000011\rangle$
$-\beta \mid 001100100\rangle + \beta \mid 001010101\rangle + \beta \mid 101110110\rangle + \beta \mid 101000111\rangle$
$+\beta \mid 100101000\rangle - \beta \mid 100011001\rangle - \beta \mid 000111010\rangle - \beta \mid 000001011\rangle$
$+\beta \mid 010101100\rangle + \beta \mid 010011101\rangle + \beta \mid 110111110\rangle - \beta \mid 110001111\rangle$
$-\beta \mid 001100000\rangle + \beta \mid 001010001\rangle + \beta \mid 101110010\rangle + \beta \mid 101000011\rangle$
$+\beta \mid 111100100\rangle + \beta \mid 111010101\rangle + \beta \mid 011110110\rangle - \beta \mid 011000111\rangle$
$+\beta \mid 010101000\rangle + \beta \mid 010011001\rangle + \beta \mid 110111010\rangle - \beta \mid 110001011\rangle$
$+\beta \mid 010101100\rangle + \beta \mid 010011101\rangle + \beta \mid 110111110\rangle - \beta \mid 110001111\rangle)$

○ <u>L'application de la Porte $H\,(6)$</u>:

$\mid \Psi_1\rangle = \frac{1}{16\sqrt{2}}(\alpha \mid 100000000\rangle + \alpha \mid 100001000\rangle - \alpha \mid 100110001\rangle - \alpha \mid 100111001\rangle$
$+\alpha \mid 000010010\rangle + \alpha \mid 000011010\rangle + \alpha \mid 000100011\rangle + \alpha \mid 000101011\rangle$
$+\alpha \mid 010000100\rangle + \alpha \mid 010001100\rangle + \alpha \mid 010110101\rangle + \alpha \mid 010111101\rangle$
$-\alpha \mid 110010110\rangle - \alpha \mid 110011110\rangle + \alpha \mid 110100111\rangle + \alpha \mid 110101111\rangle$
$-\alpha \mid 111000000\rangle + \alpha \mid 111001000\rangle - \alpha \mid 111110001\rangle + \alpha \mid 111111001\rangle$
$+\alpha \mid 011010010\rangle - \alpha \mid 011011010\rangle - \alpha \mid 011100011\rangle + \alpha \mid 011101011\rangle$
$+\alpha \mid 001000100\rangle - \alpha \mid 001001100\rangle - \alpha \mid 001110101\rangle + \alpha \mid 001111101\rangle$
$+\alpha \mid 101010110\rangle - \alpha \mid 101011110\rangle + \alpha \mid 101100111\rangle - \alpha \mid 101101111\rangle$
$-\alpha \mid 110010000\rangle - \alpha \mid 110011000\rangle + \alpha \mid 110100001\rangle + \alpha \mid 110101001\rangle$
$+\alpha \mid 010000010\rangle + \alpha \mid 010001010\rangle + \alpha \mid 010110011\rangle + \alpha \mid 010111011\rangle$
$+\alpha \mid 000010100\rangle + \alpha \mid 000011100\rangle + \alpha \mid 000100101\rangle + \alpha \mid 000101101\rangle$
$+\alpha \mid 100000110\rangle + \alpha \mid 100001110\rangle - \alpha \mid 100110111\rangle - \alpha \mid 100110111\rangle$
$+\alpha \mid 101010000\rangle - \alpha \mid 101011000\rangle + \alpha \mid 101100001\rangle - \alpha \mid 101101001\rangle$
$+\alpha \mid 001000010\rangle - \alpha \mid 001001010\rangle - \alpha \mid 001110011\rangle + \alpha \mid 001111011\rangle$
$+\alpha \mid 011010100\rangle - \alpha \mid 011011100\rangle - \alpha \mid 011100101\rangle + \alpha \mid 011101101\rangle$
$-\alpha \mid 111000110\rangle + \alpha \mid 111001110\rangle - \alpha \mid 111110111\rangle + \alpha \mid 111111111\rangle$
$+\alpha \mid 000010000\rangle + \alpha \mid 000011000\rangle + \alpha \mid 000100001\rangle + \alpha \mid 000101001\rangle$

$$+\alpha \mid 100000010\rangle + \alpha \mid 100001010\rangle - \alpha \mid 100110011\rangle - \alpha \mid 100111011\rangle$$
$$-\alpha \mid 110010100\rangle - \alpha \mid 110011100\rangle + \alpha \mid 110100101\rangle + \alpha \mid 110101101\rangle$$
$$+\alpha \mid 010000110\rangle + \alpha \mid 010001110\rangle + \alpha \mid 010110111\rangle + \alpha \mid 010111111\rangle$$
$$+\alpha \mid 011010000\rangle - \alpha \mid 011011000\rangle - \alpha \mid 011100001\rangle + \alpha \mid 011101001\rangle$$
$$-\alpha \mid 111000010\rangle + \alpha \mid 111001010\rangle - \alpha \mid 111111011\rangle + \alpha \mid 111111011\rangle$$
$$+\alpha \mid 101010100\rangle - \alpha \mid 101011100\rangle + \alpha \mid 101100101\rangle - \alpha \mid 101101101\rangle$$
$$+\alpha \mid 001000110\rangle - \alpha \mid 001001110\rangle - \alpha \mid 001110111\rangle + \alpha \mid 001111111\rangle$$
$$+\alpha \mid 100110000\rangle + \alpha \mid 100111000\rangle - \alpha \mid 100000001\rangle - \alpha \mid 100001001\rangle$$
$$-\alpha \mid 000100010\rangle - \alpha \mid 000101010\rangle - \alpha \mid 000010011\rangle - \alpha \mid 000011011\rangle$$
$$-\alpha \mid 010110100\rangle - \alpha \mid 010111100\rangle - \alpha \mid 010000101\rangle - \alpha \mid 010001101\rangle$$
$$-\alpha \mid 110100110\rangle - \alpha \mid 110101110\rangle + \alpha \mid 110010111\rangle + \alpha \mid 110011111\rangle$$
$$+\alpha \mid 111110000\rangle - \alpha \mid 111111000\rangle + \alpha \mid 111000001\rangle - \alpha \mid 111001001\rangle$$
$$+\alpha \mid 011100010\rangle - \alpha \mid 011101010\rangle - \alpha \mid 011010011\rangle + \alpha \mid 011011011\rangle$$
$$+\alpha \mid 001110100\rangle - \alpha \mid 001111100\rangle - \alpha \mid 001000101\rangle + \alpha \mid 001001101\rangle$$
$$-\alpha \mid 101100110\rangle + \alpha \mid 101101110\rangle - \alpha \mid 101010111\rangle + \alpha \mid 101011111\rangle$$
$$-\alpha \mid 010110000\rangle - \alpha \mid 010111000\rangle - \alpha \mid 010000001\rangle - \alpha \mid 010001001\rangle$$
$$-\alpha \mid 110100010\rangle - \alpha \mid 110101010\rangle + \alpha \mid 110010011\rangle + \alpha \mid 110011011\rangle$$
$$+\alpha \mid 100110100\rangle + \alpha \mid 100111100\rangle - \alpha \mid 100000101\rangle - \alpha \mid 100001101\rangle$$
$$-\alpha \mid 000100110\rangle - \alpha \mid 000101110\rangle - \alpha \mid 000010111\rangle - \alpha \mid 000011111\rangle$$
$$+\alpha \mid 001110000\rangle - \alpha \mid 001111000\rangle - \alpha \mid 001000001\rangle + \alpha \mid 001001001\rangle$$
$$-\alpha \mid 101100010\rangle + \alpha \mid 101101010\rangle - \alpha \mid 101010011\rangle + \alpha \mid 101011011\rangle$$
$$+\alpha \mid 111110100\rangle - \alpha \mid 111111100\rangle + \alpha \mid 111000101\rangle - \alpha \mid 111001101\rangle$$
$$+\alpha \mid 011100110\rangle - \alpha \mid 011101110\rangle - \alpha \mid 011010111\rangle + \alpha \mid 011011111\rangle$$
$$+\alpha \mid 010000000\rangle + \alpha \mid 010001000\rangle + \alpha \mid 010110001\rangle + \alpha \mid 010111001\rangle$$
$$-\alpha \mid 110010010\rangle - \alpha \mid 110011010\rangle + \alpha \mid 110100011\rangle + \alpha \mid 110101011\rangle$$
$$+\alpha \mid 100000100\rangle + \alpha \mid 100001100\rangle - \alpha \mid 100110101\rangle - \alpha \mid 100111101\rangle$$
$$+\alpha \mid 000010110\rangle + \alpha \mid 000011110\rangle + \alpha \mid 000100111\rangle + \alpha \mid 000101111\rangle$$
$$+\alpha \mid 001000000\rangle - \alpha \mid 001001000\rangle - \alpha \mid 001110001\rangle + \alpha \mid 001111001\rangle$$
$$+\alpha \mid 101010010\rangle - \alpha \mid 101011010\rangle + \alpha \mid 101100011\rangle - \alpha \mid 101101011\rangle$$
$$-\alpha \mid 111000100\rangle + \alpha \mid 111001100\rangle - \alpha \mid 111110101\rangle + \alpha \mid 111111101\rangle$$
$$+\alpha \mid 011010110\rangle - \alpha \mid 011011110\rangle - \alpha \mid 011100111\rangle + \alpha \mid 011101111\rangle$$
$$-\alpha \mid 110100000\rangle - \alpha \mid 110101000\rangle + \alpha \mid 110010001\rangle + \alpha \mid 110011001\rangle$$

$-\alpha \mid 010110010\rangle - \alpha \mid 010111010\rangle - \alpha \mid 010000011\rangle - \alpha \mid 010001011\rangle$

$-\alpha \mid 000100100\rangle - \alpha \mid 000101100\rangle - \alpha \mid 000010101\rangle - \alpha \mid 000011101\rangle$

$+\alpha \mid 100110110\rangle + \alpha \mid 100111110\rangle - \alpha \mid 100000111\rangle - \alpha \mid 100001111\rangle$

$-\alpha \mid 101100000\rangle + \alpha \mid 101101000\rangle - \alpha \mid 101010001\rangle + \alpha \mid 101011001\rangle$

$+\alpha \mid 001110010\rangle - \alpha \mid 001111010\rangle - \alpha \mid 001000011\rangle + \alpha \mid 001001011\rangle$

$+\alpha \mid 011100100\rangle - \alpha \mid 011101100\rangle - \alpha \mid 011010101\rangle + \alpha \mid 011011101\rangle$

$+\alpha \mid 111110110\rangle - \alpha \mid 111111110\rangle + \alpha \mid 111000111\rangle - \alpha \mid 111001111\rangle$

$-\alpha \mid 000100000\rangle - \alpha \mid 000101000\rangle - \alpha \mid 000010001\rangle - \alpha \mid 000011001\rangle$

$+\alpha \mid 100110010\rangle + \alpha \mid 100111010\rangle - \alpha \mid 100000011\rangle - \alpha \mid 100001011\rangle$

$-\alpha \mid 110100100\rangle - \alpha \mid 110101100\rangle + \alpha \mid 110010101\rangle + \alpha \mid 110011101\rangle$

$-\alpha \mid 010110110\rangle - \alpha \mid 010111110\rangle - \alpha \mid 010000111\rangle - \alpha \mid 010001111\rangle$

$+\alpha \mid 011100000\rangle - \alpha \mid 011101000\rangle - \alpha \mid 011010001\rangle + \alpha \mid 011011001\rangle$

$+\alpha \mid 111110010\rangle - \alpha \mid 111111010\rangle + \alpha \mid 111000011\rangle - \alpha \mid 111001011\rangle$

$-\alpha \mid 101100100\rangle + \alpha \mid 101101100\rangle - \alpha \mid 101010101\rangle + \alpha \mid 101011101\rangle$

$+\alpha \mid 001110110\rangle - \alpha \mid 001111110\rangle - \alpha \mid 001000111\rangle + \alpha \mid 001001111\rangle$

$+\alpha \mid 111000000\rangle + \alpha \mid 111001000\rangle + \alpha \mid 111110001\rangle + \alpha \mid 111111001\rangle$

$-\alpha \mid 011010010\rangle - \alpha \mid 011011010\rangle + \alpha \mid 011100011\rangle + \alpha \mid 011101011\rangle$

$-\alpha \mid 001000100\rangle - \alpha \mid 001001100\rangle + \alpha \mid 001110101\rangle + \alpha \mid 001111101\rangle$

$-\alpha \mid 101010110\rangle - \alpha \mid 101011110\rangle - \alpha \mid 101100111\rangle - \alpha \mid 101101111\rangle$

$-\alpha \mid 100000000\rangle + \alpha \mid 100001000\rangle + \alpha \mid 100110001\rangle - \alpha \mid 100111001\rangle$

$-\alpha \mid 000010010\rangle + \alpha \mid 000011010\rangle - \alpha \mid 000100011\rangle + \alpha \mid 000101011\rangle$

$-\alpha \mid 010000100\rangle + \alpha \mid 010001100\rangle - \alpha \mid 010110101\rangle + \alpha \mid 010111101\rangle$

$+\alpha \mid 110010110\rangle - \alpha \mid 110011110\rangle - \alpha \mid 110100111\rangle + \alpha \mid 110101111\rangle$

$-\alpha \mid 001000000\rangle - \alpha \mid 001001000\rangle + \alpha \mid 001110001\rangle + \alpha \mid 001111001\rangle$

$-\alpha \mid 101010010\rangle - \alpha \mid 101011010\rangle - \alpha \mid 101100011\rangle - \alpha \mid 101101011\rangle$

$+\alpha \mid 111000100\rangle + \alpha \mid 111001100\rangle + \alpha \mid 111110101\rangle + \alpha \mid 111111101\rangle$

$-\alpha \mid 011010110\rangle - \alpha \mid 011011110\rangle + \alpha \mid 011100111\rangle + \alpha \mid 011101111\rangle$

$-\alpha \mid 010000000\rangle + \alpha \mid 010001000\rangle - \alpha \mid 010110001\rangle + \alpha \mid 010111001\rangle$

$+\alpha \mid 110010010\rangle - \alpha \mid 110011010\rangle - \alpha \mid 110100011\rangle + \alpha \mid 110101011\rangle$

$-\alpha \mid 100000100\rangle + \alpha \mid 100001100\rangle + \alpha \mid 100110101\rangle - \alpha \mid 100111101\rangle$

$-\alpha \mid 100010110\rangle + \alpha \mid 100011110\rangle + \alpha \mid 100100111\rangle - \alpha \mid 100101111\rangle$

$+\alpha \mid 101100000\rangle + \alpha \mid 101101000\rangle + \alpha \mid 101010001\rangle + \alpha \mid 101011001\rangle$

$-\alpha \mid 001110010\rangle - \alpha \mid 001111010\rangle + \alpha \mid 001000011\rangle + \alpha \mid 001001011\rangle$

$-\alpha \mid 011100100\rangle - \alpha \mid 011101100\rangle + \alpha \mid 011010101\rangle + \alpha \mid 011011101\rangle$

$-\alpha \mid 111110110\rangle - \alpha \mid 111111110\rangle - \alpha \mid 111000111\rangle - \alpha \mid 111001111\rangle$

$+\alpha \mid 110100000\rangle - \alpha \mid 110101000\rangle - \alpha \mid 110010001\rangle + \alpha \mid 110011001\rangle$

$+\alpha \mid 010110010\rangle - \alpha \mid 010111010\rangle + \alpha \mid 010000011\rangle - \alpha \mid 010001011\rangle$

$+\alpha \mid 000100100\rangle - \alpha \mid 000101100\rangle + \alpha \mid 000010101\rangle - \alpha \mid 000011101\rangle$

$-\alpha \mid 100110110\rangle + \alpha \mid 100111110\rangle + \alpha \mid 100000111\rangle - \alpha \mid 100001111\rangle$

$-\alpha \mid 011100000\rangle - \alpha \mid 011101000\rangle + \alpha \mid 011010001\rangle + \alpha \mid 011011001\rangle$

$-\alpha \mid 111110010\rangle - \alpha \mid 111111010\rangle - \alpha \mid 111000011\rangle - \alpha \mid 111001011\rangle$

$+\alpha \mid 101100100\rangle + \alpha \mid 101101100\rangle + \alpha \mid 101010101\rangle + \alpha \mid 101011101\rangle$

$-\alpha \mid 001110110\rangle - \alpha \mid 001111110\rangle + \alpha \mid 001000111\rangle + \alpha \mid 001001111\rangle$

$+\alpha \mid 000100000\rangle - \alpha \mid 000101000\rangle + \alpha \mid 000010001\rangle - \alpha \mid 000011001\rangle$

$-\alpha \mid 100110010\rangle + \alpha \mid 100111010\rangle + \alpha \mid 100000011\rangle - \alpha \mid 100001011\rangle$

$+\alpha \mid 110100100\rangle - \alpha \mid 110101100\rangle - \alpha \mid 110010101\rangle + \alpha \mid 110011101\rangle$

$+\alpha \mid 010110110\rangle - \alpha \mid 010111110\rangle + \alpha \mid 010000111\rangle - \alpha \mid 010001111\rangle$

-$\alpha \mid 101010000\rangle - \alpha \mid 101011000\rangle - \alpha \mid 101100001\rangle - \alpha \mid 101101001\rangle$

$-\alpha \mid 001000010\rangle - \alpha \mid 001001010\rangle + \alpha \mid 001110011\rangle + \alpha \mid 001111011\rangle$

$-\alpha \mid 011010100\rangle - \alpha \mid 011011100\rangle + \alpha \mid 011100101\rangle + \alpha \mid 011101101\rangle$

$+\alpha \mid 111000110\rangle + \alpha \mid 111001110\rangle + \alpha \mid 111110111\rangle + \alpha \mid 111111111\rangle$

$+\alpha \mid 110010000\rangle - \alpha \mid 110011000\rangle - \alpha \mid 110100001\rangle + \alpha \mid 110101001\rangle$

$-\alpha \mid 010000010\rangle + \alpha \mid 010001010\rangle - \alpha \mid 010110011\rangle + \alpha \mid 010111011\rangle$

$-\alpha \mid 000010100\rangle + \alpha \mid 000011100\rangle - \alpha \mid 000100101\rangle + \alpha \mid 000101101\rangle$

$-\alpha \mid 100000110\rangle + \alpha \mid 100001110\rangle + \alpha \mid 100110111\rangle - \alpha \mid 100111111\rangle$

$-\alpha \mid 011010000\rangle - \alpha \mid 011011000\rangle + \alpha \mid 011100001\rangle + \alpha \mid 011101001\rangle$

$+\alpha \mid 111000010\rangle + \alpha \mid 111001010\rangle + \alpha \mid 111110011\rangle + \alpha \mid 111111011\rangle$

$-\alpha \mid 101010100\rangle - \alpha \mid 101011100\rangle - \alpha \mid 101100101\rangle - \alpha \mid 101101101\rangle$

$-\alpha \mid 001000110\rangle - \alpha \mid 001001110\rangle + \alpha \mid 001110111\rangle + \alpha \mid 001111111\rangle$

$-\alpha \mid 000010000\rangle + \alpha \mid 000011000\rangle - \alpha \mid 000100001\rangle + \alpha \mid 000101001\rangle$

$-\alpha \mid 100000010\rangle + \alpha \mid 100001010\rangle + \alpha \mid 100110011\rangle - \alpha \mid 100111011\rangle$

$+\alpha \mid 110010100\rangle - \alpha \mid 110011100\rangle - \alpha \mid 110100101\rangle + \alpha \mid 110101101\rangle$

$-\alpha \mid 010000110\rangle + \alpha \mid 010001110\rangle - \alpha \mid 010110111\rangle + \alpha \mid 010111111\rangle$

$-\alpha \mid 111110000\rangle - \alpha \mid 111111000\rangle - \alpha \mid 111000001\rangle - \alpha \mid 111001001\rangle$

$-\alpha \mid 011100010\rangle - \alpha \mid 011101010\rangle + \alpha \mid 011010011\rangle + \alpha \mid 011011011\rangle$

$-\alpha \mid 001110100\rangle - \alpha \mid 001111100\rangle + \alpha \mid 001000101\rangle + \alpha \mid 001001101\rangle$

$+\alpha \mid 101100110\rangle + \alpha \mid 101101110\rangle + \alpha \mid 101010111\rangle + \alpha \mid 101011111\rangle$

$-\alpha \mid 100110000\rangle + \alpha \mid 100111000\rangle + \alpha \mid 100000001\rangle - \alpha \mid 100001001\rangle$

$+\alpha \mid 000100010\rangle - \alpha \mid 000101010\rangle + \alpha \mid 000010011\rangle - \alpha \mid 000011011\rangle$

$+\alpha \mid 010110100\rangle - \alpha \mid 010111100\rangle + \alpha \mid 010000101\rangle - \alpha \mid 010001101\rangle$

$+\alpha \mid 110100110\rangle - \alpha \mid 110101110\rangle - \alpha \mid 110010111\rangle + \alpha \mid 110011111\rangle$

$-\alpha \mid 001110000\rangle - \alpha \mid 001111000\rangle + \alpha \mid 001000001\rangle + \alpha \mid 001001001\rangle$

$+\alpha \mid 101100010\rangle + \alpha \mid 101101010\rangle + \alpha \mid 101010011\rangle + \alpha \mid 101011011\rangle$

$-\alpha \mid 111110100\rangle - \alpha \mid 111111100\rangle - \alpha \mid 111000101\rangle - \alpha \mid 111001101\rangle$

$-\alpha \mid 011100110\rangle - \alpha \mid 011101110\rangle + \alpha \mid 011010111\rangle + \alpha \mid 011011111\rangle$

$+\alpha \mid 010110000\rangle - \alpha \mid 010111000\rangle + \alpha \mid 010000001\rangle - \alpha \mid 010001001\rangle$

$+\alpha \mid 110100010\rangle - \alpha \mid 110101010\rangle - \alpha \mid 110010011\rangle + \alpha \mid 110011011\rangle$

$-\alpha \mid 100110100\rangle + \alpha \mid 100111100\rangle + \alpha \mid 100000101\rangle - \alpha \mid 100001101\rangle$

$+\alpha \mid 000100110\rangle - \alpha \mid 000101110\rangle - \alpha \mid 100000111\rangle + \alpha \mid 100001111\rangle$

$-\beta \mid 110000000\rangle - \beta \mid 110001000\rangle + \beta \mid 110110001\rangle + \beta \mid 110111001\rangle$

$+\beta \mid 010010010\rangle + \beta \mid 010011010\rangle + \beta \mid 010100011\rangle + \beta \mid 010101011\rangle$

$-\beta \mid 000000100\rangle - \beta \mid 000001100\rangle - \beta \mid 000110101\rangle - \beta \mid 000111101\rangle$

$-\beta \mid 100010110\rangle - \beta \mid 100011110\rangle + \beta \mid 100100111\rangle + \beta \mid 100101111\rangle$

$+\beta \mid 101000000\rangle - \beta \mid 101001000\rangle + \beta \mid 101110001\rangle - \beta \mid 101111001\rangle$

$+\beta \mid 001010010\rangle - \beta \mid 001011010\rangle - \beta \mid 001100011\rangle + \beta \mid 001101011\rangle$

$-\beta \mid 011000100\rangle + \beta \mid 011001100\rangle + \beta \mid 011110101\rangle - \beta \mid 011111101\rangle$

$+\beta \mid 111010110\rangle - \beta \mid 111011110\rangle + \beta \mid 111100111\rangle - \beta \mid 111101111\rangle$

$-\beta \mid 000000000\rangle - \beta \mid 000001000\rangle - \beta \mid 000110001\rangle - \beta \mid 000111001\rangle$

$-\beta \mid 100010010\rangle - \beta \mid 100011010\rangle + \beta \mid 100100011\rangle + \beta \mid 100101011\rangle$

$-\beta \mid 110000100\rangle - \beta \mid 110001100\rangle + \beta \mid 110110101\rangle + \beta \mid 110111101\rangle$

$-\beta \mid 010010110\rangle - \beta \mid 010011110\rangle - \beta \mid 010100111\rangle - \beta \mid 010101111\rangle$

$-\beta \mid 011000000\rangle + \beta \mid 011001000\rangle + \beta \mid 011110001\rangle - \beta \mid 011111001\rangle$

$+\beta \mid 111010010\rangle - \beta \mid 111011010\rangle + \beta \mid 111100011\rangle - \beta \mid 111101011\rangle$

$+\beta \mid 101000100\rangle - \beta \mid 101001100\rangle + \beta \mid 101110101\rangle - \beta \mid 101111101\rangle$

$+\beta \mid 001010110\rangle - \beta \mid 001011110\rangle - \beta \mid 001100111\rangle + \beta \mid 001101111\rangle$

$-\beta \mid 100100000\rangle - \beta \mid 100101000\rangle + \beta \mid 100010001\rangle + \beta \mid 100011001\rangle$

$$+\beta \mid 000110010\rangle + \beta \mid 000111010\rangle + \beta \mid 000000011\rangle + \beta \mid 000001011\rangle$$
$$-\beta \mid 010100100\rangle - \beta \mid 010101100\rangle - \beta \mid 010010101\rangle - \beta \mid 010011101\rangle$$
$$-\beta \mid 110110110\rangle - \beta \mid 110111110\rangle + \beta \mid 110000111\rangle + \beta \mid 110001111\rangle$$
$$-\beta \mid 111100000\rangle + \beta \mid 111101000\rangle - \beta \mid 111010001\rangle + \beta \mid 111011001\rangle$$
$$-\beta \mid 011110010\rangle + \beta \mid 011111010\rangle + \beta \mid 011000011\rangle - \beta \mid 011001011\rangle$$
$$+\beta \mid 001100100\rangle - \beta \mid 001101100\rangle - \beta \mid 001010101\rangle + \beta \mid 001011101\rangle$$
$$-\beta \mid 101110110\rangle + \beta \mid 101111110\rangle - \beta \mid 101000111\rangle + \beta \mid 101001111\rangle$$
$$-\beta \mid 010100000\rangle - \beta \mid 010101000\rangle - \beta \mid 010010001\rangle - \beta \mid 010011001\rangle$$
$$-\beta \mid 110110010\rangle - \beta \mid 110111010\rangle + \beta \mid 110000011\rangle + \beta \mid 110001011\rangle$$
$$-\beta \mid 100100100\rangle - \beta \mid 100101100\rangle + \beta \mid 100010101\rangle + \beta \mid 100011101\rangle$$
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$$+\beta \mid 001100000\rangle - \beta \mid 001101000\rangle - \beta \mid 001010001\rangle + \beta \mid 001011001\rangle$$
$$-\beta \mid 101110010\rangle + \beta \mid 101111010\rangle - \beta \mid 101000011\rangle + \beta \mid 101001011\rangle$$
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$$-\beta \mid 100010000\rangle - \beta \mid 100011000\rangle + \beta \mid 100100001\rangle + \beta \mid 100101001\rangle$$
$$-\beta \mid 000000010\rangle - \beta \mid 000001010\rangle - \beta \mid 000110011\rangle - \beta \mid 000111011\rangle$$
$$+\beta \mid 010010100\rangle + \beta \mid 010011100\rangle + \beta \mid 010100101\rangle + \beta \mid 010101101\rangle$$
$$-\beta \mid 110000110\rangle - \beta \mid 110001110\rangle + \beta \mid 110110111\rangle + \beta \mid 110111111\rangle$$
$$+\beta \mid 111010000\rangle - \beta \mid 111011000\rangle + \beta \mid 111100001\rangle - \beta \mid 111101001\rangle$$
$$-\beta \mid 011000010\rangle + \beta \mid 011001010\rangle + \beta \mid 011110011\rangle - \beta \mid 011111011\rangle$$
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$$+\beta \mid 101000010\rangle - \beta \mid 101001010\rangle + \beta \mid 101110011\rangle - \beta \mid 101111011\rangle$$
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$+\beta \mid 000110100\rangle + \beta \mid 000111100\rangle + \beta \mid 000000101\rangle + \beta \mid 000001101\rangle$
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$+\beta \mid 001100010\rangle - \beta \mid 001101010\rangle - \beta \mid 001010011\rangle + \beta \mid 001011011\rangle$
$-\beta \mid 011110100\rangle + \beta \mid 011111100\rangle + \beta \mid 011000101\rangle - \beta \mid 011001101\rangle$
$-\beta \mid 111100110\rangle + \beta \mid 111101110\rangle - \beta \mid 111010111\rangle + \beta \mid 111011111\rangle$
$+\beta \mid 000110000\rangle + \beta \mid 000111000\rangle + \beta \mid 000000001\rangle + \beta \mid 000001001\rangle$
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$-\beta \mid 010100110\rangle - \beta \mid 010101110\rangle - \beta \mid 010010111\rangle - \beta \mid 010011111\rangle$
$-\beta \mid 011110000\rangle + \beta \mid 011111000\rangle + \beta \mid 01100001\rangle - \beta \mid 011001001\rangle$
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$+\beta \mid 001100110\rangle - \beta \mid 001101110\rangle - \beta \mid 001010111\rangle + \beta \mid 001011111\rangle$
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$+\beta \mid 100010000\rangle - \beta \mid 100011000\rangle - \beta \mid 100100001\rangle + \beta \mid 100101001\rangle$
$+\beta \mid 000000010\rangle - \beta \mid 000001010\rangle + \beta \mid 000110011\rangle - \beta \mid 000111011\rangle$
$-\beta \mid 010010100\rangle + \beta \mid 010011100\rangle - \beta \mid 010100101\rangle + \beta \mid 010101101\rangle$
$+\beta \mid 110000110\rangle - \beta \mid 110001110\rangle - \beta \mid 110110111\rangle + \beta \mid 110111111\rangle$
$-\beta \mid 001010000\rangle - \beta \mid 001011000\rangle + \beta \mid 001100001\rangle + \beta \mid 001101001\rangle$
$-\beta \mid 101000010\rangle - \beta \mid 101001010\rangle - \beta \mid 101110011\rangle - \beta \mid 101111011\rangle$
$-\beta \mid 111010100\rangle - \beta \mid 111011100\rangle - \beta \mid 111100101\rangle - \beta \mid 111101101\rangle$
$+\beta \mid 011000110\rangle + \beta \mid 011001110\rangle - \beta \mid 011110111\rangle - \beta \mid 011111111\rangle$
$-\beta \mid 010010000\rangle + \beta \mid 010011000\rangle - \beta \mid 010100001\rangle + \beta \mid 010101001\rangle$
$+\beta \mid 110000010\rangle - \beta \mid 110001010\rangle - \beta \mid 110110011\rangle + \beta \mid 110111011\rangle$
$+\beta \mid 100010100\rangle - \beta \mid 100011100\rangle - \beta \mid 100100101\rangle + \beta \mid 100101101\rangle$
$+\beta \mid 000000110\rangle - \beta \mid 000001110\rangle + \beta \mid 000110111\rangle - \beta \mid 000111111\rangle$
$+\beta \mid 101110000\rangle + \beta \mid 101111000\rangle + \beta \mid 101000001\rangle + \beta \mid 101001001\rangle$

$-\beta \mid 001100010\rangle - \beta \mid 001101010\rangle + \beta \mid 001010011\rangle + \beta \mid 001011011\rangle$

$+\beta \mid 011110100\rangle + \beta \mid 011111100\rangle - \beta \mid 011000101\rangle - \beta \mid 011001101\rangle$

$+\beta \mid 111100110\rangle + \beta \mid 111101110\rangle + \beta \mid 111010111\rangle + \beta \mid 111011111\rangle$

$+\beta \mid 110110000\rangle - \beta \mid 110111000\rangle - \beta \mid 110000001\rangle + \beta \mid 110001001\rangle$

$+\beta \mid 010100010\rangle - \beta \mid 010101010\rangle + \beta \mid 010010011\rangle - \beta \mid 010011011\rangle$

$-\beta \mid 000110100\rangle + \beta \mid 000111100\rangle - \beta \mid 000000101\rangle + \beta \mid 000001101\rangle$

$+\beta \mid 100100110\rangle - \beta \mid 100101110\rangle - \beta \mid 100010111\rangle + \beta \mid 100011111\rangle$

$+\beta \mid 011110000\rangle + \beta \mid 011111000\rangle - \beta \mid 011000001\rangle - \beta \mid 011001001\rangle$

$+\beta \mid 111100010\rangle + \beta \mid 111101010\rangle + \beta \mid 111010011\rangle + \beta \mid 111011011\rangle$

$+\beta \mid 101110100\rangle + \beta \mid 101111100\rangle + \beta \mid 101000101\rangle + \beta \mid 101001101\rangle$

$-\beta \mid 001100110\rangle - \beta \mid 001101110\rangle + \beta \mid 001010111\rangle + \beta \mid 001011111\rangle$

$-\beta \mid 000110000\rangle + \beta \mid 000111000\rangle - \beta \mid 000000001\rangle + \beta \mid 000001001\rangle$

$+\beta \mid 100100010\rangle - \beta \mid 100101010\rangle - \beta \mid 100010011\rangle + \beta \mid 100011011\rangle$

$+\beta \mid 110110100\rangle - \beta \mid 110111100\rangle - \beta \mid 110000101\rangle + \beta \mid 110001101\rangle$

$+\beta \mid 010100110\rangle - \beta \mid 010101110\rangle + \beta \mid 010010111\rangle - \beta \mid 010011111\rangle$

-$\beta \mid 101000000\rangle - \beta \mid 101001000\rangle - \beta \mid 101110001\rangle - \beta \mid 101111001\rangle$

$-\beta \mid 001010010\rangle - \beta \mid 001011010\rangle + \beta \mid 001100011\rangle + \beta \mid 001101011\rangle$

$+\beta \mid 011000100\rangle + \beta \mid 011001100\rangle - \beta \mid 011110101\rangle - \beta \mid 011111101\rangle$

$-\beta \mid 111010110\rangle - \beta \mid 111011110\rangle - \beta \mid 111100111\rangle - \beta \mid 111101111\rangle$

$+\beta \mid 110000000\rangle - \beta \mid 110001000\rangle - \beta \mid 110110001\rangle + \beta \mid 110111001\rangle$

$-\beta \mid 010010010\rangle + \beta \mid 010011010\rangle - \beta \mid 010100011\rangle + \beta \mid 010101011\rangle$

$-\beta \mid 000000100\rangle + \beta \mid 000001100\rangle - \beta \mid 000110101\rangle + \beta \mid 000111101\rangle$

$-\beta \mid 100010110\rangle + \beta \mid 100011110\rangle + \beta \mid 100100111\rangle - \beta \mid 100101111\rangle$

$+\beta \mid 011000000\rangle + \beta \mid 011001000\rangle - \beta \mid 011110001\rangle - \beta \mid 011111001\rangle$

$-\beta \mid 111010010\rangle - \beta \mid 111011010\rangle - \beta \mid 111100011\rangle - \beta \mid 111101011\rangle$

$-\beta \mid 101000100\rangle - \beta \mid 101001100\rangle - \beta \mid 101110101\rangle - \beta \mid 101111101\rangle$

$-\beta \mid 001010110\rangle - \beta \mid 001011110\rangle + \beta \mid 001100111\rangle + \beta \mid 001101111\rangle$

$+\beta \mid 000000000\rangle - \beta \mid 000001000\rangle + \beta \mid 000110001 - \beta \mid 000111001\rangle$

$+\beta \mid 100010010\rangle - \beta \mid 100011010\rangle - \beta \mid 100100011\rangle + \beta \mid 100101011\rangle$

$+\beta \mid 110000100\rangle - \beta \mid 110001100\rangle - \beta \mid 110110101\rangle + \beta \mid 110111101\rangle$

$-\beta \mid 010010110\rangle + \beta \mid 010011110\rangle - \beta \mid 010100111\rangle + \beta \mid 010101111\rangle$

$+\beta \mid 111100000\rangle + \beta \mid 111101000\rangle + \beta \mid 111010001\rangle + \beta \mid 111011001\rangle$

$+\beta \mid 011110010\rangle + \beta \mid 011111010\rangle - \beta \mid 011000011\rangle - \beta \mid 011001011\rangle$

$-\beta \mid 001100100\rangle - \beta \mid 001101100\rangle + \beta \mid 001010101\rangle + \beta \mid 001011101\rangle$

$+\beta \mid 101110110\rangle + \beta \mid 101111110\rangle + \beta \mid 101000111\rangle + \beta \mid 101001111\rangle$

$+\beta \mid 100100000\rangle - \beta \mid 100101000\rangle - \beta \mid 100010001\rangle + \beta \mid 100011001\rangle$

$-\beta \mid 000110010\rangle + \beta \mid 000111010\rangle - \beta \mid 000000011\rangle + \beta \mid 000001011\rangle$

$+\beta \mid 010100100\rangle - \beta \mid 010101100\rangle + \beta \mid 010010101\rangle - \beta \mid 010011101\rangle$

$+\beta \mid 110110110\rangle - \beta \mid 110111110\rangle - \beta \mid 110000111\rangle + \beta \mid 110001111\rangle$

$-\beta \mid 001100000\rangle - \beta \mid 001101000\rangle + \beta \mid 001010001\rangle + \beta \mid 001011001\rangle$

$+\beta \mid 101110010\rangle + \beta \mid 101111010\rangle + \beta \mid 101000011\rangle + \beta \mid 101001011\rangle$

$+\beta \mid 111100100\rangle + \beta \mid 111101100\rangle + \beta \mid 111010101\rangle + \beta \mid 111011101\rangle$

$+\beta \mid 011110110\rangle + \beta \mid 011111110\rangle - \beta \mid 011000111\rangle - \beta \mid 011001111\rangle$

$+\beta \mid 010100000\rangle - \beta \mid 010101000\rangle + \beta \mid 010010001\rangle - \beta \mid 010011001\rangle$

$+\beta \mid 110110010\rangle - \beta \mid 110111010\rangle - \beta \mid 110000011\rangle + \beta \mid 110001011\rangle$

$+\beta \mid 010100100\rangle - \beta \mid 010101100\rangle + \beta \mid 010010101\rangle - \beta \mid 010011101\rangle$

$+\beta \mid 110110110\rangle - \beta \mid 110111110\rangle - \beta \mid 110000111\rangle + \beta \mid 110001111\rangle)$

○ L'application de la Porte $H\,(7)$:

$\mid \Psi_1\rangle = \frac{1}{16}(+\alpha \mid 100001000\rangle + \alpha \mid 100001100\rangle - \alpha \mid 100111001\rangle - \alpha \mid 100111101\rangle$

$+\alpha \mid 000011010\rangle + \alpha \mid 000011110\rangle + \alpha \mid 000101011\rangle + \alpha \mid 000101111\rangle$

$+\alpha \mid 010001000\rangle - \alpha \mid 010001100\rangle + \alpha \mid 010111001\rangle - \alpha \mid 010111101\rangle$

$-\alpha \mid 110011010\rangle + \alpha \mid 110011110\rangle + \alpha \mid 110101011\rangle - \alpha \mid 110101111\rangle$

$+\alpha \mid 111001000\rangle + \alpha \mid 111001100\rangle + \alpha \mid 111111001\rangle + \alpha \mid 111111101\rangle$

$-\alpha \mid 011011010\rangle - \alpha \mid 011011110\rangle + \alpha \mid 011101011\rangle + \alpha \mid 011101111\rangle$

$-\alpha \mid 001001000\rangle + \alpha \mid 001001100\rangle + \alpha \mid 001111001\rangle - \alpha \mid 001111101\rangle$

$-\alpha \mid 101011010\rangle + \alpha \mid 101011110\rangle - \alpha \mid 101101011\rangle + \alpha \mid 101101111\rangle$

$-\alpha \mid 110011000\rangle - \alpha \mid 110011100\rangle + \alpha \mid 110101001\rangle + \alpha \mid 110101101\rangle$

$+\alpha \mid 010001010\rangle + \alpha \mid 010001110\rangle + \alpha \mid 010111011\rangle + \alpha \mid 010111111\rangle$

$+\alpha \mid 000011000\rangle - \alpha \mid 000011100\rangle + \alpha \mid 000101001\rangle - \alpha \mid 000101101\rangle$

$+\alpha \mid 100001010\rangle - \alpha \mid 100001110\rangle - \alpha \mid 100111011\rangle + \alpha \mid 100111111\rangle$

$-\alpha \mid 101011000\rangle - \alpha \mid 101011100\rangle - \alpha \mid 101101001\rangle - \alpha \mid 101101101\rangle$

$-\alpha \mid 001001010\rangle - \alpha \mid 001001110\rangle + \alpha \mid 001111011\rangle + \alpha \mid 001111111\rangle$

$-\alpha \mid 011011000\rangle + \alpha \mid 011011100\rangle + \alpha \mid 011101001\rangle - \alpha \mid 011101101\rangle$

$+\alpha \mid 111001010\rangle - \alpha \mid 111001110\rangle + \alpha \mid 111111011\rangle - \alpha \mid 111111111\rangle$

$$+\alpha \mid 000011000\rangle + \alpha \mid 000011100\rangle \ + \alpha \mid 000101001\rangle + \alpha \mid 000101101\rangle$$
$$+\alpha \mid 100001010\rangle + \alpha \mid 100001110\rangle \ - \alpha \mid \ 100111011\rangle - \alpha \mid \ 100111111\rangle$$
$$-\alpha \mid 110011000\rangle + \alpha \mid 110011100\rangle \ + \alpha \mid 110101001\rangle - \alpha \mid 110101101\rangle$$
$$+\alpha \mid 010001010\rangle - \alpha \mid 010001110\rangle \ + \alpha \mid \ 010111011\rangle - \alpha \mid \ 010111111\rangle$$
$$-\alpha \mid 011011000\rangle - \alpha \mid 011011100\rangle \ + \alpha \mid 011101001\rangle + \alpha \mid 011101101\rangle$$
$$+\alpha \mid 111001010\rangle + \alpha \mid 111001110\rangle \ + \alpha \mid \ 111111011\rangle + \alpha \mid \ 111111111\rangle$$
$$-\alpha \mid 101011000\rangle + \alpha \mid 101011100\rangle \ - \alpha \mid 101101001\rangle + \alpha \mid 101101101\rangle$$
$$-\alpha \mid 001001010\rangle + \alpha \mid 001001110\rangle \ + \alpha \mid \ 001111011\rangle - \alpha \mid \ 001111111\rangle$$
$$+\alpha \mid 100111000\rangle + \alpha \mid 100111100\rangle \ - \alpha \mid 100001001\rangle - \alpha \mid 100001101\rangle$$
$$-\alpha \mid 000101010\rangle - \alpha \mid 000101110\rangle \ - \alpha \mid \ 000011011\rangle - \alpha \mid \ 000011111\rangle$$
$$-\alpha \mid 010111000\rangle + \alpha \mid 010111100\rangle \ - \alpha \mid 010001001\rangle + \alpha \mid 010001101\rangle$$
$$-\alpha \mid 110101010\rangle + \alpha \mid 110101110\rangle \ + \alpha \mid \ 110011011\rangle - \alpha \mid \ 110011111\rangle$$
$$-\alpha \mid 111111000\rangle - \alpha \mid 111111100\rangle \ -\alpha \mid 111001001\rangle - \alpha \mid 111001101\rangle$$
$$-\alpha \mid 011101010\rangle - \alpha \mid 011101110\rangle \ +\alpha \mid \ 011011011\rangle + \alpha \mid \ 011011111\rangle$$
$$-\alpha \mid 001111000\rangle + \alpha \mid 001111100\rangle \ + \alpha \mid 001001001 \ \rangle - \alpha \mid 001001101 \ \rangle$$
$$+\alpha \mid 101101010 \ \rangle - \alpha \mid 101101110 \ \rangle \ +\alpha \mid \ 101011011\rangle - \alpha \mid \ 101011111\rangle$$
$$-\alpha \mid 010111000\rangle \ - \alpha \mid 010111100\rangle \ - \alpha \mid 010001001\rangle - \alpha \mid 010001101\rangle$$
$$-\alpha \mid 110101010\rangle \ - \alpha \mid 110101110\rangle \ + \alpha \mid \ 110011011\rangle + \alpha \mid \ 110011111\rangle$$
$$+\alpha \mid 100111000\rangle \ - \alpha \mid 100111100\rangle \ - \alpha \mid 100001001\rangle + \alpha \mid 100001101\rangle$$
$$-\alpha \mid 000101010\rangle + \alpha \mid 000101110\rangle \ - \alpha \mid \ 000011011\rangle + \alpha \mid \ 000011111\rangle$$
$$-\alpha \mid 001111000\rangle - \alpha \mid 001111100\rangle + \alpha \mid 001001001\rangle + \alpha \mid 001001101\rangle$$
$$+\alpha \mid 101101010\rangle + \alpha \mid 101101110\rangle \ + \alpha \mid \ 101011011\rangle + \alpha \mid \ 101011111\rangle$$
$$-\alpha \mid 111111000\rangle + \alpha \mid 111111100\rangle \ - \alpha \mid 111001001\rangle + \alpha \mid 111001101\rangle$$
$$-\alpha \mid 011101010\rangle + \alpha \mid 011101110\rangle \ + \alpha \mid \ 011011011\rangle - \alpha \mid \ 011011111\rangle$$
$$+\alpha \mid 010001000\rangle + \alpha \mid 010001100\rangle \ + \alpha \mid 010111001\rangle + \alpha \mid 010111101\rangle$$
$$-\alpha \mid 110011010\rangle \ - \alpha \mid 110011110\rangle \ + \alpha \mid \ 110101011\rangle + \alpha \mid \ 110101111\rangle$$
$$+\alpha \mid 100001000\rangle - \alpha \mid 100001100\rangle \ - \alpha \mid 100111001\rangle + \alpha \mid 100111101\rangle$$
$$+\alpha \mid 000011010\rangle - \alpha \mid 000011110\rangle \ + \alpha \mid \ 000101011\rangle - \alpha \mid \ 000101111\rangle$$
$$-\alpha \mid 001001000\rangle \ - \alpha \mid 001001100\rangle \ + \alpha \mid 001111001\rangle + \alpha \mid 001111101\rangle$$
$$-\alpha \mid 101011010\rangle - \alpha \mid 101011110\rangle - \alpha \mid \ 101101011\rangle - \alpha \mid \ 101101111\rangle$$
$$+\alpha \mid 111001000\rangle - \alpha \mid 111001100\rangle + \alpha \mid 111111001\rangle - \alpha \mid 111111101\rangle$$
$$-\alpha \mid 011011010\rangle + \alpha \mid 011011110\rangle + \alpha \mid \ 011101011\rangle \ - \alpha \mid \ 011101111\rangle$$

$$-\alpha \mid 110101000\rangle \ -\alpha \mid 110101100\rangle \ + \ \alpha \mid 110011001\rangle \ \ +\alpha \mid 110011101\rangle$$
$$-\alpha \mid 010111010\rangle - \alpha \mid 010111110\rangle \ -\alpha \mid \ 010001011\rangle - \alpha \mid \ 010001111\rangle$$
$$-\alpha \mid 000101000\rangle + \alpha \mid 000101100\rangle \ -\alpha \mid 000011001\rangle + \alpha \mid 000011101\rangle$$
$$+\alpha \mid 100111010\rangle - \alpha \mid 100111110\rangle \ -\alpha \mid \ 100001011\rangle + \alpha \mid \ 100001111\rangle$$
$$+\alpha \mid 101101000\rangle + \alpha \mid 101101100\rangle \ +\alpha \mid 101011001\rangle + \alpha \mid 101011101\rangle$$
$$-\alpha \mid 001111010\rangle - \alpha \mid 001111110\rangle \ +\alpha \mid \ 001001011\rangle + \alpha \mid \ 001001111\rangle$$
$$-\alpha \mid 011101000\rangle + \alpha \mid 011101100\rangle \ \ +\alpha \mid 011011001\rangle - \alpha \mid 011011101\rangle$$
$$-\alpha \mid 111111010\rangle + \alpha \mid 111111110\rangle \ -\alpha \mid \ 111001011\rangle + \alpha \mid \ 111001111\rangle$$
$$-\alpha \mid 000101000\rangle - \alpha \mid 000101100\rangle \ -\alpha \mid 000011001\rangle - \alpha \mid 000011101\rangle$$
$$+\alpha \mid 100111010\rangle + \alpha \mid 100111110\rangle \ -\alpha \mid \ 100001011\rangle - \alpha \mid \ 100001111\rangle$$
$$-\alpha \mid 110101000\rangle + \alpha \mid 110101100\rangle \ +\alpha \mid 110011001\rangle - \alpha \mid 110011101\rangle$$
$$-\alpha \mid 010111010\rangle + \alpha \mid 010111110\rangle \ -\alpha \mid \ 010001011\rangle + \alpha \mid \ 010001111\rangle$$
$$-\alpha \mid 011101000\rangle - \alpha \mid 011101100\rangle \ +\alpha \mid 011011001\rangle + \alpha \mid 011011101\rangle$$
$$-\alpha \mid 111111010\rangle - \alpha \mid 111111110\rangle \ -\alpha \mid \ 111001011\rangle - \alpha \mid \ 111001111\rangle$$
$$+\alpha \mid 101101000\rangle - \alpha \mid 101101100\rangle \ +\alpha \mid 101011001\rangle - \alpha \mid 101011101\rangle$$
$$-\alpha \mid 001111010\rangle + \alpha \mid 001111110\rangle \ +\alpha \mid \ 001001011\rangle - \alpha \mid \ 001001111\rangle$$
$$-\beta \mid 110001000\rangle - \ \beta \mid 11000100\rangle \ +\beta \mid 110111001\rangle + \beta \mid 110111001\rangle$$
$$+\beta \mid 010011010\rangle + \beta \mid 010011110\rangle \ +\beta \mid 010101011\rangle + \beta \mid 010101011\rangle$$
$$-\beta \mid 000001000\rangle + \beta \mid 000001100\rangle \ -\beta \mid 000111001\rangle + \beta \mid 000111101\rangle$$
$$-\beta \mid 100011010\rangle + \beta \mid 100011110\rangle \ +\beta \mid 100101011\rangle - \beta \mid 100101111\rangle$$
$$-\beta \mid 101001000\rangle - \beta \mid 101001100\rangle \ -\beta \mid 101111001\rangle - \beta \mid 101111101\rangle$$
$$-\beta \mid 001011010\rangle - \beta \mid 001011110\rangle \ +\beta \mid 001101011\rangle + \beta \mid 001101111\rangle$$
$$+\beta \mid 011001000\rangle \ -\beta \mid 011001100\rangle \ -\beta \mid 011111001\rangle + \beta \mid 011111101\rangle$$
$$-\beta \mid 111011010\rangle \ +\beta \mid 111011110\rangle \ -\beta \mid 111101011\rangle + \beta \mid 111101111\rangle$$
$$-\beta \mid 000001000\rangle \ -\beta \mid 000001100\rangle \ -\beta \mid 000111001\rangle - \beta \mid 000111101\rangle$$
$$-\beta \mid 100011010\rangle - \beta \mid 100011110\rangle \ +\beta \mid 100101011\rangle + \beta \mid 100101111\rangle$$
$$-\beta \mid 11000100\rangle + \beta \mid 110001100\rangle \ +\beta \mid 110111001\rangle - \beta \mid 110111101\rangle$$
$$-\beta \mid 010011010\rangle + \beta \mid 010011110\rangle \ -\beta \mid 010101011\rangle + \beta \mid 010101111\rangle$$
$$+\beta \mid 011001000\rangle + \beta \mid 011001100\rangle \ -\beta \mid 011111001\rangle - \beta \mid 011111101\rangle$$
$$-\beta \mid 111011010\rangle - \beta \mid 111011110\rangle \ -\beta \mid 111101011\rangle - \beta \mid 111101111\rangle$$
$$-\beta \mid 101001000\rangle + \beta \mid 101001100\rangle \ -\beta \mid 101111001\rangle + \beta \mid 101111101\rangle$$
$$-\beta \mid 001011010\rangle + \beta \mid 001011110\rangle \ +\beta \mid 001101011\rangle - \beta \mid 001101111\rangle$$

$$-\beta\mid 100101000\rangle-\beta\mid 100101100\rangle+\beta\mid 100011001\rangle+\beta\mid 100011101\rangle$$
$$+\beta\mid 000111010\rangle+\beta\mid 000111110\rangle+\beta\mid 000001011\rangle+\beta\mid 000001111\rangle$$
$$-\beta\mid 010101000\rangle+\beta\mid 010101100\rangle-\beta\mid 010011001\rangle+\beta\mid 010011101\rangle$$
$$-\beta\mid 110111010\rangle+\beta\mid 110111110\rangle+\beta\mid 110001011\rangle-\beta\mid 110001111\rangle$$
$$+\beta\mid 111101000\rangle+\beta\mid 111101100\rangle+\beta\mid 111011001\rangle+\beta\mid 111011101\rangle$$
$$+\beta\mid 011111010\rangle+\beta\mid 011111110\rangle-\beta\mid 011001011\rangle-\beta\mid 011001111\rangle$$
$$-\beta\mid 001101000\rangle+\beta\mid 001101100\rangle+\beta\mid 001011001\rangle-\beta\mid 001011101\rangle$$
$$+\beta\mid 101111010\rangle-\beta\mid 101111110\rangle+\beta\mid 101001011\rangle-\beta\mid 101001111\rangle$$
$$-\beta\mid 010101000\rangle-\beta\mid 010101100\rangle-\beta\mid 010011001\rangle-\beta\mid 010011101\rangle$$
$$-\beta\mid 110111010\rangle-\beta\mid 110111110\rangle+\beta\mid 110001011\rangle+\beta\mid 110001111\rangle$$
$$-\beta\mid 100101000\rangle+\beta\mid 100101100\rangle+\beta\mid 100011001\rangle-\beta\mid 100011101\rangle$$
$$+\beta\mid 000111010\rangle-\beta\mid 000111110\rangle+\beta\mid 000001011\rangle-\beta\mid 000001111\rangle$$
$$-\beta\mid 001101000\rangle-\beta\mid 001101100\rangle+\beta\mid 001011001\rangle+\beta\mid 001011101\rangle$$
$$+\beta\mid 101111010\rangle+\beta\mid 101111110\rangle+\beta\mid 101001011\rangle+\beta\mid 101001111\rangle$$
$$+\beta\mid 111101000\rangle-\beta\mid 111101100\rangle+\beta\mid 011011001\rangle-\beta\mid 011011101\rangle$$
$$+\beta\mid 011111010\rangle-\beta\mid 011111110\rangle-\beta\mid 011001011\rangle+\beta\mid 011001111\rangle$$
$$-\beta\mid 100011000\rangle-\beta\mid 100011100\rangle+\beta\mid 100101001\rangle+\beta\mid 100101101\rangle$$
$$-\beta\mid 000001010\rangle-\beta\mid 000001110\rangle-\beta\mid 000111011\rangle-\beta\mid 000111111\rangle$$
$$+\beta\mid 010011000\rangle-\beta\mid 010011100\rangle+\beta\mid 010101001\rangle-\beta\mid 010101101\rangle$$
$$-\beta\mid 110001010\rangle+\beta\mid 110001110\rangle+\beta\mid 110111011\rangle-\beta\mid 110111111\rangle$$
$$-\beta\mid 111011000\rangle-\beta\mid 111011100\rangle-\beta\mid 111101001\rangle-\beta\mid 111101101\rangle$$
$$+\beta\mid 011001010\rangle+\beta\mid 011001110\rangle-\beta\mid 011111011\rangle-\beta\mid 011111111\rangle$$
$$-\beta\mid 001011000\rangle+\beta\mid 001011100\rangle+\beta\mid 001101001\rangle-\beta\mid 001101101\rangle$$
$$-\beta\mid 101001010\rangle+\beta\mid 101001110\rangle-\beta\mid 101111011\rangle+\beta\mid 101111111\rangle$$
$$+\beta\mid 010011000\rangle+\beta\mid 010011100\rangle+\beta\mid 010101001\rangle+\beta\mid 010101101\rangle$$
$$-\beta\mid 110001010\rangle-\beta\mid 110001110\rangle+\beta\mid 110111011\rangle+\beta\mid 110111111\rangle$$
$$-\beta\mid 100011000\rangle+\beta\mid 100011100\rangle+\beta\mid 100101001\rangle-\beta\mid 100101101\rangle$$
$$-\beta\mid 000001010\rangle+\beta\mid 000001110\rangle-\beta\mid 000111011\rangle+\beta\mid 000111111\rangle$$
$$-\beta\mid 001011000\rangle-\beta\mid 001011100\rangle+\beta\mid 001101001\rangle+\beta\mid 001101101\rangle$$
$$-\beta\mid 101001010\rangle-\beta\mid 101001110\rangle-\beta\mid 101111011\rangle-\beta\mid 101111111\rangle$$
$$-\beta\mid 111011000\rangle+\beta\mid 111011100\rangle-\beta\mid 111101001\rangle+\beta\mid 111101101\rangle$$
$$+\beta\mid 011001010\rangle-\beta\mid 011001110\rangle-\beta\mid 011111011\rangle+\beta\mid 011111111\rangle$$

$-\beta \mid 110111000\rangle - \beta \mid 110111100\rangle \; + \beta \mid 110001001\rangle + \beta \mid 110001101\rangle$

$-\beta \mid 010101010\rangle - \beta \mid 010101110\rangle \; - \beta \mid 010011011\rangle - \beta \mid 010011111\rangle$

$+\beta \mid 000111000\rangle - \beta \mid 000111100\rangle \; + \beta \mid 000001001\rangle - \beta \mid 000001101\rangle$

$-\beta \mid 100101010\rangle + \beta \mid 100101110\rangle \; + \beta \mid 100011011\rangle - \beta \mid 100011111\rangle$

$+\beta \mid 101111000\rangle + \beta \mid 101111100\rangle \; + \beta \mid 101001001\rangle + \beta \mid 101001101\rangle$

$-\beta \mid 001101010\rangle - \beta \mid 001101110\rangle \; + \beta \mid 001011011\rangle + \beta \mid 001011111\rangle$

$+\beta \mid 011111000\rangle - \beta \mid 011111100\rangle \; - \beta \mid 011001001\rangle + \beta \mid 011001101\rangle$

$+\beta \mid 111101010\rangle - \beta \mid 111101110\rangle \; + \beta \mid 111011011\rangle - \beta \mid 111011111\rangle$

$+\beta \mid 000111000\rangle + \beta \mid 000111100\rangle \; + \beta \mid 000001001\rangle + \beta \mid 000001101\rangle$

$-\beta \mid 100101010\rangle - \beta \mid 100101110\rangle \; + \beta \mid 100011011\rangle + \beta \mid 100011111\rangle$

$-\beta \mid 110111000\rangle + \beta \mid 110111100\rangle \; + \beta \mid 110001001\rangle - \beta \mid 110001101\rangle$

$-\beta \mid 010101010\rangle + \beta \mid 010101110\rangle \; - \beta \mid 010011011\rangle + \beta \mid 010011111\rangle$

$+\beta \mid 011111000\rangle \; + \beta \mid 011111100\rangle \; - \beta \mid 011001001\rangle - \beta \mid 011001101\rangle$

$+\beta \mid 111101010\rangle + \beta \mid 111101110\rangle \; + \beta \mid 111011011\rangle + \beta \mid 111011111\rangle$

$+\beta \mid 101111000\rangle - \beta \mid 101111100\rangle \; + \beta \mid 101001001\rangle - \beta \mid 101001101\rangle$

$-\beta \mid 001101010\rangle + \beta \mid 001101110\rangle \; + \beta \mid 001011011\rangle - \beta \mid 001011111\rangle)$

∘ <u>L'application de la Porte $H\,(8)$</u>:

$\mid \; \Psi_1\rangle \; = \frac{1}{8\sqrt{2}}(+\alpha \mid 100001000\rangle + \alpha \mid 100001010\rangle \; - \alpha \mid 100111001\rangle - \alpha \mid 100111011\rangle$

$+\alpha \mid 000011000\rangle \; - \alpha \mid 000011010\rangle + \alpha \mid \; 000101001\rangle - \alpha \mid \; 000101011\rangle$

$+\alpha \mid 010001000\rangle + \alpha \mid 010001010\rangle \; + \alpha \mid 010111001\rangle + \alpha \mid 010111011\rangle$

$-\alpha \mid 110011000\rangle + \alpha \mid 110011010\rangle \; + \alpha \mid \; 110101001\rangle - \alpha \mid \; 110101011\rangle$

$+\alpha \mid 111001000\rangle + \alpha \mid 111001010\rangle \; + \alpha \mid 111111001\rangle + \alpha \mid 111111011\rangle$

$-\alpha \mid 011011000\rangle + \alpha \mid 011011010\rangle \; + \alpha \mid \; 011101001\rangle - \alpha \mid \; 011101011\rangle$

$-\alpha \mid 001001000\rangle - \alpha \mid 001001010\rangle \; + \alpha \mid 001111001\rangle + \alpha \mid 001111011\rangle$

$-\alpha \mid 101011000\rangle + \alpha \mid 101011010\rangle \; - \alpha \mid \; 101101001\rangle + \alpha \mid \; 101101011\rangle$

$-\alpha \mid 110011000\rangle - \alpha \mid 110011010\rangle \; + \alpha \mid 110101001\rangle + \alpha \mid 110101011\rangle$

$+\alpha \mid 010001000\rangle - \alpha \mid 010001010\rangle \; + \alpha \mid \; 010111001\rangle - \alpha \mid \; 010111011\rangle$

$+\alpha \mid 000011000\rangle + \alpha \mid 000011010\rangle \; + \alpha \mid 000101001\rangle + \alpha \mid 000101011\rangle$

$+\alpha \mid 100001000\rangle - \alpha \mid 100001010\rangle \; - \alpha \mid \; 100111001\rangle + \alpha \mid \; 100111011\rangle$

$-\alpha \mid 101011000\rangle - \alpha \mid 101011010\rangle \; - \alpha \mid 101101001\rangle - \alpha \mid 101101011\rangle$

$-\alpha \mid 001001000\rangle + \alpha \mid 001001010\rangle \; + \alpha \mid \; 001111001\rangle - \alpha \mid \; 001111011\rangle$

$-\alpha \mid 011011000\rangle - \alpha \mid 011011010\rangle \; + \alpha \mid 011101001\rangle + \alpha \mid 011101011\rangle$

$$+\alpha\mid 111001000\rangle - \alpha\mid 111001010\rangle \ + \alpha\mid \ 111111001\rangle - \alpha\mid \ 111111011\rangle$$
$$+\alpha\mid 100111000\rangle + \alpha\mid 100111010\rangle \ - \alpha\mid 100001001\rangle - \alpha\mid 100001011\rangle$$
$$-\alpha\mid 000101000\rangle + \alpha\mid 000101010\rangle \ - \alpha\mid \ 000011001\rangle + \alpha\mid \ 000011011\rangle$$
$$-\alpha\mid 010111000\rangle \ - \alpha\mid 010111010\rangle - \alpha\mid 010001001\rangle \ - \alpha\mid 010001011\rangle$$
$$-\alpha\mid 110101000\rangle \ + \alpha\mid 110101010\rangle + \alpha\mid \ 110011001\rangle \ - \alpha\mid \ 110011011\rangle$$
$$-\alpha\mid 111111000\rangle - \alpha\mid 111111010\rangle \quad -\alpha\mid 111001001\rangle - \alpha\mid 111001011\rangle$$
$$-\alpha\mid 011101000\rangle + \alpha\mid 011101010\rangle \ +\alpha\mid \ 011011001\rangle - \alpha\mid \ 011011011\rangle$$
$$-\alpha\mid 001111000\rangle \ - \alpha\mid 001111010\rangle \ + \alpha\mid 001001001\ \rangle \ + \alpha\mid 001001011\ \rangle$$
$$+\alpha\mid 101101000\ \rangle \ - \alpha\mid 101101010\ \rangle \ +\alpha\mid \ 101011001\rangle \ - \alpha\mid \ 101011011\rangle$$
$$-\alpha\mid 110101000\rangle \ - \alpha\mid 110101010\rangle \ + \ \alpha\mid 110011001\rangle \ \ + \alpha\mid 110011011\rangle$$
$$-\alpha\mid 010111000\rangle + \alpha\mid 010111010\rangle \ - \alpha\mid \ 010001001\rangle + \alpha\mid \ 010001011\rangle$$
$$-\alpha\mid 000101000\rangle \ - \alpha\mid 000101010\rangle \ - \alpha\mid 000011001\rangle \ - \alpha\mid 000011011\rangle$$
$$+\alpha\mid 100111000\rangle \ - \alpha\mid 100111010\rangle - \alpha\mid \ 100001001\rangle \ + \alpha\mid \ 100001011\rangle$$
$$+\alpha\mid 101101000\rangle + \alpha\mid 101101010\rangle + \alpha\mid 101011001\rangle + \alpha\mid 101011011\rangle$$
$$-\alpha\mid 001111000\rangle + \alpha\mid 001111010\rangle + \alpha\mid \ 001001001\rangle - \alpha\mid \ 001001011\rangle$$
$$-\alpha\mid 011101000\rangle - \alpha\mid 011101010\rangle + \alpha\mid 011011001\rangle \ + \alpha\mid 011011011\rangle$$
$$-\alpha\mid 111111000\rangle + \alpha\mid 111111010\rangle - \alpha\mid \ 111001001\rangle \ + \alpha\mid \ 111001011\rangle$$
$$-\beta\mid 110001000\rangle - \ \beta\mid 110001010\rangle \ + \beta\mid 110111001\rangle + \beta\mid 110111011\rangle$$
$$+\beta\mid 010011000\rangle - \beta\mid 010011010\rangle \ + \beta\mid 010101001\rangle - \beta\mid 010101011\rangle$$
$$-\beta\mid 000001000\rangle - \beta\mid 000001010\rangle \ - \beta\mid 000111001\rangle - \beta\mid 000111011\rangle$$
$$-\beta\mid 100011000\rangle + \beta\mid 100011010\rangle \ + \beta\mid 100101001\rangle - \beta\mid 100101011\rangle$$
$$-\beta\mid 101001000\rangle - \beta\mid 101001010\rangle \ - \beta\mid 101111001\rangle - \beta\mid 101111011\rangle$$
$$-\beta\mid 001011000\rangle + \beta\mid 001011010\rangle \ + \beta\mid 001101001\rangle - \beta\mid 001101011\rangle$$
$$+\beta\mid 011001000\rangle \ + \beta\mid 011001010\rangle \ - \beta\mid 011111001\rangle - \beta\mid 011111011\rangle$$
$$-\beta\mid 111011000\rangle \ + \beta\mid 111011010\rangle \ - \beta\mid 111101001\rangle + \beta\mid 111101011\rangle$$
$$-\beta\mid 100101000\rangle - \beta\mid 100101010\rangle \ + \beta\mid 100011001\rangle + \beta\mid 100011011\rangle$$
$$+\beta\mid 000111000\rangle - \beta\mid 000111010\rangle \ + \beta\mid 000001001\rangle - \beta\mid 000001011\rangle$$
$$-\beta\mid 010101000\rangle - \beta\mid 010101010\rangle \quad - \beta\mid 010011001\rangle - \beta\mid 010011011\rangle$$
$$-\beta\mid 110111000\rangle + \beta\mid 110111010\rangle \ + \beta\mid 110001001\rangle - \beta\mid 110001011\rangle$$
$$+\beta\mid 111101000\rangle - \beta\mid 111101010\rangle \ + \beta\mid 111011001\rangle + \beta\mid 111011011\rangle$$
$$+\beta\mid 011111000\rangle - \beta\mid 011111010\rangle \ - \beta\mid 011001001\rangle + \beta\mid 011001011\rangle$$
$$-\beta\mid 001101000\rangle - \beta\mid 001101010\rangle \ + \beta\mid 001011001\rangle + \beta\mid 001011011\rangle$$

$+\beta \mid 101111000\rangle - \beta \mid 101111010\rangle + \beta \mid 101001001\rangle - \beta \mid 101001011\rangle$
$-\beta \mid 100011000\rangle - \beta \mid 100011010\rangle + \beta \mid 100101001\rangle + \beta \mid 100101011\rangle$
$-\beta \mid 000001000\rangle + \beta \mid 000001010\rangle - \beta \mid 000111001\rangle + \beta \mid 000111011\rangle$
$+\beta \mid 010011000\rangle + \beta \mid 010011010\rangle + \beta \mid 010101001\rangle + \beta \mid 010101011\rangle$
$-\beta \mid 110001000\rangle + \beta \mid 110001010\rangle + \beta \mid 110111001\rangle - \beta \mid 110111011\rangle$
$-\beta \mid 111011000\rangle - \beta \mid 111011010\rangle - \beta \mid 111101001\rangle - \beta \mid 111101011\rangle$
$+\beta \mid 011001000\rangle - \beta \mid 011001010\rangle - \beta \mid 011111001\rangle + \beta \mid 011111011\rangle$
$-\beta \mid 001011000\rangle - \beta \mid 001011010\rangle + \beta \mid 001101001\rangle + \beta \mid 001101011\rangle$
$-\beta \mid 101001000\rangle + \beta \mid 101001010\rangle - \beta \mid 101111001\rangle + \beta \mid 101111011\rangle$
$-\beta \mid 110111000\rangle - \beta \mid 110111010\rangle + \beta \mid 110001001\rangle + \beta \mid 110001011\rangle$
$-\beta \mid 010101000\rangle + \beta \mid 010101010\rangle - \beta \mid 010011001\rangle + \beta \mid 010011011\rangle$
$+\beta \mid 000111000\rangle + \beta \mid 000111010\rangle + \beta \mid 000001001\rangle + \beta \mid 000001011\rangle$
$-\beta \mid 100101000\rangle + \beta \mid 100101010\rangle + \beta \mid 100011001\rangle - \beta \mid 100011011\rangle$
$+\beta \mid 101111000\rangle + \beta \mid 101111010\rangle + \beta \mid 101001001\rangle + \beta \mid 101001011\rangle$
$-\beta \mid 001101000\rangle + \beta \mid 001101010\rangle + \beta \mid 001011001\rangle - \beta \mid 001011011\rangle$
$+\beta \mid 011111000\rangle + \beta \mid 011111010\rangle - \beta \mid 011001001\rangle - \beta \mid 011001011\rangle$
$+\beta \mid 111101000\rangle - \beta \mid 111101010\rangle + \beta \mid 111011001\rangle - \beta \mid 111011011\rangle)$

∘ L'application de la Porte$H$ (9):

$\mid \Psi_1\rangle = \frac{1}{8}(+\alpha \mid 100001000\rangle + \alpha \mid 100001001\rangle - \alpha \mid 100111000\rangle + \alpha \mid 100111001\rangle$
$+\alpha \mid 000011000\rangle + \alpha \mid 000011001\rangle + \alpha \mid 000101000\rangle - \alpha \mid 000101001\rangle + \alpha \mid 010001000\rangle$
$+\alpha \mid 010001001\rangle + \alpha \mid 010111000\rangle - \alpha \mid 010111001\rangle - \alpha \mid 110011000\rangle - \alpha \mid 110011001\rangle$
$+\alpha \mid 110101000\rangle - \alpha \mid 110101001\rangle + \alpha \mid 111001000\rangle + \alpha \mid 111001001\rangle + \alpha \mid 111111000\rangle$
$-\alpha \mid 111111001\rangle - \alpha \mid 011011000\rangle - \alpha \mid 011011001\rangle + \alpha \mid 011101000\rangle - \alpha \mid 011101001\rangle$
$-\alpha \mid 001001000\rangle - \alpha \mid 001001001\rangle + \alpha \mid 001111000\rangle - \alpha \mid 001111001\rangle - \alpha \mid 101011000\rangle$
$-\alpha \mid 101011001\rangle - \alpha \mid 101101000\rangle + \alpha \mid 101101001\rangle + \alpha \mid 100111000\rangle + \alpha \mid 100111001\rangle$
$-\alpha \mid 100001000\rangle + \alpha \mid 100001001\rangle - \alpha \mid 000101000\rangle - \alpha \mid 000101001\rangle - \alpha \mid 000011000\rangle$
$+\alpha \mid 000011001\rangle - \alpha \mid 010111000\rangle - \alpha \mid 010111001\rangle - \alpha \mid 010001000\rangle + \alpha \mid 010001001\rangle$
$-\alpha \mid 110101000\rangle - \alpha \mid 110101001\rangle + \alpha \mid 110011000\rangle - \alpha \mid 110011001\rangle - \alpha \mid 111111000\rangle$
$-\alpha \mid 111111001\rangle - \alpha \mid 111001000\rangle + \alpha \mid 111001001\rangle - \alpha \mid 011101000\rangle - \alpha \mid 011101001\rangle$
$+\alpha \mid 011011000\rangle - \alpha \mid 011011001\rangle - \alpha \mid 001111000\rangle - \alpha \mid 001111001\rangle + \alpha \mid 001001000\rangle$
$-\alpha \mid 001001001\rangle + \alpha \mid 101101000\rangle + \alpha \mid 101101001\rangle + \alpha \mid 101011000\rangle - \alpha \mid 101011001\rangle$
$-\beta \mid 110001000\rangle - \beta \mid 110001001\rangle + \beta \mid 110111000\rangle - \beta \mid 110111001\rangle + \beta \mid 010011000\rangle$

$+\beta \mid 010011001\rangle \ + \beta \mid 010101000\rangle - \beta \mid 010101001\rangle - \beta \mid 000001000\rangle - \beta \mid 000001001\rangle$
$-\beta \mid 000111000\rangle + \beta \mid 000111001\rangle - \beta \mid 100011000\rangle - \beta \mid 100011001\rangle \ + \beta \mid 100101000\rangle$
$-\beta \mid 100101001\rangle - \beta \mid 101001000\rangle - \beta \mid 101001001\rangle \ - \beta \mid 101111000\rangle + \beta \mid 101111001\rangle$
$-\beta \mid 001011000\rangle - \beta \mid 001011001\rangle \ + \beta \mid 001101000\rangle - \beta \mid 001101001\rangle + \beta \mid 011001000\rangle$
$+\beta \mid 011001001\rangle \ - \beta \mid 011111000\rangle + \beta \mid 011111001\rangle - \beta \mid 111011000\rangle \ - \beta \mid 111011001\rangle$
$-\beta \mid 111101000\rangle + \beta \mid 111101001\rangle \ - \beta \mid 100101000\rangle - \beta \mid 100101001\rangle \ + \beta \mid 100011000\rangle$
$-\beta \mid 100011001\rangle + \beta \mid 000111000\rangle + \beta \mid 000111001\rangle \ + \beta \mid 000001000\rangle - \beta \mid 000001001\rangle$
$-\beta \mid 010101000\rangle - \beta \mid 010101001\rangle \ - \beta \mid 010011000\rangle + \beta \mid 010011001\rangle - \beta \mid 110111000\rangle$
$-\beta \mid 110111001\rangle \ + \beta \mid 110001000\rangle - \beta \mid 110001001\rangle + \beta \mid 111101000\rangle + \beta \mid 111101001\rangle$
$+\beta \mid 111011000\rangle - \beta \mid 111011001\rangle + \beta \mid 011111000\rangle + \beta \mid 011111001\rangle \ - \beta \mid 011001000\rangle$
$+\beta \mid 011001001\rangle - \beta \mid 001101000\rangle - \beta \mid 001101001\rangle \ + \beta \mid 001011000\rangle - \beta \mid 001011001\rangle$
$+\beta \mid 101111000\rangle + \beta \mid 101111001\rangle \ + \beta \mid 101001000\rangle - \beta \mid 101001001\rangle)$

Et après toutes les simplifications, on aura

$\mid \Psi_1\rangle \ =\frac{1}{4}(\alpha \mid 100001001\rangle \ + \alpha \mid 100111001\rangle + \alpha \mid 000011001\rangle \ - \alpha \mid \ 000101001\rangle$
$+\alpha \mid 010001101\rangle \ - \alpha \mid 010111001\rangle - \alpha \mid 110011001\rangle \ - \alpha \mid \ 110101001\rangle + \alpha \mid 111001001\rangle$
$-\alpha \mid 111111001\rangle - \alpha \mid 011011001\rangle \ - \alpha \mid \ 011101001\rangle - \alpha \mid 001001001\rangle \ - \alpha \mid 001111001\rangle$
$-\alpha \mid 101011001\rangle \ + \alpha \mid \ 101101001\rangle - \ \beta \mid 110001001\rangle \ - \beta \mid 110111001\rangle + \beta \mid 010011001\rangle$
$-\beta \mid 010101001\rangle - \beta \mid 000001001\rangle \ + \beta \mid 000111001\rangle - \beta \mid 100011001\rangle \ - \beta \mid 100101001\rangle$
$-\beta \mid 101001001\rangle \ + \beta \mid 101111001\rangle - \beta \mid 001011001\rangle \ - \beta \mid 001101001\rangle + \beta \mid 011001001\rangle$
$+\beta \mid 011111001\rangle - \beta \mid 111011001\rangle \ + \beta \mid 111101001\rangle)$

<u>Correction d'erreur</u> :

Résultat de mesure=1001, donc erreur de type X_1

La correction : appliquer la porte X sur le premier qubit

$\mid \Psi_1\rangle \ =\frac{1}{4}(\alpha \mid 000001001\rangle \ + \alpha \mid 000111001\rangle + \alpha \mid 100011001\rangle \ - \alpha \mid \ 100101001\rangle$
$+\alpha \mid 110001101\rangle \ - \alpha \mid 110111001\rangle - \alpha \mid 010011001\rangle \ - \alpha \mid \ 010101001\rangle + \alpha \mid 011001001\rangle$
$-\alpha \mid 011111001\rangle - \alpha \mid 111011001\rangle \ - \alpha \mid \ 111101001\rangle - \alpha \mid 101001001\rangle \ - \alpha \mid 101111001\rangle$
$-\alpha \mid 001011001\rangle \ + \alpha \mid \ 001101001\rangle - \ \beta \mid 010001001\rangle \ - \beta \mid 010111001\rangle + \beta \mid 110011001\rangle$
$-\beta \mid 110101001\rangle - \beta \mid 100001001\rangle \ + \beta \mid 100111001\rangle - \beta \mid 000011001\rangle \ - \beta \mid 000101001\rangle$
$-\beta \mid 001001001\rangle \ + \beta \mid 001111001\rangle - \beta \mid 101011001\rangle \ - \beta \mid 101101001\rangle + \beta \mid 111001001\rangle$
$+\beta \mid 111111001\rangle - \beta \mid 011011001\rangle \ + \beta \mid 011101001\rangle)$

◦ **<u>Suppression des quatre qubits du syndrome :</u>**

$\mid \Psi_1\rangle \ =\frac{1}{4}(\alpha \mid 00000 \ \rangle \ + \alpha \mid 00011 \ \rangle + \alpha \mid 10001 \ \rangle \ - \alpha \mid \ 10010 \ \rangle + \alpha \mid 11000 \ \rangle \ - \alpha \mid 11011 \ \rangle$

$-\alpha \mid 01001\rangle -\alpha \mid 01010\rangle +\alpha \mid 01100\rangle -\alpha \mid 01111\rangle -\alpha \mid 11101\rangle -\alpha \mid 11110\rangle -\alpha \mid 10100\rangle$
$-\alpha \mid 10111\rangle -\alpha \mid 00101\rangle +\alpha \mid 00110\rangle -\beta \mid 01000\rangle -\beta \mid 01011\rangle +\beta \mid 11001\rangle -\beta \mid 11010\rangle$
$-\beta \mid 10000\rangle +\beta \mid 10011\rangle -\beta \mid 00001\rangle -\beta \mid 00010\rangle -\beta \mid 00100\rangle +\beta \mid 00111\rangle -\beta \mid 10101\rangle$
$-\beta \mid 10110\rangle +\beta \mid 11100\rangle +\beta \mid 11111\rangle -\beta \mid 01101\rangle +\beta \mid 01110\rangle)$

∘ L'application de la porte $H\,(1)$:

$\mid \Psi_1\rangle = \frac{1}{4\sqrt{2}}(\alpha \mid 00000\rangle +\alpha \mid 10000\rangle +\alpha \mid 00011\rangle +\alpha \mid 10011\rangle +\alpha \mid 00001\rangle -\alpha \mid 10001\rangle$
$-\alpha \mid 00010\rangle +\alpha \mid 10010\rangle +\alpha \mid 01000\rangle -\alpha \mid 11000\rangle -\alpha \mid 01011\rangle +\alpha \mid 11011\rangle -\alpha \mid 01001\rangle$
$-\alpha \mid 11001\rangle -\alpha \mid 01010\rangle -\alpha \mid 11010\rangle +\alpha \mid 01100\rangle +\alpha \mid 11100\rangle -\alpha \mid 01111\rangle -\alpha \mid 11111\rangle$
$-\alpha \mid 01101\rangle +\alpha \mid 11101\rangle -\alpha \mid 01110\rangle +\alpha \mid 11110\rangle -\alpha \mid 00100\rangle +\alpha \mid 10100\rangle -\alpha \mid 00111\rangle$
$+\alpha \mid 10111\rangle -\alpha \mid 00101\rangle -\alpha \mid 10101\rangle +\alpha \mid 00110\rangle +\alpha \mid 10110\rangle -\beta \mid 01000\rangle -\beta \mid 11000\rangle$
$-\beta \mid 01011\rangle -\beta \mid 11011\rangle +\beta \mid 01001\rangle -\beta \mid 11001\rangle -\beta \mid 01010\rangle +\beta \mid 11010\rangle -\beta \mid 00000\rangle$
$+\beta \mid 10000\rangle +\beta \mid 00011\rangle -\beta \mid 10011\rangle -\beta \mid 00001\rangle -\beta \mid 10001\rangle -\beta \mid 00010\rangle -\beta \mid 10010\rangle$
$-\beta \mid 00100\rangle -\beta \mid 10100\rangle +\beta \mid 00111\rangle +\beta \mid 10111\rangle -\beta \mid 00101\rangle +\beta \mid 10101\rangle -\beta \mid 00110\rangle$
$+\beta \mid 10110\rangle +\beta \mid 01100\rangle -\beta \mid 11100\rangle +\beta \mid 01111\rangle -\beta \mid 11111\rangle -\beta \mid 01101\rangle -\beta \mid 11101\rangle$
$+\beta \mid 01110\rangle +\beta \mid 11110\rangle)$

∘ L'application de la porte $H\,(2)$:

$\mid \Psi_1\rangle = \frac{1}{8}((\alpha \mid 00000\rangle +\alpha \mid 01000\rangle +\alpha \mid 10000\rangle +\alpha \mid 11000\rangle +\alpha \mid 00011\rangle +\alpha \mid 01011\rangle$
$+\alpha \mid 10011\rangle +\alpha \mid 11011\rangle +\alpha \mid 00001\rangle +\alpha \mid 01001\rangle -\alpha \mid 10001\rangle -\alpha \mid 11001\rangle -\alpha \mid 00010\rangle$
$-\alpha \mid 01010\rangle +\alpha \mid 10010\rangle +\alpha \mid 11010\rangle +\alpha \mid 00000\rangle -\alpha \mid 01000\rangle -\alpha \mid 10000\rangle +\alpha \mid 11000\rangle$
$-\alpha \mid 00011\rangle +\alpha \mid 01011\rangle +\alpha \mid 10011\rangle -\alpha \mid 11011\rangle -\alpha \mid 00001\rangle +\alpha \mid 01001\rangle -\alpha \mid 10001\rangle$
$+\alpha \mid 11001\rangle -\alpha \mid 00010\rangle +\alpha \mid 01010\rangle -\alpha \mid 10010\rangle +\alpha \mid 11010\rangle +\alpha \mid 00100\rangle -\alpha \mid 01100\rangle$
$+\alpha \mid 10100\rangle -\alpha \mid 11100\rangle -\alpha \mid 00111\rangle +\alpha \mid 01111\rangle -\alpha \mid 10111\rangle +\alpha \mid 11111\rangle -\alpha \mid 00101\rangle$
$+\alpha \mid 01101\rangle +\alpha \mid 10101\rangle -\alpha \mid 11101\rangle -\alpha \mid 00110\rangle +\alpha \mid 01110\rangle +\alpha \mid 10110\rangle -\alpha \mid 11110\rangle$
$-\alpha \mid 00100\rangle -\alpha \mid 01100\rangle +\alpha \mid 10100\rangle +\alpha \mid 11100\rangle -\alpha \mid 00111\rangle -\alpha \mid 01111\rangle +\alpha \mid 10111\rangle$
$+\alpha \mid 11111\rangle -\alpha \mid 00101\rangle -\alpha \mid 01101\rangle -\alpha \mid 10101\rangle -\alpha \mid 11101\rangle +\alpha \mid 00110\rangle +\alpha \mid 01110\rangle$
$+\alpha \mid 10110\rangle +\alpha \mid 11110\rangle -\beta \mid 00000\rangle +\beta \mid 01000\rangle -\beta \mid 10000\rangle +\beta \mid 11000\rangle -\beta \mid 00011\rangle$
$+\beta \mid 01011\rangle -\beta \mid 10011\rangle +\beta \mid 11011\rangle +\beta \mid 00001\rangle -\beta \mid 01001\rangle -\beta \mid 10001\rangle +\beta \mid 11001\rangle$
$-\beta \mid 00010\rangle +\beta \mid 01010\rangle +\beta \mid 10010\rangle -\beta \mid 11010\rangle -\beta \mid 00000\rangle -\beta \mid 01000\rangle +\beta \mid 10000\rangle$
$+\beta \mid 11000\rangle +\beta \mid 00011\rangle +\beta \mid 01011\rangle -\beta \mid 10011\rangle -\beta \mid 11011\rangle -\beta \mid 00001\rangle -\beta \mid 01001\rangle$
$-\beta \mid 10001\rangle -\beta \mid 11001\rangle -\beta \mid 00010\rangle -\beta \mid 01010\rangle -\beta \mid 10010\rangle -\beta \mid 11010\rangle -\beta \mid 00100\rangle$
$-\beta \mid 01100\rangle -\beta \mid 10100\rangle -\beta \mid 11100\rangle +\beta \mid 00111\rangle +\beta \mid 01111\rangle +\beta \mid 10111\rangle +\beta \mid 11111\rangle$
$-\beta \mid 00101\rangle -\beta \mid 01101\rangle +\beta \mid 10101\rangle +\beta \mid 11101\rangle -\beta \mid 00110\rangle -\beta \mid 01110\rangle +\beta \mid 10110\rangle$

$+\beta \mid 11110 \rangle + \beta \mid 00100 \rangle - \beta \mid 01100 \rangle - \beta \mid 10100 \rangle + \beta \mid 11100 \rangle + \beta \mid 00111\rangle - \beta \mid 01111\rangle$
$-\beta \mid 10111\rangle + \beta \mid 11111\rangle - \beta \mid 00101 \rangle + \beta \mid 01101 \rangle - \beta \mid 10101 \rangle + \beta \mid 11101 \rangle + \beta \mid 00110 \rangle$
$-\beta \mid 01110 \rangle + \beta \mid 10110 \rangle - \beta \mid 11110 \rangle)$

Apres toutes les simplifications, on aura :

$\mid \Psi_1\rangle = \frac{1}{4}(\alpha \mid 00000\rangle + \alpha \mid 11000\rangle + \alpha \mid 01011\rangle + \alpha \mid 10011\rangle + \alpha \mid 01001\rangle - \alpha \mid 10001\rangle$
$-\alpha \mid 00010\rangle + \alpha \mid 11010\rangle - \alpha \mid 01100\rangle + \alpha \mid 10100\rangle - \alpha \mid 00111\rangle + \alpha \mid 11111\rangle - \alpha \mid 00101\rangle$
$-\alpha \mid 11101\rangle + \alpha \mid 0110\rangle + \alpha \mid 10110\rangle - \beta \mid 00000\rangle + \beta \mid 11000\rangle + \beta \mid 01011\rangle - \beta \mid 10011\rangle$
$-\beta \mid 01001\rangle - \beta \mid 10001\rangle - \beta \mid 00010\rangle - \beta \mid 11010\rangle - \beta \mid 01100\rangle - \beta \mid 10100\rangle + \beta \mid 00111\rangle$
$+\beta \mid 11111\rangle - \beta \mid 00101\rangle + \beta \mid 11101\rangle - \beta \mid 01110\rangle + \beta \mid 10110\rangle)$

○ L'application de la porte $CNOT\,(3,5)$:

$\mid \Psi_1\rangle = \frac{1}{4}(\alpha \mid 00000\rangle + \alpha \mid 11000\rangle + \alpha \mid 01011\rangle + \alpha \mid 10011\rangle + \alpha \mid 01001\rangle - \alpha \mid 10001\rangle$
$-\alpha \mid 00010\rangle + \alpha \mid 11010\rangle - \alpha \mid 01101\rangle + \alpha \mid 10101\rangle - \alpha \mid 00110\rangle + \alpha \mid 11110\rangle - \alpha \mid 00100\rangle$
$-\alpha \mid 11100\rangle + \alpha \mid 01110\rangle + \alpha \mid 10111\rangle - \beta \mid 00000\rangle + \beta \mid 11000\rangle + \beta \mid 01011\rangle - \beta \mid 10011\rangle$
$-\beta \mid 01001\rangle - \beta \mid 10001\rangle - \beta \mid 00010\rangle - \beta \mid 11010\rangle - \beta \mid 01101\rangle - \beta \mid 10101\rangle + \beta \mid 00110\rangle$
$+\beta \mid 11110\rangle - \beta \mid 00100\rangle + \beta \mid 11100\rangle - \beta \mid 01111\rangle + \beta \mid 10111\rangle)$

○ L'application de la porte $CNOT\,(2,5)$:

$\mid \Psi_1\rangle = \frac{1}{4}(\alpha \mid 00000\rangle + \alpha \mid 11001\rangle + \alpha \mid 01010\rangle + \alpha \mid 10011\rangle + \alpha \mid 01000\rangle - \alpha \mid 10001\rangle$
$-\alpha \mid 00010\rangle + \alpha \mid 11011\rangle - \alpha \mid 01100\rangle + \alpha \mid 10101\rangle - \alpha \mid 00110\rangle + \alpha \mid 11111\rangle - \alpha \mid 00100\rangle$
$-\alpha \mid 11101\rangle + \alpha \mid 01111\rangle + \alpha \mid 10111\rangle - \beta \mid 00000\rangle + \beta \mid 11001\rangle + \beta \mid 01010\rangle - \beta \mid 10011\rangle$
$-\beta \mid 01000\rangle - \beta \mid 10001\rangle - \beta \mid 00010\rangle - \beta \mid 11011\rangle - \beta \mid 01100\rangle - \beta \mid 10101\rangle + \beta \mid 00110\rangle$
$+\beta \mid 11111\rangle - \beta \mid 00100\rangle + \beta \mid 11101\rangle - \beta \mid 01110\rangle + \beta \mid 10111\rangle)$

○ L'application de la porte $CNOT\,(1,5)$:

$\mid \Psi_1\rangle = \frac{1}{4}(\alpha \mid 00000\rangle + \alpha \mid 11000\rangle + \alpha \mid 01010\rangle + \alpha \mid 10010\rangle + \alpha \mid 01000\rangle - \alpha \mid 10000\rangle$
$-\alpha \mid 00010\rangle + \alpha \mid 11010\rangle - \alpha \mid 01100\rangle + \alpha \mid 10100\rangle - \alpha \mid 00110\rangle + \alpha \mid 11110\rangle - \alpha \mid 00100\rangle$
$-\alpha \mid 11100\rangle + \alpha \mid 01111\rangle + \alpha \mid 10110\rangle - \beta \mid 00000\rangle + \beta \mid 11000\rangle + \beta \mid 01010\rangle - \beta \mid 10010\rangle$
$-\beta \mid 01000\rangle - \beta \mid 10000\rangle - \beta \mid 00010\rangle - \beta \mid 11010\rangle - \beta \mid 01100\rangle - \beta \mid 10100\rangle + \beta \mid 00110\rangle$
$+\beta \mid 11110\rangle - \beta \mid 00100\rangle + \beta \mid 11100\rangle - \beta \mid 01110\rangle + \beta \mid 10110\rangle)$

○ L'application de la porte $H(1)$:

$\mid \Psi_1\rangle = \frac{1}{4\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 10000\rangle + \alpha \mid 01000\rangle - \alpha \mid 11000\rangle + \alpha \mid 01010\rangle + \alpha \mid 11010\rangle$
$+\alpha \mid 00010\rangle - \alpha \mid 10010\rangle + \alpha \mid 01000\rangle + \alpha \mid 11000\rangle - \alpha \mid 00000\rangle + \alpha \mid 10000\rangle - \alpha \mid 00010\rangle$
$-\alpha \mid 10010\rangle + \alpha \mid 01010\rangle - \alpha \mid 11010\rangle - \alpha \mid 01100\rangle - \alpha \mid 11100\rangle + \alpha \mid 00100\rangle - \alpha \mid 10100\rangle$
$-\alpha \mid 00110\rangle - \alpha \mid 10110\rangle + \alpha \mid 01110\rangle - \alpha \mid 11110\rangle - \alpha \mid 00100\rangle - \alpha \mid 10100\rangle - \alpha \mid 01100\rangle$

$+\alpha \mid 11100\rangle + \alpha \mid 01111\rangle + \alpha \mid 11111\rangle + \alpha \mid 00110\rangle - \alpha \mid 10110\rangle$-$\beta \mid 00000\rangle - \beta \mid 10000\rangle$
$+\beta \mid 01000\rangle - \beta \mid 11000\rangle + \beta \mid 01010\rangle + \beta \mid 11010\rangle - \beta \mid 00010\rangle + \beta \mid 10010\rangle$-$\beta \mid 01000\rangle$
$-\beta \mid 11000\rangle - \beta \mid 00000\rangle + \beta \mid 10000\rangle - \beta \mid 00010\rangle - \beta \mid 10010\rangle - \beta \mid 01010\rangle + \beta \mid 11010\rangle$
-$\beta \mid 01100\rangle - \beta \mid 11100\rangle - \beta \mid 00100\rangle + \beta \mid 10100\rangle + \beta \mid 00110\rangle + \beta \mid 10110\rangle + \beta \mid 01110\rangle$
$-\beta \mid 11110\rangle$-$\beta \mid 00100\rangle - \beta \mid 10100\rangle + \beta \mid 01100\rangle - \beta \mid 11100\rangle - \beta \mid 01110\rangle - \beta \mid 11110\rangle$
$+\beta \mid 00110\rangle - \beta \mid 10110\rangle)$

Aprés toutes les simplifications, on aura :

$$\mid \Psi_1\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 10000\rangle + \alpha \mid 01000\rangle + \alpha \mid 01010\rangle - \alpha \mid 10010\rangle - \alpha \mid 01100\rangle - \alpha \mid 10100\rangle$$
$$-\alpha \mid 10110\rangle + \alpha \mid 01110\rangle - \beta \mid 00000\rangle - \beta \mid 11000\rangle + \beta \mid 11010\rangle - \beta \mid 00010\rangle - \beta \mid 11100\rangle$$
$$-\beta \mid 00100\rangle + \beta \mid 00110\rangle - \beta \mid 11110\rangle$$

○ L'application de la porte H(4):

$\mid \Psi_1\rangle = \frac{1}{4}(\alpha \mid 10000\rangle + \alpha \mid 10010\rangle + \alpha \mid 01000\rangle + \alpha \mid 01010\rangle + \alpha \mid 01000\rangle - \alpha \mid 01010\rangle$
$-\alpha \mid 10000\rangle + \alpha \mid 10010\rangle$-$\alpha \mid 01100\rangle - \alpha \mid 01110\rangle - \alpha \mid 10100\rangle - \alpha \mid 10110\rangle - \alpha \mid 10100\rangle$
$+\alpha \mid 10110\rangle + \alpha \mid 01100\rangle - \alpha \mid 01110\rangle$-$\beta \mid 00000\rangle - \beta \mid 00010\rangle - \beta \mid 11000\rangle - \beta \mid 11010\rangle$
$+\beta \mid 11000\rangle - \beta \mid 11010\rangle - \beta \mid 00000\rangle + \beta \mid 00010\rangle$-$\beta \mid 11100\rangle - \beta \mid 11110\rangle - \beta \mid 00100\rangle$
$-\beta \mid 00110\rangle + \beta \mid 00100\rangle - \beta \mid 00110\rangle - \beta \mid 11100\rangle + \beta \mid 11110\rangle$

Aprés la simplification :

$$\mid \Psi_1\rangle = \frac{1}{2}(\alpha \mid 10010\rangle + \alpha \mid 01000\rangle - \alpha \mid 01110\rangle - \alpha \mid 10100\rangle - \beta \mid 00000\rangle - \beta \mid 11010\rangle$$
$$-\beta \mid 11100\rangle - \beta \mid 00110\rangle)$$

○ L'application de la porte$CNOT\,(3,4)$:

$$\mid \Psi_1\rangle = \frac{1}{2}(\alpha \mid 10010\rangle + \alpha \mid 01000\rangle - \alpha \mid 01100\rangle - \alpha \mid 10110\rangle - \beta \mid 00000\rangle - \beta \mid 11010\rangle$$
$$-\beta \mid 11110\rangle - \beta \mid 00100\rangle)$$

○L'application de la porte $CNOT\,(1,4)$:

$$|\Psi_1\rangle = \frac{1}{2}(\alpha \mid 10000\rangle + \alpha \mid 01000\rangle - \alpha \mid 01100\rangle - \alpha \mid 10100\rangle - \beta \mid 00000\rangle - \beta \mid 11000\rangle$$
$$-\beta \mid 11100\rangle - \beta \mid 00100\rangle)$$

○<u>L'application de la porte $H\,(3)$</u>:

$$|\Psi_1\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 10000\rangle + \alpha \mid 10100\rangle + \alpha \mid 01000\rangle + \alpha \mid 01100\rangle - \alpha \mid 01000\rangle + \alpha \mid 01100\rangle$$
$$-\alpha \mid 10000\rangle + \alpha \mid 10100\rangle - \beta \mid 00000\rangle - \beta \mid 00100\rangle - \beta \mid 11000\rangle - \beta \mid 11100\rangle - \beta \mid 11000\rangle$$
$$+\beta \mid 11100\rangle - \beta \mid 00000\rangle + \beta \mid 00100\rangle)$$

Aprés la simplification, on aura :

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(\alpha \mid 10100\rangle + \alpha \mid 01100\rangle - \beta \mid 00000\rangle - \beta \mid 11000\rangle)$$

○ <u>L'application de la porte $CNOT\,(2,3)$</u>:

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(\alpha \mid 10100\rangle + \alpha \mid 01000\rangle - \beta \mid 00000\rangle - \beta \mid 11100\rangle)$$

○ <u>L'application de la porte $CNOT\,(1,3)$</u>:

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(\alpha \mid 10000\rangle + \alpha \mid 01000\rangle - \beta \mid 00000\rangle - \beta \mid 11000\rangle)$$

○ <u>L'application de la porte $H\,(1)$</u>:

$$|\Psi_1\rangle = \frac{1}{2}(\alpha \mid 00000\rangle - \alpha \mid 10000\rangle + \alpha \mid 01000\rangle + \alpha \mid 11000\rangle$$
$$-\beta \mid 00000\rangle - \beta \mid 10000\rangle - \beta \mid 01000\rangle + \beta \mid 11000\rangle)$$

○ <u>L'application de la porte $H\,(2)$</u>:

$$\mid \Psi_1\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 01000\rangle - \alpha \mid 10000\rangle - \alpha \mid 11000\rangle + \alpha \mid 00000\rangle - \alpha \mid 01000\rangle$$
$$+ \alpha \mid 10000\rangle - \alpha \mid 11000\rangle - \beta \mid 00000\rangle - \beta \mid 01000\rangle - \beta \mid 10000\rangle - \beta \mid 11000\rangle - \beta \mid 00000\rangle$$
$$+\beta \mid 01000\rangle + \beta \mid 10000\rangle - \beta \mid 11000\rangle)$$

Aprés la simplification, on aura :

$$\mid \Psi_1\rangle = \frac{1}{\sqrt{2}}(\alpha \mid 00000\rangle - \alpha \mid 11000\rangle - \beta \mid 00000\rangle - \beta \mid 11000\rangle)$$

◦ L'application de la porte $CNOT\,(1,2)$:

$$\mid \Psi_1\rangle = \frac{1}{\sqrt{2}}(\alpha \mid 00000\rangle - \alpha \mid 10000\rangle - \beta \mid 00000\rangle - \beta \mid 10000\rangle)$$

◦ L'application de la porte $Z\,(1)$:

$$\mid \Psi_1\rangle = \frac{1}{\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 10000\rangle - \beta \mid 00000\rangle + \beta \mid 10000\rangle)$$

◦ L'application de la porte $H\,(1)$:

$$\mid \Psi_1\rangle = \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 10000\rangle + \alpha \mid 00000\rangle - \alpha \mid 10000\rangle$$
$$-\beta \mid 00000\rangle - \beta \mid 10000\rangle + \beta \mid 00000\rangle - \beta \mid 10000\rangle$$

Apres la simplification, on aura :

$$\mid \Psi_1\rangle = (\alpha \mid 00000\rangle - \beta \mid 10000\rangle)$$

◦ L'application de la porte $Z\,(1)$:

$$\mid \Psi_1\rangle = (\alpha \mid 00000\rangle + \beta \mid 10000\rangle) = (\alpha \mid 0\rangle + \beta \mid 1\rangle) \otimes \mid 0000\rangle$$
$$= \mid \Psi\rangle \otimes \mid 0000\rangle$$

Remarques :

*Le code à cinq qubits est le code quantique permettant la correction d'erreurs le plus optimal. C'est le code le plus court et donc il est d'un intérêt immense .

*Il n'y a pas de $M_4 = Z_0X_1X_2Z_3$ car $M_4 = M_0M_1M_2M_3$

*$M_i^2 = \mathbb{I}$ pour tous car : $X_k^2 = Z_k^2 = \mathbb{I}$.

*$M_iM_j = M_jM_i$ donc tout couple M_i, M_j commute.

* $\mid 0\rangle_L$ et$\mid 1\rangle_L$ sont des vecteurs propres de tous les M_i de valeur propre ± 1 ; ceci est dû au fait que tous les M_i commutent et

$$M_i(1 + M_i) = (1 + M_i).$$

*Il est possible de vérifier qu'en appliquant une erreur$X_k(Y_kouZ_k)$, les $X_k \mid \Psi\rangle$, $Y_k \mid \Psi\rangle$ et $Z_k \mid \Psi\rangle$ sont également des vecteurs propres de tous les M_i, mais avec des ensembles différents d'états propres .

2.5 Conclusion :

Dans ce chapitre, on a présenté quelques algorithmes de correction d'erreurs basés sur les codes stabilisateurs. Le plus intéressant est celui à 5 qubits. Il ne nécessite que 4 qubits supplémentaires et permet la détection et la correction des erreurs de type X, Y et Z. L'idée de base est de trouver un encodage tel que :

•On peut définir des opérateurs de syndrome qui ont la particularité d'avoir comme vecteurs propres les états erronés ainsi que l´état sans erreur.

•La mesure de ces opérateurs ne modifie pas l'état traité.

•Le résultat de la mesure (les valeurs propres) signe sans ambiguïté la position de l'erreur.

Une fois l'erreur détectée, il suffit d'apporter la correction.

Chapitre 3

Nouvelle Solution proposée

La théorie des codes correcteurs s'intéresse à la protection de l'information. Cette tâche est réalisée généralement en s'appuyant sur un encodage préalable bien choisi basé sur la redondance.

Dans ce chapitre nous allons étudier un code stabilisateur qui contient 5 qubits mais ne nécessite que dix portes quantiques.

Encodage :

Le circuit d'encodage est le suivant :

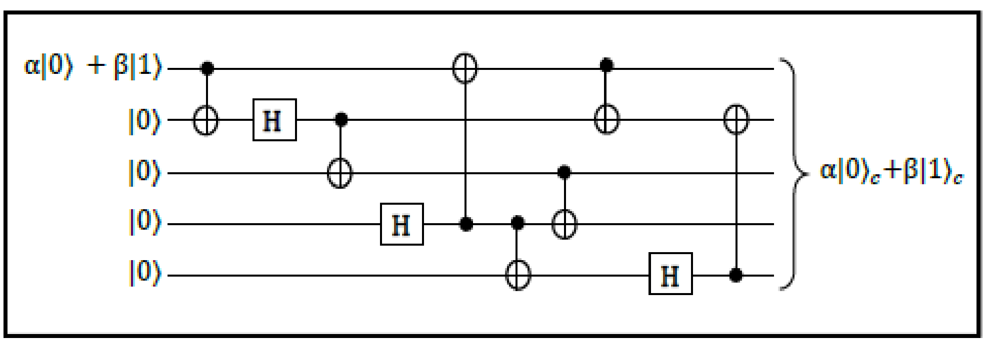

Fig.3.1 : Le circuit d'encodage.

Groupe stabilisateur :

Il existe 5 groupes stabilisareurs, mais dans ce calcul on utilise un seul groupe stabilisateur donné dans le tableau suivant :

	5	4	3	2	1
M_0	I	Z	X	Z	X
M_1	Z	I	Z	Z	Z
M_2	X	Z	Z	X	I
M_3	X	X	X	I	Z

Fig.3.2 :Les stabilisateurs.

Calcul des syndromes :

Le syndrome associé à chaque type d'erreur est donné par le circuit suivant :

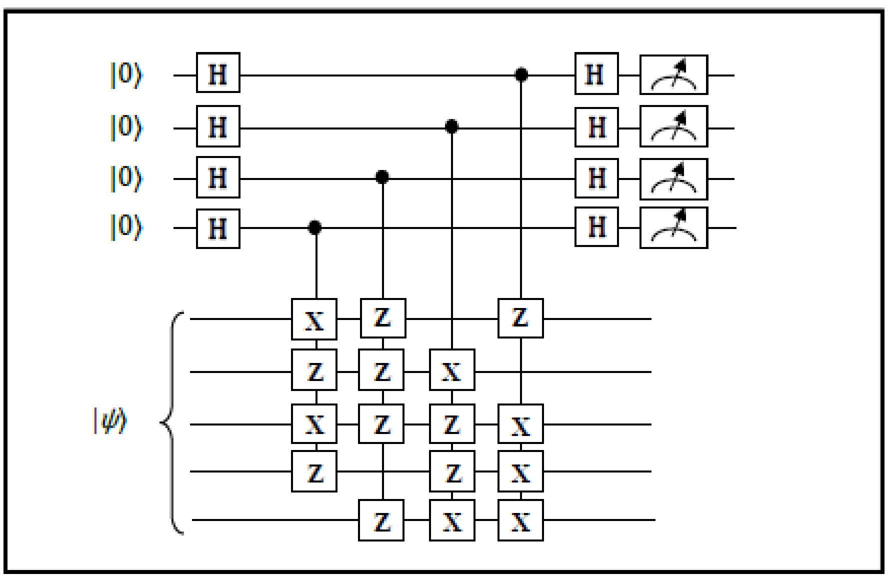

Fig.3.3 : Calcul des syndromes d'erreur.

Les syndromes d'erreurs sont résumés dans le tableau qui suit :

	$X_0Y_0Z_0$	$X_1Y_1Z_1$	$X_2Y_2Z_2$	$X_3Y_3Z_3$	$X_4Y_4Z_4$	1
$M_0 = X_0Z_1X_2Z_3$	$+--$	$--+$	$+--$	$--+$	$+++$	$+$
$M_1 = Z_0Z_1Z_2Z_4$	$--+$	$--+$	$--+$	$+++$	$--+$	$+$
$M_2 = X_1Z_2Z_3X_4$	$+++$	$+--$	$--+$	$--+$	$+--$	$+$
$M_3 = Z_0X_2X_3X_4$	$--+$	$+++$	$+--$	$+--$	$+--$	$+$

Fig.3.4 :Syndromes d'erreurs.

Décodage :

Le circuit de décodage est le suivant :

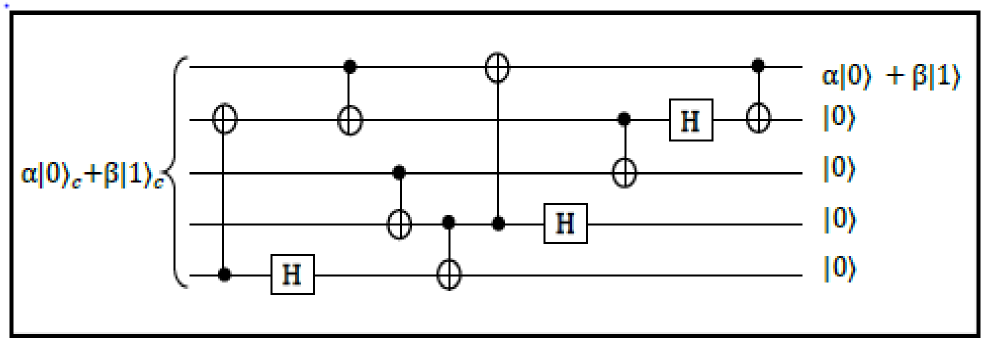

Fig.3.5 : Le circuit de dcodage.

3.1 Vérification formelle

3.1.1 L'encodage

On considère le qubit $\mid \Psi\rangle$ défini par :

$$\mid \Psi\rangle \quad = \alpha \mid 0\rangle + \beta \mid 1\rangle$$

En premier lieu, on fait une étape d'encodage. Elle consiste d'abord à l'ajout de quatre qubits supplémentaires. Elle est réalisée en appliquant sept portes CNOT et trois portes H.

∘Létat initial :

$$\mid \Psi_1\rangle = \alpha \mid 00000\rangle + \beta \mid 10000\rangle$$

○ <u>L'application du $CNOT\,(1,2)$</u>:

$$| \Psi_2\rangle = \alpha \mid 00000\rangle + \beta \mid 11000\rangle$$

○ <u>L'application du $H\,(2)$</u>:

$$| \Psi_3\rangle = \frac{1}{\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 0100\rangle + \beta \mid 10000\rangle - \beta \mid 11000\rangle)$$

○ <u>L'application du $CNOT\,(2,3)$</u>:

$$| \Psi_4\rangle = \frac{1}{\sqrt{2}}\,(\alpha \mid 00000\rangle + \alpha \mid 01100\rangle + \beta \mid 10000\rangle - \beta \mid 11100\rangle)$$

○ <u>L'application du $H\,(4)$</u>:

$$\begin{aligned} | \quad \Psi_5\rangle &= \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 00010\rangle + \alpha \mid 01100\rangle + \alpha \mid 01110\rangle \\ +\beta \quad | \quad & 10000\rangle + \beta \mid 10010\rangle - \beta \mid 11100\rangle - \beta \mid 11110\rangle \end{aligned}$$

○ <u>L'application du $CNOT\,(4,1)$</u>:

$$\begin{aligned} | \quad \Psi_6\rangle &= \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 10010\rangle + \alpha \mid 01100\rangle + \alpha \mid 11110\rangle \\ +\beta \quad | \quad & 10000\rangle + \beta \mid 00010\rangle - \beta \mid 11100\rangle - \beta \mid 01110\rangle) \end{aligned}$$

○ <u>L'application du$CNOT\,(4,5)$</u> :

$$\begin{aligned} | \quad \Psi_7\rangle &= \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 10011\rangle + \alpha \mid 01100\rangle + \alpha \mid 11111\rangle \\ +\beta \quad | \quad & 10000\rangle + \beta \mid 00011\rangle - \beta \mid 11100\rangle - \beta \mid 01101\rangle) \end{aligned}$$

○ <u>L'application du $CNOT\,(3,4)$</u> :

$$\begin{aligned} | \quad \Psi_8\rangle &= \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 10011\rangle + \alpha \mid 01110\rangle + \alpha \mid 11101\rangle \\ +\beta \quad | \quad & 10000\rangle + \beta \mid 00011\rangle - \beta \mid 11110\rangle - \beta \mid 01101\rangle) \end{aligned}$$

○ <u>L'application du $CNOT\,(1,2)$</u> :

$$\begin{aligned} | \quad \Psi_9\rangle &= \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 11011\rangle + \alpha \mid 01110\rangle + \alpha \mid 10101\rangle \\ +\beta \quad | \quad & 11000\rangle + \beta \mid 00011\rangle - \beta \mid 10110\rangle - \beta \mid 01101\rangle) \end{aligned}$$

∘ L'application du $H\,(5)$:

$$\begin{aligned} &\mid \Psi_{10}\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 00001\rangle + \alpha \mid 11010\rangle - \alpha \mid 11011\rangle + \alpha \mid 01110\rangle + \alpha \mid 01111\rangle \\ +\alpha &\mid 10100\rangle - \alpha \mid 10101\rangle + \beta \mid 11000\rangle + \beta \mid 11001\rangle + \beta \mid 00011\rangle - \beta \mid 10110\rangle + \beta \mid 11000\rangle \\ +\beta &\mid 11001\rangle + \beta \mid 00011\rangle - \beta \mid 10110\rangle + \beta \mid 11000\rangle + \beta \mid 11001\rangle + \beta \mid 00011\rangle - \beta \mid 10110\rangle \\ -\beta &\mid 10110\rangle - \beta \mid 10111\rangle - \beta \mid 01100\,\rangle + \beta \mid 01101\,\rangle) \end{aligned}$$

∘ L'application du$CNOT\,(5,2)$:

$$\begin{aligned} &\mid \Psi_{11}\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 01001\rangle + \alpha \mid 11010\rangle - \alpha \mid 10011\rangle + \alpha \mid 01110\rangle + \alpha \mid 00111\rangle \\ +\alpha &\mid 10100\rangle - \alpha \mid 11101\rangle + \beta \mid 11000\rangle + \beta \mid 10001\rangle + \beta \mid 00010\rangle - \beta \mid 01011\rangle - \beta \mid 10110\rangle \\ -\beta &\mid 11111\rangle - \beta \mid 01100\rangle + \beta \mid 00101\rangle) \end{aligned}$$

Donc :

$$\begin{aligned} &\mid \Psi_{11}\rangle = \alpha(\frac{1}{2\sqrt{2}} \mid 00000\rangle + \frac{1}{2\sqrt{2}} \mid 01001\rangle + \frac{1}{2\sqrt{2}} \mid 11010\rangle - \frac{1}{2\sqrt{2}} \mid 10011\rangle + \frac{1}{2\sqrt{2}} \mid 01110\rangle \\ +\frac{1}{2\sqrt{2}} &\mid 00111\rangle + \frac{1}{2\sqrt{2}} \mid 10100\rangle - \frac{1}{2\sqrt{2}} \mid 11101\rangle) + \beta(\frac{1}{2\sqrt{2}} \mid 11000\rangle + \frac{1}{2\sqrt{2}} \mid 10001\rangle + \frac{1}{2\sqrt{2}} \mid 00010\rangle \\ -\frac{1}{2\sqrt{2}} &\mid 01011\rangle - \frac{1}{2\sqrt{2}} \mid 10110\rangle - \frac{1}{2\sqrt{2}} \mid 11111\rangle - \frac{1}{2\sqrt{2}} \mid 01100\rangle + \frac{1}{2\sqrt{2}} \mid 00101\rangle) \end{aligned}$$

Et on écrit aussi

$$\mid \Psi_{11}\rangle = \alpha \mid 0\rangle_c + \beta \mid 1\rangle_c$$

Avec

$$\mid 0\rangle_c = \frac{1}{2\sqrt{2}}(00000\rangle + \mid 01001\rangle + \mid 11010\rangle - \mid 10011\rangle \mid 01110\rangle + \mid 00111\rangle + \mid 10100\rangle - \mid 11101\rangle)$$

$$\mid 1\rangle_c = \frac{1}{2\sqrt{2}}(\mid 11000\rangle + \mid 10001\rangle + \mid 00010\rangle - \mid 01011\rangle - \mid 10110\rangle - \mid 11111\rangle - \mid 01100\rangle + \mid 00101\rangle)$$

3.1.2 Simulation d'une errreur

$*$**Cas 01 : sans erreur**

$| \quad \Psi_1\rangle = \dfrac{1}{2\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 01001\rangle + \alpha \mid 11010\rangle - \alpha \mid 10011\rangle \ + \alpha \mid 01110\rangle + \alpha \mid 00111\rangle$
$+\alpha \quad | \quad 10100\rangle - \alpha \mid 11101\rangle + \ \beta \mid 11000\rangle + \ \beta \mid 10001\rangle + \ \beta \mid 00010\rangle - \ \beta \mid 01011\rangle - \ \beta \mid 10110\rangle$
$- \beta \quad | \quad 11111\rangle \ - \beta \mid 01100\rangle \ + \beta \mid 00101\rangle)$

3.1.3 Mesure du syndrome

L'ajout des quatre qubits du syndrome :

$| \Psi_1\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 000000000\rangle + \alpha \mid 010010000\rangle + \alpha \mid 110100000\rangle - \alpha \mid 100110000\rangle$
$+\alpha \mid 011100000\rangle + \alpha \mid 001110000\rangle + \alpha \mid 101000000\rangle - \alpha \mid 111010000\rangle$
$+ \beta \mid 110000000\rangle + \ \beta \mid 100010000\rangle + \ \beta \mid 000100000\rangle - \ \beta \mid 010110000\rangle$
$- \beta \mid 101100000\rangle - \ \beta \mid 111110000\rangle \ - \beta \mid 011000000\rangle \ + \beta \mid 001010000\rangle)$

· L'application de la porte $H(6)$:

$| \Psi_1\rangle = \frac{1}{4}(\alpha \mid 000000000\rangle + \alpha \mid 000001000\rangle + \alpha \mid 010010000\rangle + \alpha \mid 010011000\rangle$
$+\alpha \mid 110100000\rangle + \alpha \mid 110101000\rangle - \alpha \mid 100110000\rangle - \alpha \mid 100111000\rangle$
$+\alpha \mid 011100000\rangle + \alpha \mid 011101000\rangle + \alpha \mid 001110000\rangle + \alpha \mid 001111000\rangle$
$+\alpha \mid 101000000\rangle + \alpha \mid 101001000\rangle - \alpha \mid 111010000\rangle - \alpha \mid 111011000\rangle$
$+\beta \mid 110000000\rangle + \beta \mid 110001000\rangle + \beta \mid 100010000\rangle + \beta \mid 100011000\rangle$
$+\beta \mid 000100000\rangle + \beta \mid 000101000\rangle - \beta \mid 010110000\rangle - \beta \mid 010111000\rangle$
$-\beta \mid 101100000\rangle - \beta \mid 101101000\rangle - \beta \mid 111110000\rangle - \beta \mid 111111000\rangle$
$-\beta \mid 011000000\rangle - \beta \mid 011001000\rangle + \beta \mid 001010000\rangle + \beta \mid 001011000\rangle)$

· L'application de la porte $H(7)$:

$| \Psi_1\rangle = \frac{1}{4\sqrt{2}}(\alpha \mid 000000000\rangle + \alpha \mid 000000100\rangle + \alpha \mid 000001000\rangle + \alpha \mid 000001100\rangle$
$+ \alpha \mid 010010000 \rangle + \alpha \mid 010010100 \rangle + \alpha \mid 010011000 \rangle + \alpha \mid 010011100 \rangle$
$+\alpha \mid 110100000 \rangle + \alpha \mid 110100100 \rangle + \alpha \mid 110101000 \rangle + \alpha \mid 110101100 \rangle$
$-\alpha \mid 100110000 \rangle - \alpha \mid 100110100 \rangle - \alpha \mid 100111000 \rangle - \alpha \mid 100111100 \rangle$
$+\alpha \mid 011100000 \rangle + \alpha \mid 011100100 \rangle + \alpha \mid 011101000 \rangle + \alpha \mid 011101000 \rangle$
$+\alpha \mid 001110000 \rangle + \alpha \mid 001110100 \rangle + \alpha \mid 001111000 \rangle + \alpha \mid 001111100 \rangle$
$+\alpha \mid 101000000 \rangle + \alpha \mid 101000100 \rangle + \alpha \mid 101001000 \rangle + \alpha \mid 101001100 \rangle$
$-\alpha \mid 111010000 \rangle - \alpha \mid 111010100 \rangle - \alpha \mid 111011000 \rangle - \alpha \mid 111011100\rangle$

$+ \beta \mid 110000000 \rangle + \beta \mid 110000100 \rangle + \beta \mid 110001000 \rangle + \beta \mid 110001100 \rangle$

$+\beta \mid 100010000 \rangle + \beta \mid 100010100 \rangle + \beta \mid 100011000 \rangle + \beta \mid 100011100 \rangle$

$+\beta \mid 000100000 \rangle + \beta \mid 000100100 \rangle + \beta \mid 000101000 \rangle + \beta \mid 000101100 \rangle$

$-\beta \mid 010110000 \rangle - \beta \mid 010110100 \rangle - \beta \mid 010111000 \rangle - \beta \mid 010111100 \rangle$

$-\beta \mid 101100000 \rangle - \beta \mid 101100100 \rangle - \beta \mid 101101000 \rangle - \beta \mid 101101100 \rangle$

$-\beta \mid 111110000 \rangle - \beta \mid 111110100 \rangle - \beta \mid 111111000 \rangle - \beta \mid 111111100\rangle$

$-\beta \mid 011000000 \rangle - \beta \mid 011000100 \rangle - \beta \mid 011001000 \rangle - \beta \mid 011001100\rangle$

$+\beta \mid 001010000\rangle + \beta \mid 001010100\rangle + \beta \mid 001011000\rangle + \beta \mid 001011100 \rangle$

· <u>L'application de la porte $H(8)$:</u>

$\mid \Psi_1\rangle = \frac{1}{8}(\alpha \mid 000000000\rangle + \alpha \mid 000000010\rangle + \alpha \mid 000000100\rangle + \alpha \mid 000000110\rangle$

$+\alpha \mid 000001000\rangle + \alpha \mid 000001010\rangle + \alpha \mid 000001100\rangle + \alpha \mid 000001110\rangle$

$+ \alpha \mid 010010000 \rangle + \alpha \mid 010010010 \rangle + \alpha \mid 010010100 \rangle + \alpha \mid 010010110 \rangle$

$+\alpha \mid 010011000 \rangle + \alpha \mid 010011010 \rangle + \alpha \mid 010011100 \rangle + \alpha \mid 010011110 \rangle$

$+\alpha \mid 110100000 \rangle + \alpha \mid 110100010 \rangle + \alpha \mid 110100100 \rangle + \alpha \mid 110100110 \rangle$

$+\alpha \mid 110101000 \rangle + \alpha \mid 110101010 \rangle + \alpha \mid 110101100 \rangle + \alpha \mid 110101110 \rangle$

$-\alpha \mid 100110000 \rangle - \alpha \mid 100110010 \rangle - \alpha \mid 100110100 \rangle - \alpha \mid 100110110 \rangle$

$-\alpha \mid 100111000 \rangle - \alpha \mid 100111010 \rangle - \alpha \mid 100111100 \rangle - \alpha \mid 100111110 \rangle$

$+\alpha \mid 011100000 \rangle + \alpha \mid 011100010 \rangle + \alpha \mid 011100100 \rangle + \alpha \mid 011100110 \rangle$

$+\alpha \mid 011101000 \rangle + \alpha \mid 011101010 \rangle + \alpha \mid 011101100 \rangle + \alpha \mid 011101110 \rangle$

$+\alpha \mid 001110000 \rangle + \alpha \mid 001110010 \rangle + \alpha \mid 001110100 \rangle + \alpha \mid 001110110 \rangle$

$+\alpha \mid 001111000 \rangle + \alpha \mid 001111010 \rangle + \alpha \mid 001111100 \rangle + \alpha \mid 001111110 \rangle$

$+\alpha \mid 101000000 \rangle + \alpha \mid 101000010 \rangle + \alpha \mid 101000100 \rangle + \alpha \mid 101000110 \rangle$

$+\alpha \mid 101001000 \rangle + \alpha \mid 101001010 \rangle + \alpha \mid 101001100 \rangle + \alpha \mid 101001110 \rangle$

$-\alpha \mid 111010000 \rangle - \alpha \mid 111010010 \rangle - \alpha \mid 111010100 \rangle - \alpha \mid 111010110 \rangle$

$-\alpha \mid 111011000 \rangle - \alpha \mid 111011010\rangle - \alpha \mid 111011100 \rangle - \alpha \mid 111011110\rangle$

$+ \beta \mid 110000000 \rangle + \beta \mid 110000010 \rangle + \beta \mid 110000100 \rangle + \beta \mid 110000110 \rangle$

$+\beta \mid 110001000 \rangle + \beta \mid 110001010 \rangle + \beta \mid 110001100 \rangle + \beta \mid 110001110 \rangle$

$+\beta \mid 100010000 \rangle + \beta \mid 100010010 \rangle + \beta \mid 100010100 \rangle + \beta \mid 100010110 \rangle$

$+\beta \mid 100011000 \rangle + \beta \mid 100011010 \rangle + \beta \mid 100011100 \rangle + \beta \mid 100011110 \rangle$

$+\beta \mid 000100000 \rangle + \beta \mid 000100010 \rangle + \beta \mid 000100100 \rangle + \beta \mid 000100110 \rangle$

$+\beta \mid 000101000 \rangle + \beta \mid 000101010 \rangle + \beta \mid 000101100 \rangle + \beta \mid 000101110 \rangle$

$-\beta \mid 010110000 \rangle - \beta \mid 010110010 \rangle - \beta \mid 010110100 \rangle - \beta \mid 010110110 \rangle$

$-\beta \mid 010111000\ \rangle - \beta \mid 010111010\ \rangle - \beta \mid 010111100\ \rangle - \beta \mid 010111110\ \rangle$

-$\beta \mid 101100000\ \rangle - \beta \mid 101100010\ \rangle - \beta \mid 101100100\ \rangle - \beta \mid 101100110\ \rangle$

$-\beta \mid 101101000\ \rangle - \beta \mid 101101010\ \rangle - \beta \mid 101101100\ \rangle - \beta \mid 101101110\ \rangle$

-$\beta \mid 111110000\ \rangle - \beta \mid 111110010\ \rangle - \beta \mid 111110100\ \rangle - \beta \mid 111110110\ \rangle$

$-\beta \mid 111111000\ \rangle - \beta \mid 111111010\ \rangle - \beta \mid\ 111111100\rangle - \beta \mid\ 111111110\rangle$

-$\beta \mid 011000000\ \rangle - \beta \mid 011000010\ \rangle - \beta \mid 011000100\ \rangle - \beta \mid 011000110\ \rangle$

$-\beta \mid 011001000\ \rangle - \beta \mid 011001010\ \rangle - \beta \mid 011001100\rangle - \beta \mid 011001110\rangle$

$+\beta \mid 001010000\rangle + \beta \mid 001010010\rangle + \beta \mid 001010100\rangle + \beta \mid 001010110\rangle$

$+\beta \mid 001011000\rangle + \beta \mid 001011010\rangle + \beta \mid 001011100\ \rangle + \beta \mid 001011110\ \rangle)$

· L'application de la porte $H(9)$

$\mid \Psi_1\rangle = \frac{1}{8\sqrt{2}}(\alpha \mid 000000000\rangle + \alpha \mid 000000001\rangle + \alpha \mid 000000010\rangle + \alpha \mid 000000011\rangle$

$+\alpha \mid 000000100\rangle + \alpha \mid 000000101\rangle + \alpha \mid 000000110\rangle + \alpha \mid 000000111\rangle$

$+\alpha \mid 000001000\rangle + \alpha \mid 000001001\rangle + \alpha \mid 000001010\rangle + \alpha \mid 000001011\rangle$

$+\alpha \mid 000001100\rangle + \alpha \mid 000001101\rangle + \alpha \mid 000001110\rangle + \alpha \mid 000001111\rangle$

$+\ \alpha \mid 010010000\ \rangle +\ \alpha \mid 010010001\ \rangle + \alpha \mid 010010010\ \rangle +\ \alpha \mid 010010011\ \rangle$

$+\alpha \mid 010010100\ \rangle + \alpha \mid 010010101\ \rangle + \alpha \mid 010010110\ \rangle + \alpha \mid 010010111\ \rangle$

$+\alpha \mid 010011000\ \rangle + \alpha \mid 010011001\ \rangle + \alpha \mid 010011010\ \rangle + \alpha \mid 010011011\ \rangle$

$+\alpha \mid 010011100\ \rangle + \alpha \mid 010011101\ \rangle + \alpha \mid 010011110\ \rangle + \alpha \mid 010011111\ \rangle$

$+\alpha \mid\ 110100000\ \rangle\ + \alpha \mid\ 110100001\ \rangle + \alpha \mid\ 110100010\ \rangle\ + \alpha \mid\ 110100011\ \rangle$

$+\alpha \mid\ 110100100\ \rangle + \alpha \mid\ 110100101\rangle + \alpha \mid\ 110100110\ \rangle + \alpha \mid\ 110100111\ \rangle$

$+\alpha \mid 110101000\ \rangle + \alpha \mid 110101001\ \rangle + \alpha \mid 110101010\ \rangle + \alpha \mid 110101011\ \rangle$

$+\alpha \mid 110101100\ \rangle\ + \alpha \mid 110101101\ \rangle\ + \alpha \mid 110101110\ \rangle\ + \alpha \mid 110101111\ \rangle$

-$\alpha \mid 100110000\ \rangle - \alpha \mid 100110001\ \rangle$-$\alpha \mid 100110010\ \rangle - \alpha \mid 100110011\ \rangle$

$-\alpha \mid\ 100110100\ \rangle - \alpha \mid\ 100110101\ \rangle - \alpha \mid\ 100110110\ \rangle - \alpha \mid\ 100110111\ \rangle$

$-\alpha \mid 100111000\ \rangle - \alpha \mid 100111001\ \rangle\ - \alpha \mid 100111010\ \rangle - \alpha \mid 100111011\ \rangle$

$-\alpha \mid 100111100\ \rangle - \alpha \mid 100111101\ \rangle - \alpha \mid 100111110\ \rangle - \alpha \mid 100111111\ \rangle$

$+\alpha \mid 011100000\ \rangle + \alpha \mid 011100001\ \rangle + \alpha \mid 011100010\ \rangle + \alpha \mid 011100011\ \rangle$

$+\alpha \mid 011100100\ \rangle + \alpha \mid 011100101\ \rangle + \alpha \mid 011100110\ \rangle + \alpha \mid 011100111\ \rangle$

$+\alpha \mid 011101000\ \rangle + \alpha \mid 011101001\ \rangle + \alpha \mid 011101010\ \rangle + \alpha \mid 011101011\ \rangle$

$+\alpha \mid 011101100\ \rangle + \alpha \mid 011101101\ \rangle + \alpha \mid 011101110\ \rangle + \alpha \mid 011101111\ \rangle$

$+\alpha \mid 001110000\ \rangle + \alpha \mid 001110001\ \rangle + \alpha \mid 001110010\ \rangle + \alpha \mid 001110011\ \rangle$

$+\alpha \mid 001110100\ \rangle + \alpha \mid 001110101\ \rangle + \alpha \mid 001110110\ \rangle + \alpha \mid 001110111\ \rangle$

$+\alpha \mid 001111000 \rangle + \alpha \mid 001111001 \rangle + \alpha \mid 001111010 \rangle + \alpha \mid 001111011 \rangle$
$+\alpha \mid 001111100 \rangle + \alpha \mid 001111101 \rangle + \alpha \mid 001111110 \rangle + \alpha \mid 001111111 \rangle$
$+\alpha \mid 101000000 \rangle + \alpha \mid 101000001\rangle + \alpha \mid 101000010 \rangle + \alpha \mid 101000011 \rangle$
$+\alpha \mid 101000100 \rangle + \alpha \mid 101000101 \rangle + \alpha \mid 101000110 \rangle + \alpha \mid 101000111 \rangle$
$+\alpha \mid 101001000 \rangle + \alpha \mid 101001001 \rangle + \alpha \mid 101001010 \rangle + \alpha \mid 101001011 \rangle$
$+\alpha \mid 101001100 \rangle + \alpha \mid 101001101 \rangle + \alpha \mid 101001110 \rangle + \alpha \mid 101001111 \rangle$
-$\alpha \mid 111010000 \rangle - \alpha \mid 111010001 \rangle$-$\alpha \mid 111010010 \rangle - \alpha \mid 111010011 \rangle$
-$\alpha \mid 111010100 \rangle - \alpha \mid 111010101 \rangle$-$\alpha \mid 111010110 \rangle - \alpha \mid 111010111 \rangle$
$-\alpha \mid 111011000 \rangle - \alpha \mid 111011001\rangle - \alpha \mid 111011010 \rangle - \alpha \mid 111011011\rangle$
$-\alpha \mid 111011100 \rangle - \alpha \mid 111011101\rangle - \alpha \mid 111011110 \rangle - \alpha \mid 111011111\rangle$
$+ \beta \mid 110000000 \rangle + \beta \mid 110000001 \rangle + \beta \mid 110000010 \rangle + \beta \mid 110000011\rangle$
$+\beta \mid 110000100 \rangle + \beta \mid 110000101 \rangle + \beta \mid 110000110 \rangle + \beta \mid 110000111 \rangle$
$+\beta \mid 110001000 \rangle + \beta \mid 110001001 \rangle + \beta \mid 110001010 \rangle + \beta \mid 110001011 \rangle$
$+\beta \mid 110001100 \rangle + \beta \mid 110001101 \rangle + \beta \mid 110001110 \rangle + \beta \mid 110001111 \rangle$
$+\beta \mid 100010000 \rangle + \beta \mid 100010001 \rangle + \beta \mid 100010010 \rangle + \beta \mid 100010011 \rangle$
$+\beta \mid 100010100 \rangle + \beta \mid 100010101 \rangle + \beta \mid 100010110 \rangle + \beta \mid 100010111 \rangle$
$+\beta \mid 100011000 \rangle + \beta \mid 100011001 \rangle + \beta \mid 100011010 \rangle + \beta \mid 100011011 \rangle$
$+\beta \mid 100011100 \rangle + \beta \mid 100011101 \rangle + \beta \mid 100011110 \rangle + \beta \mid 100011111 \rangle$
$+\beta \mid 000100000 \rangle + \beta \mid 000100001 \rangle + \beta \mid 000100010 \rangle + \beta \mid 000100011 \rangle$
$+\beta \mid 000100100 \rangle + \beta \mid 000100101 \rangle + \beta \mid 000100110 \rangle + \beta \mid 000100111 \rangle$
$+\beta \mid 000101000 \rangle + \beta \mid 000101001 \rangle + \beta \mid 000101010 \rangle + \beta \mid 000101011 \rangle$
$+\beta \mid 000101100 \rangle + \beta \mid 000101101 \rangle + \beta \mid 000101110 \rangle + \beta \mid 000101111 \rangle$
-$\beta \mid 010110000 \rangle - \beta \mid 010110001 \rangle - \beta \mid 010110010 \rangle - \beta \mid 010110011 \rangle$
$-\beta \mid 010110100 \rangle - \beta \mid 010110101 \rangle - \beta \mid 010110110 \rangle - \beta \mid 010110111 \rangle$
$-\beta \mid 010111000 \rangle - \beta \mid 010111001 \rangle - \beta \mid 010111010 \rangle - \beta \mid 010111011 \rangle$
$-\beta \mid 010111100 \rangle - \beta \mid 010111101 \rangle - \beta \mid 010111110 \rangle - \beta \mid 010111111 \rangle$
-$\beta \mid 101100000 \rangle - \beta \mid 101100001 \rangle - \beta \mid 101100010 \rangle - \beta \mid 101100011 \rangle$
$-\beta \mid 101100100 \rangle - \beta \mid 101100101 \rangle - \beta \mid 101100110 \rangle - \beta \mid 101100111 \rangle$
$-\beta \mid 101101000 \rangle - \beta \mid 101101001 \rangle - \beta \mid 101101010 \rangle - \beta \mid 101101011\rangle$
$-\beta \mid 101101100 \rangle - \beta \mid 101101101 \rangle - \beta \mid 101101110 \rangle - \beta \mid 101101111 \rangle$
-$\beta \mid 111110000 \rangle - \beta \mid 111110001 \rangle - \beta \mid 111110010 \rangle - \beta \mid 111110011 \rangle$
$-\beta \mid 111110100 \rangle - \beta \mid 111110101 \rangle - \beta \mid 111110110 \rangle - \beta \mid 111110111 \rangle$

$-\beta \mid 111111000 \rangle - \beta \mid 111111001 \rangle - \beta \mid 111111010 \rangle - \beta \mid 111111011 \rangle$

$-\beta \mid 111111100\rangle - \beta \mid 111111101\rangle - \beta \mid 111111110\rangle - \beta \mid 111111111\rangle$

-$\beta \mid 011000000 \rangle - \beta \mid 011000001 \rangle - \beta \mid 011000010 \rangle - \beta \mid 011000011 \rangle$

$-\beta \mid 011000100 \rangle - \beta \mid 011000101 \rangle - \beta \mid 011000110 \rangle - \beta \mid 011000111 \rangle$

$-\beta \mid 011001000 \rangle - \beta \mid 011001001 \rangle - \beta \mid 011001010 \rangle - \beta \mid 011001011\rangle$

$-\beta \mid 011001100\rangle - \beta \mid 011001101\rangle - \beta \mid 011001110\rangle - \beta \mid 011001111\rangle$

$+\beta \mid 001010000\rangle + \beta \mid 001010001\rangle + \beta \mid 001010010\rangle + \beta \mid 001010011\rangle$

$+\beta \mid 001010100\rangle + \beta \mid 001010101\rangle + \beta \mid 001010110\rangle + \beta \mid 001010111\rangle$

$+\beta \mid 001011000\rangle + \beta \mid 001011001\rangle + \beta \mid 001011010\rangle + \beta \mid 001011011\rangle$

$+\beta \mid 001011100 \rangle + \beta \mid 001011101 \rangle + \beta \mid 001011110 \rangle + \beta \mid 001011111 \rangle)$

∘ **Application controlée des stabilisateurs :**

· Pour $M_0 = X(1)Z(2)X(3)Z(4)I(5)$

$\mid \Psi_1\rangle = \frac{1}{8\sqrt{2}}(\alpha \mid 000000000\rangle + \alpha \mid 000000001\rangle + \alpha \mid 000000010\rangle + \alpha \mid 000000011\rangle$

$+\alpha \mid 000000100\rangle + \alpha \mid 000000101\rangle + \alpha \mid 000000110\rangle + \alpha \mid 000000111\rangle$

$+\alpha \mid 101001000\rangle + \alpha \mid 101001001\rangle + \alpha \mid 101001010\rangle + \alpha \mid 101001011\rangle$

$+\alpha \mid 101001100\rangle + \alpha \mid 101001101\rangle + \alpha \mid 101001110\rangle + \alpha \mid 101001111\rangle$

$+ \alpha \mid 010010000 \rangle + \alpha \mid 010010001 \rangle + \alpha \mid 010010010 \rangle + \alpha \mid 010010011 \rangle$

$+\alpha \mid 010010100 \rangle + \alpha \mid 010010101 \rangle + \alpha \mid 010010110 \rangle + \alpha \mid 010010111 \rangle$

$-\alpha \mid 111011000 \rangle - \alpha \mid 111011001 \rangle - \alpha \mid 111011010 \rangle - \alpha \mid 111011011 \rangle$

$-\alpha \mid 111011100 \rangle - \alpha \mid 111011101 \rangle - \alpha \mid 111011110 \rangle - \alpha \mid 111011111 \rangle$

$+\alpha \mid 110100000 \rangle + \alpha \mid 110100001 \rangle + \alpha \mid 110100010 \rangle + \alpha \mid 110100011 \rangle$

$+\alpha \mid 110100100 \rangle + \alpha \mid 110100101\rangle + \alpha \mid 110100110 \rangle + \alpha \mid 110100111 \rangle$

$+\alpha \mid 011101000 \rangle + \alpha \mid 011101001 \rangle + \alpha \mid 011101010 \rangle + \alpha \mid 011101011 \rangle$

$+\alpha \mid 011101100 \rangle + \alpha \mid 011101101 \rangle + \alpha \mid 011101110 \rangle + \alpha \mid 011101111 \rangle$

-$\alpha \mid 100110000 \rangle - \alpha \mid 100110001 \rangle$-$\alpha \mid 100110010 \rangle - \alpha \mid 100110011 \rangle$

$-\alpha \mid 100110100 \rangle - \alpha \mid 100110101 \rangle - \alpha \mid 100110110 \rangle - \alpha \mid 100110111 \rangle$

$+\alpha \mid 001111000 \rangle + \alpha \mid 001111001 \rangle + \alpha \mid 001111010 \rangle + \alpha \mid 001111011 \rangle$

$+\alpha \mid 001111100 \rangle - \alpha \mid 001111101 \rangle + \alpha \mid 001111110 \rangle + \alpha \mid 001111111 \rangle$

$+\alpha \mid 011100000 \rangle + \alpha \mid 011100001 \rangle + \alpha \mid 011100010 \rangle + \alpha \mid 011100011 \rangle$

$+\alpha \mid 011100100 \rangle + \alpha \mid 011100101 \rangle + \alpha \mid 011100110 \rangle + \alpha \mid 011100111 \rangle$

$+\alpha \mid 110101000 \rangle + \alpha \mid 110101001 \rangle + \alpha \mid 110101010 \rangle + \alpha \mid 110101011 \rangle$

$+\alpha \mid 110101100 \rangle + \alpha \mid 110101101 \rangle + \alpha \mid 110101110 \rangle + \alpha \mid 110101111 \rangle$

$+\alpha \mid 001110000 \ \rangle + \alpha \mid 001110001 \ \rangle + \alpha \mid 001110010 \ \rangle + \alpha \mid 001110011 \ \rangle$

$+\alpha \mid 001110100 \ \rangle + \alpha \mid 001110101 \ \rangle + \alpha \mid 001110110 \ \rangle + \alpha \mid 001110111 \ \rangle$

$-\alpha \mid 100111000 \ \rangle - \alpha \mid 100111001 \ \rangle - \alpha \mid 100111010 \ \rangle - \alpha \mid 100111011 \ \rangle$

$-\alpha \mid 100111100 \ \rangle - \alpha \mid 100111101 \ \rangle - \alpha \mid 100111110 \ \rangle - \alpha \mid 100111111 \ \rangle$

$+\alpha \mid 101000000 \rangle + \alpha \mid 101000001 \rangle + \alpha \mid 101000010 \rangle + \alpha \mid 101000011 \rangle$

$+\alpha \mid 101000100 \rangle + \alpha \mid 101000101 \rangle + \alpha \mid 101000110 \rangle + \alpha \mid 101000111 \rangle$

$+\alpha \mid 000001000 \rangle + \alpha \mid 000001001 \rangle + \alpha \mid 000001010 \rangle + \alpha \mid 000001011 \rangle$

$+\alpha \mid 000001100 \rangle + \alpha \mid 000001101 \rangle + \alpha \mid 000001110 \rangle + \alpha \mid 000001111 \rangle$

-$\alpha \mid 111010000 \rangle - \alpha \mid 111010001 \rangle$-$\alpha \mid 111010010 \rangle - \alpha \mid 111010011 \rangle$

-$\alpha \mid 111010100 \rangle - \alpha \mid 111010101 \rangle$-$\alpha \mid 111010110 \rangle - \alpha \mid 111010111 \rangle$

$+\alpha \mid 010011000 \rangle + \alpha \mid 010011001 \rangle + \alpha \mid 010011010 \rangle + \alpha \mid 010011011 \rangle$

$+\alpha \mid 010011100 \rangle + \alpha \mid 010011101 \rangle + \alpha \mid 010011110 \rangle + \alpha \mid 010011111 \rangle$

$+ \beta \mid 110000000 \rangle + \beta \mid 110000001 \rangle + \beta \mid 110000010 \rangle + \beta \mid 110000011 \rangle$

$+\beta \mid 110000100 \rangle + \beta \mid 110000101 \rangle + \beta \mid 110000110 \rangle + \beta \mid 110000111 \rangle$

$-\beta \mid 011001000 \rangle - \beta \mid 011001001 \rangle - \beta \mid 011001010 \rangle - \beta \mid 011001011 \rangle$

$-\beta \mid 011001100 \rangle - \beta \mid 011001101 \rangle - \beta \mid 011001110 \rangle - \beta \mid 011001111 \rangle$

$+\beta \mid 100010000 \rangle + \beta \mid 100010001 \rangle + \beta \mid 100010010 \rangle + \beta \mid 100010011 \rangle$

$+\beta \mid 100010100 \rangle + \beta \mid 100010101 \rangle + \beta \mid 100010110 \rangle + \beta \mid 100010111 \rangle$

$+\beta \mid 001011000 \rangle + \beta \mid 001011001 \rangle + \beta \mid 001011010 \rangle + \beta \mid 001011011 \rangle$

$+\beta \mid 001011100 \rangle + \beta \mid 001011101 \rangle + \beta \mid 001011110 \rangle + \beta \mid 001011111 \rangle$

$+\beta \mid 000100000 \rangle + \beta \mid 000100001 \rangle + \beta \mid 000100010 \rangle + \beta \mid 000100011 \rangle$

$+\beta \mid 000100100 \rangle + \beta \mid 000100101 \rangle + \beta \mid 000100110 \rangle + \beta \mid 000100111 \rangle$

$-\beta \mid 101101000 \rangle - \beta \mid 101101001 \rangle - \beta \mid 101101010 \rangle - \beta \mid 101101011 \rangle$

$-\beta \mid 101101100 \rangle - \beta \mid 101101101 \rangle - \beta \mid 101101110 \rangle - \beta \mid 101101111 \rangle$

-$\beta \mid 010110000 \rangle \beta \mid 010110001 \rangle - \beta \mid 010110010 \rangle - \beta \mid 010110011 \rangle$

$-\beta \mid 010110100 \rangle - \beta \mid 010110101 \rangle - \beta \mid 010110110 \rangle - \beta \mid 010110111 \rangle$

$-\beta \mid 111111000 \rangle - \beta \mid 111111001 \rangle - \beta \mid 111111010 \rangle - \beta \mid 111111011 \rangle$

$-\beta \mid 111111100 \rangle - \beta \mid 111111101 \rangle - \beta \mid 111111110 \rangle - \beta \mid 111111111 \rangle$

-$\beta \mid 101100000 \rangle - \beta \mid 101100001 \rangle - \beta \mid 101100010 \rangle - \beta \mid 101100011 \rangle$

$-\beta \mid 101100100 \rangle - \beta \mid 101100101 \rangle - \beta \mid 101100110 \rangle - \beta \mid 101100111 \rangle$

$+\beta \mid 000101000 \rangle + \beta \mid 000101001 \rangle + \beta \mid 000101010 \rangle + \beta \mid 000101011 \rangle$

$+\beta \mid 000101100 \rangle + \beta \mid 000101101 \rangle + \beta \mid 000101110 \rangle + \beta \mid 000101111 \rangle$

$-\beta \mid 111110000\rangle - \beta \mid 111110001\rangle - \beta \mid 111110010\rangle - \beta \mid 111110011\rangle$
$-\beta \mid 111110100\rangle - \beta \mid 111110101\rangle - \beta \mid 111110110\rangle - \beta \mid 111110111\rangle$
$-\beta \mid 010111000\rangle - \beta \mid 010111001\rangle - \beta \mid 010111010\rangle - \beta \mid 010111011\rangle$
$-\beta \mid 010111100\rangle - \beta \mid 010111101\rangle - \beta \mid 010111110\rangle - \beta \mid 010111111\rangle$
$-\beta \mid 011000000\rangle - \beta \mid 011000001\rangle - \beta \mid 011000010\rangle - \beta \mid 011000011\rangle$
$-\beta \mid 011000100\rangle - \beta \mid 011000101\rangle - \beta \mid 011000110\rangle - \beta \mid 011000111\rangle$
$+\beta \mid 110001000\rangle + \beta \mid 110001001\rangle + \beta \mid 110001010\rangle + \beta \mid 110001011\rangle$
$+\beta \mid 110001100\rangle + \beta \mid 110001101\rangle + \beta \mid 110001110\rangle + \beta \mid 110001111\rangle$
$+\beta \mid 001010000\rangle + \beta \mid 001010001\rangle + \beta \mid 001010010\rangle + \beta \mid 001010011\rangle$
$+\beta \mid 001010100\rangle + \beta \mid 001010101\rangle + \beta \mid 001010110\rangle + \beta \mid 001010111\rangle$
$+\beta \mid 100011000\rangle + \beta \mid 100011001\rangle + \beta \mid 100011010\rangle + \beta \mid 100011011\rangle$
$+\beta \mid 100011100\rangle + \beta \mid 100011101\rangle + \beta \mid 100011110\rangle + \beta \mid 100011111\rangle)$

· Pour $M_1 = Z(1)\,Z(2)\,Z(3)\,I(4)\,Z(5)$

$\mid \Psi_1\rangle = \frac{1}{8\sqrt{2}}(\alpha \mid 000000000\rangle + \alpha \mid 000000001\rangle + \alpha \mid 000000010\rangle + \alpha \mid 000000011\rangle$
$+\alpha \mid 000000100\rangle + \alpha \mid 000000101\rangle + \alpha \mid 000000110\rangle + \alpha \mid 000000111\rangle$
$+\alpha \mid 101001000\rangle + \alpha \mid 101001001\rangle + \alpha \mid 101001010\rangle + \alpha \mid 101001011\rangle$
$+\alpha \mid 101001100\rangle + \alpha \mid 101001101\rangle + \alpha \mid 101001110\rangle + \alpha \mid 101001111\rangle$
$+\alpha \mid 010010000\rangle + \alpha \mid 010010001\rangle + \alpha \mid 010010010\rangle + \alpha \mid 010010011\rangle$
$+\alpha \mid 010010100\rangle + \alpha \mid 010010101\rangle + \alpha \mid 010010110\rangle + \alpha \mid 010010111\rangle$
$-\alpha \mid 111011000\rangle - \alpha \mid 111011001\rangle - \alpha \mid 111011010\rangle - \alpha \mid 111011011\rangle$
$-\alpha \mid 111011100\rangle - \alpha \mid 111011101\rangle - \alpha \mid 111011110\rangle - \alpha \mid 111011111\rangle$
$+\alpha \mid 110100000\rangle + \alpha \mid 110100001\rangle + \alpha \mid 110100010\rangle + \alpha \mid 110100011\rangle$
$+\alpha \mid 110100100\rangle + \alpha \mid 110100101\rangle + \alpha \mid 110100110\rangle + \alpha \mid 110100111\rangle$
$+\alpha \mid 011101000\rangle + \alpha \mid 011101001\rangle + \alpha \mid 011101010\rangle + \alpha \mid 011101011\rangle$
$+\alpha \mid 011101100\rangle + \alpha \mid 011101101\rangle + \alpha \mid 011101110\rangle + \alpha \mid 011101111\rangle$
$-\alpha \mid 100110000\rangle - \alpha \mid 100110001\rangle - \alpha \mid 100110010\rangle - \alpha \mid 100110011\rangle$
$-\alpha \mid 100110100\rangle - \alpha \mid 100110101\rangle - \alpha \mid 100110110\rangle - \alpha \mid 100110111\rangle$
$+\alpha \mid 001111000\rangle + \alpha \mid 001111001\rangle + \alpha \mid 001111010\rangle + \alpha \mid 001111011\rangle$
$+\alpha \mid 001111100\rangle - \alpha \mid 001111101\rangle + \alpha \mid 001111110\rangle + \alpha \mid 001111111\rangle$
$+\alpha \mid 011100000\rangle + \alpha \mid 011100001\rangle + \alpha \mid 011100010\rangle + \alpha \mid 011100011\rangle$
$+\alpha \mid 011100100\rangle + \alpha \mid 011100101\rangle + \alpha \mid 011100110\rangle + \alpha \mid 011100111\rangle$
$+\alpha \mid 110101000\rangle + \alpha \mid 110101001\rangle + \alpha \mid 110101010\rangle + \alpha \mid 110101011\rangle$

$+\alpha \mid 110101100 \rangle + \alpha \mid 110101101 \rangle + \alpha \mid 110101110 \rangle + \alpha \mid 110101111 \rangle$
$+\alpha \mid 001110000 \rangle + \alpha \mid 001110001 \rangle + \alpha \mid 001110010 \rangle + \alpha \mid 001110011 \rangle$
$+\alpha \mid 001110100 \rangle + \alpha \mid 001110101 \rangle + \alpha \mid 001110110 \rangle + \alpha \mid 001110111 \rangle$
$-\alpha \mid 100111000 \rangle - \alpha \mid 100111001 \rangle - \alpha \mid 100111010 \rangle - \alpha \mid 100111011 \rangle$
$-\alpha \mid 100111100 \rangle - \alpha \mid 100111101 \rangle - \alpha \mid 100111110 \rangle - \alpha \mid 100111111 \rangle$
$+\alpha \mid 101000000 \rangle + \alpha \mid 101000001 \rangle + \alpha \mid 101000010 \rangle + \alpha \mid 101000011 \rangle$
$+\alpha \mid 101000100 \rangle + \alpha \mid 101000101 \rangle + \alpha \mid 101000110 \rangle + \alpha \mid 101000111 \rangle$
$+\alpha \mid 000001000 \rangle + \alpha \mid 000001001 \rangle + \alpha \mid 000001010 \rangle + \alpha \mid 000001011 \rangle$
$+\alpha \mid 000001100 \rangle + \alpha \mid 000001101 \rangle + \alpha \mid 000001110 \rangle + \alpha \mid 000001111 \rangle$
$-\alpha \mid 111010000 \rangle - \alpha \mid 111010001 \rangle - \alpha \mid 111010010 \rangle - \alpha \mid 111010011 \rangle$
$-\alpha \mid 111010100 \rangle - \alpha \mid 111010101 \rangle - \alpha \mid 111010110 \rangle - \alpha \mid 111010111 \rangle$
$+\alpha \mid 010011000 \rangle + \alpha \mid 010011001 \rangle + \alpha \mid 010011010 \rangle + \alpha \mid 010011011 \rangle$
$+\alpha \mid 010011100 \rangle + \alpha \mid 010011101 \rangle + \alpha \mid 010011110 \rangle + \alpha \mid 010011111 \rangle$
$+ \beta \mid 110000000 \rangle + \beta \mid 110000001 \rangle + \beta \mid 110000010 \rangle + \beta \mid 110000011 \rangle$
$+\beta \mid 110000100 \rangle + \beta \mid 110000101 \rangle + \beta \mid 110000110 \rangle + \beta \mid 110000111 \rangle$
$-\beta \mid 011001000 \rangle - \beta \mid 011001001 \rangle - \beta \mid 011001010 \rangle - \beta \mid 011001011 \rangle$
$-\beta \mid 011001100 \rangle - \beta \mid 011001101 \rangle - \beta \mid 011001110 \rangle - \beta \mid 011001111 \rangle$
$+\beta \mid 100010000 \rangle + \beta \mid 100010001 \rangle + \beta \mid 100010010 \rangle + \beta \mid 100010011 \rangle$
$+\beta \mid 100010100 \rangle + \beta \mid 100010101 \rangle + \beta \mid 100010110 \rangle + \beta \mid 100010111 \rangle$
$+\beta \mid 001011000 \rangle + \beta \mid 001011001 \rangle + \beta \mid 001011010 \rangle + \beta \mid 001011011 \rangle$
$+\beta \mid 001011100 \rangle + \beta \mid 001011101 \rangle + \beta \mid 001011110 \rangle + \beta \mid 001011111 \rangle$
$+\beta \mid 000100000 \rangle + \beta \mid 000100001 \rangle + \beta \mid 000100010 \rangle + \beta \mid 000100011 \rangle$
$+\beta \mid 000100100 \rangle + \beta \mid 000100101 \rangle + \beta \mid 000100110 \rangle + \beta \mid 000100111 \rangle$
$-\beta \mid 101101000 \rangle - \beta \mid 101101001 \rangle - \beta \mid 101101010 \rangle - \beta \mid 101101011 \rangle$
$-\beta \mid 101101100 \rangle - \beta \mid 101101101 \rangle - \beta \mid 101101110 \rangle - \beta \mid 101101111 \rangle$
$-\beta \mid 010110000 \rangle \beta \mid 010110001 \rangle - \beta \mid 010110010 \rangle - \beta \mid 010110011 \rangle$
$-\beta \mid 010110100 \rangle - \beta \mid 010110101 \rangle - \beta \mid 010110110 \rangle - \beta \mid 010110111 \rangle$
$-\beta \mid 111111000 \rangle - \beta \mid 111111001 \rangle - \beta \mid 111111010 \rangle - \beta \mid 111111011 \rangle$
$-\beta \mid 111111100 \rangle - \beta \mid 111111101 \rangle - \beta \mid 111111110 \rangle - \beta \mid 111111111 \rangle$
$-\beta \mid 101100000 \rangle - \beta \mid 101100001 \rangle - \beta \mid 101100010 \rangle - \beta \mid 101100011 \rangle$
$-\beta \mid 101100100 \rangle - \beta \mid 101100101 \rangle - \beta \mid 101100110 \rangle - \beta \mid 101100111 \rangle$
$+\beta \mid 000101000 \rangle + \beta \mid 000101001 \rangle + \beta \mid 000101010 \rangle + \beta \mid 000101011 \rangle$

$+\beta \mid 000101100 \rangle + \beta \mid 000101101 \rangle + \beta \mid 000101110 \rangle + \beta \mid 000101111 \rangle$

$-\beta \mid 111110000 \rangle - \beta \mid 111110001 \rangle - \beta \mid 111110010 \rangle - \beta \mid 111110011 \rangle$

$-\beta \mid 111110100 \rangle - \beta \mid 111110101 \rangle - \beta \mid 111110110 \rangle - \beta \mid 111110111 \rangle$

$-\beta \mid 010111000 \rangle - \beta \mid 010111001 \rangle - \beta \mid 010111010 \rangle - \beta \mid 010111011 \rangle$

$-\beta \mid 010111100\rangle - \beta \mid 010111101\rangle - \beta \mid 010111110\rangle - \beta \mid 010111111\rangle$

$-\beta \mid 011000000 \rangle - \beta \mid 011000001 \rangle - \beta \mid 011000010 \rangle - \beta \mid 011000011 \rangle$

$-\beta \mid 011000100 \rangle - \beta \mid 011000101 \rangle - \beta \mid 011000110 \rangle - \beta \mid 011000111 \rangle$

$+\beta \mid 110001000 \rangle + \beta \mid 110001001 \rangle + \beta \mid 110001010 \rangle + \beta \mid 110001011\rangle$

$+\beta \mid 110001100\rangle + \beta \mid 110001101\rangle + \beta \mid 110001110\rangle + \beta \mid 110001111\rangle$

$+\beta \mid 001010000\rangle + \beta \mid 001010001\rangle + \beta \mid 001010010\rangle + \beta \mid 001010011\rangle$

$+\beta \mid 001010100\rangle + \beta \mid 001010101\rangle + \beta \mid 001010110\rangle + \beta \mid 001010111\rangle$

$+\beta \mid 100011000\rangle + \beta \mid 100011001\rangle + \beta \mid 100011010\rangle + \beta \mid 100011011\rangle$

$+\beta \mid 100011100 \rangle + \beta \mid 100011101 \rangle + \beta \mid 100011110 \rangle + \beta \mid 100011111 \rangle)$

· <u>Pour$M_2 = I(1)\,X(2)\,Z(3)\,Z(4)\,X(5)$</u>

$\mid \Psi_1\rangle = \frac{1}{8\sqrt{2}}(\alpha \mid 000000000\rangle + \alpha \mid 000000001\rangle + \alpha \mid 010010010\rangle + \alpha \mid 010010011\rangle$

$+\alpha \mid 000000100\rangle + \alpha \mid 000000101\rangle + \alpha \mid 010010110\rangle + \alpha \mid 010010111\rangle$

$+\alpha \mid 101001000\rangle + \alpha \mid 101001001\rangle - \alpha \mid 111011010\rangle - \alpha \mid 111011011\rangle$

$+\alpha \mid 101001100\rangle + \alpha \mid 101001101\rangle - \alpha \mid 111011110\rangle - \alpha \mid 111011111\rangle$

$+ \alpha \mid 010010000 \rangle + \alpha \mid 010010001 \rangle + \alpha \mid 000000010 \rangle + \alpha \mid 000000011 \rangle$

$+\alpha \mid 010010100 \rangle + \alpha \mid 010010101 \rangle + \alpha \mid 000000110 \rangle + \alpha \mid 000000111 \rangle$

$-\alpha \mid 111011000 \rangle - \alpha \mid 111011001 \rangle + \alpha \mid 101001010 \rangle + \alpha \mid 101001011 \rangle$

$-\alpha \mid 111011100 \rangle - \alpha \mid 111011101 \rangle + \alpha \mid 101001110 \rangle + \alpha \mid 101001111 \rangle$

$+\alpha \mid 110100000\rangle + \alpha \mid 110100001 \rangle - \alpha \mid 100110010 \rangle - \alpha \mid 100110011 \rangle$

$+\alpha \mid 110100100\rangle + \alpha \mid 110100101\rangle - \alpha \mid 100110110 \rangle - \alpha \mid 100110111 \rangle$

$+\alpha \mid 011101000\rangle + \alpha \mid 011101001 \rangle + \alpha \mid 001111010 \rangle + \alpha \mid 001111011 \rangle$

$+\alpha \mid 011101100\rangle + \alpha \mid 011101101 \rangle + \alpha \mid 001111110 \rangle + \alpha \mid 001111111 \rangle$

$-\alpha \mid 100110000\rangle - \alpha \mid 100110001 \rangle + \alpha \mid 110100010 \rangle + \alpha \mid 110100011 \rangle$

$-\alpha \mid 100110100\rangle - \alpha \mid 100110101 \rangle + \alpha \mid 110100110 \rangle + \alpha \mid 110100111 \rangle$

$+\alpha \mid 001111000 \rangle + \alpha \mid 001111001 \rangle + \alpha \mid 011101010 \rangle + \alpha \mid 011101011 \rangle$

$+\alpha \mid 001111100 \rangle - \alpha \mid 001111101 \rangle + \alpha \mid 011101110 \rangle + \alpha \mid 011101111 \rangle$

$+\alpha \mid 011100000 \rangle + \alpha \mid 011100001 \rangle + \alpha \mid 001110010 \rangle + \alpha \mid 001110011 \rangle$

$+\alpha \mid 011100100 \rangle + \alpha \mid 011100101 \rangle + \alpha \mid 001110110 \rangle + \alpha \mid 001110111 \rangle$

$+\alpha \mid 110101000 \rangle + \alpha \mid 110101001 \rangle - \alpha \mid 100111010 \rangle - \alpha \mid 100111011 \rangle$

$+\alpha \mid 110101100 \rangle + \alpha \mid 110101101 \rangle - \alpha \mid 100111110 \rangle - \alpha \mid 100111111 \rangle$

$+\alpha \mid 001110000 \rangle + \alpha \mid 001110001 \rangle + \alpha \mid 011100010 \rangle + \alpha \mid 011100011 \rangle$

$+\alpha \mid 001110100 \rangle + \alpha \mid 001110101 \rangle + \alpha \mid 011100110 \rangle + \alpha \mid 011100111 \rangle$

$-\alpha \mid 100111000 \rangle - \alpha \mid 100111001 \rangle + \alpha \mid 110101010 \rangle + \alpha \mid 110101011 \rangle$

$-\alpha \mid 100111100 \rangle - \alpha \mid 100111101 \rangle + \alpha \mid 110101110 \rangle + \alpha \mid 110101111 \rangle$

$+\alpha \mid 101000000 \rangle + \alpha \mid 101000001 \rangle$-$\alpha \mid 111010010 \rangle - \alpha \mid 111010011 \rangle$

$+\alpha \mid 101000100 \rangle + \alpha \mid 101000101 \rangle$-$\alpha \mid 111010110 \rangle - \alpha \mid 111010111 \rangle$

$+\alpha \mid 000001000 \rangle + \alpha \mid 000001001 \rangle + \alpha \mid 010011010 \rangle + \alpha \mid 010011011 \rangle$

$+\alpha \mid 000001100 \rangle + \alpha \mid 000001101 \rangle + \alpha \mid 010011110 \rangle + \alpha \mid 010011111 \rangle$

-$\alpha \mid 111010000 \rangle - \alpha \mid 111010001 \rangle + \alpha \mid 101000010 \rangle + \alpha \mid 101000011 \rangle$

-$\alpha \mid 111010100 \rangle - \alpha \mid 111010101 \rangle + \alpha \mid 101000110 \rangle + \alpha \mid 101000111 \rangle$

$+\alpha \mid 010011000 \rangle + \alpha \mid 010011001 \rangle + \alpha \mid 000001010 \rangle + \alpha \mid 000001011 \rangle$

$+\alpha \mid 010011100 \rangle + \alpha \mid 010011101 \rangle + \alpha \mid 000001110 \rangle + \alpha \mid 000001111 \rangle$

$+ \beta \mid 110000000 \rangle + \beta \mid 110000001 \rangle + \beta \mid 100010010 \rangle + \beta \mid 100010011 \rangle$

$+\beta \mid 110000100 \rangle + \beta \mid 110000101 \rangle + \beta \mid 100010110 \rangle + \beta \mid 100010111 \rangle$

$-\beta \mid 011001000 \rangle - \beta \mid 011001001 \rangle + \beta \mid 001011010 \rangle + \beta \mid 001011011 \rangle$

$-\beta \mid 011001100 \rangle - \beta \mid 011001101 \rangle + \beta \mid 001011110 \rangle + \beta \mid 001011111 \rangle$

$+\beta \mid 100010000 \rangle + \beta \mid 100010001 \rangle + \beta \mid 110000010 \rangle + \beta \mid 110000011 \rangle$

$+\beta \mid 100010100 \rangle + \beta \mid 100010101 \rangle + \beta \mid 110000110 \rangle + \beta \mid 110000111 \rangle$

$+\beta \mid 001011000 \rangle + \beta \mid 001011001 \rangle - \beta \mid 011001010 \rangle - \beta \mid 011001011 \rangle$

$+\beta \mid 001011100 \rangle + \beta \mid 001011101 \rangle - \beta \mid 011001110 \rangle - \beta \mid 011001111 \rangle$

$+\beta \mid 000100000 \rangle + \beta \mid 000100001 \rangle - \beta \mid 010110010 \rangle - \beta \mid 010110011 \rangle$

$+\beta \mid 000100100 \rangle + \beta \mid 000100101 \rangle - \beta \mid 010110110 \rangle - \beta \mid 010110111 \rangle$

$-\beta \mid 101101000 \rangle - \beta \mid 101101001 \rangle - \beta \mid 111111010 \rangle - \beta \mid 111111011 \rangle$

$-\beta \mid 101101100 \rangle - \beta \mid 101101101 \rangle - \beta \mid 111111110 \rangle - \beta \mid 111111111 \rangle$

-$\beta \mid 010110000 \rangle \beta \mid 010110001 \rangle + \beta \mid 000100010 \rangle + \beta \mid 000100011 \rangle$

$-\beta \mid 010110100 \rangle - \beta \mid 010110101 \rangle + \beta \mid 000100110 \rangle + \beta \mid 000100111 \rangle$

$-\beta \mid 111111000 \rangle - \beta \mid 111111001 \rangle - \beta \mid 101101010 \rangle - \beta \mid 101101011 \rangle$

$-\beta \mid 111111100 \rangle - \beta \mid 111111101 \rangle - \beta \mid 101101110 \rangle - \beta \mid 101101111 \rangle$

-$\beta \mid 101100000 \rangle - \beta \mid 101100001 \rangle - \beta \mid 111110010 \rangle - \beta \mid 111110011 \rangle$

$-\beta \mid 101100100 \rangle - \beta \mid 101100101 \rangle - \beta \mid 111110110 \rangle - \beta \mid 111110111 \rangle$

$+\beta \mid 000101000 \rangle + \beta \mid 000101001 \rangle - \beta \mid 010111010 \rangle - \beta \mid 010111011\rangle$

$+\beta \mid 000101100 \rangle + \beta \mid 000101101 \rangle - \beta \mid 010111110 \rangle - \beta \mid 010111111 \rangle$

-$\beta \mid 111110000 \rangle - \beta \mid 111110001 \rangle - \beta \mid 101100010 \rangle - \beta \mid 101100011 \rangle$

$-\beta \mid 111110100 \rangle - \beta \mid 111110101 \rangle - \beta \mid 101100110 \rangle - \beta \mid 101100111 \rangle$

$-\beta \mid 010111000 \rangle - \beta \mid 010111001 \rangle + \beta \mid 000101010 \rangle + \beta \mid 000101011 \rangle$

$-\beta \mid 010111100\rangle - \beta \mid 010111101\rangle + \beta \mid 000101110\rangle + \beta \mid 000101111\rangle$

-$\beta \mid 011000000 \rangle - \beta \mid 011000001 \rangle + \beta \mid 001010010 \rangle + \beta \mid 001010011 \rangle$

$-\beta \mid 011000100 \rangle - \beta \mid 011000101 \rangle + \beta \mid 001010110 \rangle + \beta \mid 001010111 \rangle$

$+\beta \mid 110001000 \rangle + \beta \mid 110001001 \rangle + \beta \mid 100011010 \rangle + \beta \mid 100011011\rangle$

$+\beta \mid 110001100\rangle + \beta \mid 110001101\rangle + \beta \mid 100011110\rangle + \beta \mid 100011111\rangle$

$+\beta \mid 001010000\rangle + \beta \mid 001010001\rangle - \beta \mid 011000010\rangle - \beta \mid 011000011\rangle$

$+\beta \mid 001010100\rangle + \beta \mid 001010101\rangle - \beta \mid 011000110\rangle - \beta \mid 011000111\rangle$

$+\beta \mid 100011000\rangle + \beta \mid 100011001\rangle + \beta \mid 110001010\rangle + \beta \mid 110001011\rangle$

$+\beta \mid 100011100 \rangle + \beta \mid 100011101 \rangle + \beta \mid 110001110 \rangle + \beta \mid 110001111 \rangle\ )$

· Pour $M_3 = Z\,(1)\,I\,(2)\,X\,(3)\,X\,(4)\,X\,(5)$

$\mid \Psi_1\rangle = \frac{1}{8\sqrt{2}}(\ \alpha \mid 000000000\rangle + \alpha \mid 001110001\rangle + \alpha \mid 010010010\rangle + \alpha \mid 011100011\rangle$

$+\alpha \mid 000000100\rangle + \alpha \mid 001110101\rangle + \alpha \mid 010010110\rangle + \alpha \mid 011100111\rangle$

$+\alpha \mid 101001000\rangle - \alpha \mid 100111001\rangle - \alpha \mid 111011010\rangle + \alpha \mid 110101011\rangle$

$+\alpha \mid 101001100\rangle - \alpha \mid 100111101\rangle - \alpha \mid 111011110\rangle + \alpha \mid 110101111\rangle$

$+ \alpha \mid 010010000 \rangle + \alpha \mid 011100001 \rangle + \alpha \mid 000000010 \rangle + \alpha \mid 001110011 \rangle$

$+\alpha \mid 010010100 \rangle + \alpha \mid 011100101 \rangle + \alpha \mid 000000110 \rangle + \alpha \mid 001110111 \rangle$

$-\alpha \mid 111011000 \rangle + \alpha \mid 110101001 \rangle + \alpha \mid 101001010 \rangle - \alpha \mid 100111011 \rangle$

$-\alpha \mid 111011100 \rangle + \alpha \mid 110101101 \rangle + \alpha \mid 101001110 \rangle - \alpha \mid 100111111 \rangle$

$+\alpha \mid 110100000\rangle - \alpha \mid 111010001 \rangle$ -$\alpha \mid 100110010 \rangle + \alpha \mid 101000011 \rangle$

$+\alpha \mid 110100100\rangle - \alpha \mid 111010101\rangle - \alpha \mid 100110110 \rangle + \alpha \mid 101000111 \rangle$

$+\alpha \mid 011101000\rangle + \alpha \mid 010011001 \rangle + \alpha \mid 001111010 \rangle + \alpha \mid 000001011 \rangle$

$+\alpha \mid 011101100\rangle + \alpha \mid 010011101 \rangle + \alpha \mid 001111110 \rangle + \alpha \mid 000001111 \rangle$

-$\alpha \mid 100110000\rangle + \alpha \mid 101000001 \rangle + \alpha \mid 110100010 \rangle - \alpha \mid 111010011 \rangle$

$-\alpha \mid 100110100\rangle + \alpha \mid 101000101 \rangle + \alpha \mid 110100110 \rangle - \alpha \mid 111010111 \rangle$

$+\alpha \mid 001111000 \rangle + \alpha \mid 000001001 \rangle + \alpha \mid 011101010 \rangle + \alpha \mid 010011011 \rangle$

$+\alpha \mid 001111100 \rangle + \alpha \mid 000001101 \rangle + \alpha \mid 011101110 \rangle + \alpha \mid 010011111 \rangle$

$+\alpha \mid 011100000 \rangle + \alpha \mid 010010001 \rangle + \alpha \mid 001110010 \rangle + \alpha \mid 000000011 \rangle$

$+\alpha \mid 011100100 \rangle + \alpha \mid 010010101 \rangle + \alpha \mid 001110110 \rangle + \alpha \mid 000000111 \rangle$

$+\alpha \mid 110101000 \rangle - \alpha \mid 111011001 \rangle - \alpha \mid 100111010 \rangle + \alpha \mid 101001011 \rangle$

$+\alpha \mid 110101100 \rangle - \alpha \mid 111011101 \rangle - \alpha \mid 100111110 \rangle + \alpha \mid 101001111 \rangle$

$+\alpha \mid 001110000 \rangle + \alpha \mid 000000001 \rangle + \alpha \mid 011100010 \rangle + \alpha \mid 010010011 \rangle$

$+\alpha \mid 001110100 \rangle + \alpha \mid 000000101 \rangle + \alpha \mid 011100110 \rangle + \alpha \mid 010010111 \rangle$

$-\alpha \mid 100111000 \rangle + \alpha \mid 101001001 \rangle + \alpha \mid 110101010 \rangle - \alpha \mid 111011011 \rangle$

$-\alpha \mid 100111100 \rangle + \alpha \mid 101001101 \rangle + \alpha \mid 110101110 \rangle - \alpha \mid 111011111 \rangle$

$+\alpha \mid 101000000 \rangle - \alpha \mid 100110001 \rangle$-$\alpha \mid 111010010 \rangle + \alpha \mid 110100011 \rangle$

$+\alpha \mid 101000100 \rangle - \alpha \mid 100110101 \rangle$-$\alpha \mid 111010110 \rangle + \alpha \mid 110100111 \rangle$

$+\alpha \mid 000001000 \rangle + \alpha \mid 001111001 \rangle + \alpha \mid 010011010 \rangle + \alpha \mid 011101011 \rangle$

$+\alpha \mid 000001100 \rangle + \alpha \mid 001111101 \rangle + \alpha \mid 010011110 \rangle + \alpha \mid 011101111 \rangle$

-$\alpha \mid 111010000 \rangle + \alpha \mid 110100001 \rangle + \alpha \mid 101000010 \rangle - \alpha \mid 100110011 \rangle$

-$\alpha \mid 111010100 \rangle + \alpha \mid 110100101 \rangle + \alpha \mid 101000110 \rangle - \alpha \mid 100110111 \rangle$

$+\alpha \mid 010011000 \rangle + \alpha \mid 011101001 \rangle + \alpha \mid 000001010 \rangle + \alpha \mid 001111011 \rangle$

$+\alpha \mid 010011100 \rangle + \alpha \mid 011101101 \rangle + \alpha \mid 000001110 \rangle + \alpha \mid 001111111 \rangle$

$+ \beta \mid 110000000 \rangle - \beta \mid 111110001 \rangle + \beta \mid 100010010 \rangle - \beta \mid 101100011 \rangle$

$+\beta \mid 110000100 \rangle - \beta \mid 111110101 \rangle + \beta \mid 100010110 \rangle - \beta \mid 101100111 \rangle$

$-\beta \mid 011001000 \rangle - \beta \mid 010111001 \rangle + \beta \mid 001011010 \rangle + \beta \mid 000101011 \rangle$

$-\beta \mid 011001100 \rangle - \beta \mid 010111101 \rangle + \beta \mid 001011110 \rangle + \beta \mid 000101111 \rangle$

$+\beta \mid 100010000 \rangle - \beta \mid 101100001 \rangle + \beta \mid 110000010 \rangle - \beta \mid 111110011 \rangle$

$+\beta \mid 100010100 \rangle - \beta \mid 101100101 \rangle + \beta \mid 110000110 \rangle - \beta \mid 111110111 \rangle$

$+\beta \mid 001011000 \rangle + \beta \mid 000101001 \rangle - \beta \mid 011001010 \rangle - \beta \mid 010111011 \rangle$

$+\beta \mid 001011100 \rangle + \beta \mid 000101101 \rangle - \beta \mid 011001110 \rangle - \beta \mid 010111111 \rangle$

$+\beta \mid 000100000 \rangle + \beta \mid 001010001 \rangle - \beta \mid 010110010 \rangle - \beta \mid 011000011 \rangle$

$+\beta \mid 000100100 \rangle + \beta \mid 001010101 \rangle - \beta \mid 010110110 \rangle - \beta \mid 011000111 \rangle$

$-\beta \mid 101101000 \rangle + \beta \mid 100011001 \rangle - \beta \mid 111111010 \rangle + \beta \mid 110001011 \rangle$

$-\beta \mid 101101100 \rangle + \beta \mid 100011101 \rangle - \beta \mid 111111110 \rangle + \beta \mid 110001111 \rangle$

-$\beta \mid 010110000 \rangle \beta \mid 011000001 \rangle + \beta \mid 000100010 \rangle + \beta \mid 001010011 \rangle$

$-\beta \mid 010110100 \rangle - \beta \mid 011000101 \rangle + \beta \mid 000100110 \rangle + \beta \mid 001010111 \rangle$

$-\beta \mid 111111000 \rangle + \beta \mid 110001001 \rangle - \beta \mid 101101010 \rangle + \beta \mid 100011011 \rangle$

$-\beta \mid 111111100 \rangle + \beta \mid 110001101 \rangle - \beta \mid 101101110 \rangle + \beta \mid 100011111 \rangle$

-$\beta \mid 101100000 \rangle + \beta \mid 100010001 \rangle - \beta \mid 111110010 \rangle + \beta \mid 110000011 \rangle$

$-\beta \mid 101100100 \rangle + \beta \mid 100010101 \rangle - \beta \mid 111110110 \rangle + \beta \mid 110000111 \rangle$

$+\beta \mid 000101000 \rangle + \beta \mid 001011001 \rangle - \beta \mid 010111010 \rangle - \beta \mid 011001011\rangle$

$+\beta \mid 000101100 \rangle + \beta \mid 001011101 \rangle - \beta \mid 010111110 \rangle - \beta \mid 011001111 \rangle$

-$\beta \mid 111110000 \rangle + \beta \mid 110000001 \rangle - \beta \mid 101100010 \rangle + \beta \mid 100010011 \rangle$

$-\beta \mid 111110100 \rangle + \beta \mid 110000101 \rangle - \beta \mid 101100110 \rangle + \beta \mid 100010111 \rangle$

$-\beta \mid 010111000 \rangle - \beta \mid 011001001 \rangle + \beta \mid 000101010 \rangle + \beta \mid 001011011 \rangle$

$-\beta \mid 010111100\rangle - \beta \mid 011001101\rangle + \beta \mid 000101110\rangle + \beta \mid 001011111\rangle$

-$\beta \mid 011000000 \rangle - \beta \mid 010110001 \rangle + \beta \mid 001010010 \rangle + \beta \mid 000100011 \rangle$

$-\beta \mid 011000100 \rangle - \beta \mid 010110101 \rangle + \beta \mid 001010110 \rangle + \beta \mid 000100111 \rangle$

$+\beta \mid 110001000 \rangle - \beta \mid 111111001 \rangle + \beta \mid 100011010 \rangle - \beta \mid 101101011\rangle$

$+\beta \mid 110001100\rangle - \beta \mid 111111101\rangle + \beta \mid 100011110\rangle - \beta \mid 101101111\rangle$

$+\beta \mid 001010000\rangle + \beta \mid 000100001\rangle - \beta \mid 011000010\rangle - \beta \mid 010110011\rangle$

$+\beta \mid 001010100\rangle + \beta \mid 000100101\rangle - \beta \mid 011000110\rangle - \beta \mid 010110111\rangle$

$+\beta \mid 100011000\rangle - \beta \mid 101101001\rangle + \beta \mid 110001010\rangle - \beta \mid 111111011\rangle$

$+\beta \mid 100011100 \rangle - \beta \mid 101101101 \rangle + \beta \mid 110001110 \rangle - \beta \mid 111111111 \rangle)$

· <u>L'application de la porte $H\,(6)$:</u>

$\mid \Psi_1\rangle = \frac{1}{16}(\alpha \mid 000000000\rangle + \alpha \mid 000001000\rangle + \alpha \mid 001110001\rangle + \alpha \mid 001111001\rangle$

$+\alpha \mid 010010010\rangle + \alpha \mid 010011010\rangle + \alpha \mid 011100011\rangle + \alpha \mid 011101011\rangle$

$+\alpha \mid 000000100\rangle + \alpha \mid 000001100\rangle + \alpha \mid 001110101\rangle + \alpha \mid 001111101\rangle$

$+\alpha \mid 010010110\rangle + \alpha \mid 010011110\rangle + \alpha \mid 011100111\rangle + \alpha \mid 011101111\rangle$

$+\alpha \mid 101000000\rangle - \alpha \mid 101001000\rangle - \alpha \mid 100110001\rangle + \alpha \mid 100111001\rangle$

$-\alpha \mid 111010010\rangle + \alpha \mid 111011010\rangle + \alpha \mid 110100011\rangle - \alpha \mid 110101011\rangle$

$+\alpha \mid 101000100\rangle - \alpha \mid 101001100\rangle - \alpha \mid 100110101\rangle + \alpha \mid 100111101\rangle$

$-\alpha \mid 111010110\rangle + \alpha \mid 111011110\rangle + \alpha \mid 110100111\rangle - \alpha \mid 110101111\rangle$

$+\alpha \mid 010010000 \rangle + \alpha \mid 010011000 \rangle + \alpha \mid 011100001 \rangle + \alpha \mid 011101001 \rangle$

$+\alpha \mid 000000010 \rangle + \alpha \mid 000001010 \rangle + \alpha \mid 001110011 \rangle + \alpha \mid 001111011 \rangle$

$+\alpha \mid 010010100 \rangle + \alpha \mid 010011100 \rangle + \alpha \mid 011100101 \rangle + \alpha \mid 011101101 \rangle$

$+\alpha \mid 000000110 \rangle + \alpha \mid 000001110 \rangle + \alpha \mid 001110111 \rangle + \alpha \mid 001111111 \rangle$

$-\alpha \mid 111010000 \rangle + \alpha \mid 111011000 \rangle + \alpha \mid 110100001 \rangle - \alpha \mid 110101001 \rangle$

$+\alpha \mid 101000010 \rangle - \alpha \mid 101001010 \rangle - \alpha \mid 100110011 \rangle + \alpha \mid 100111011 \rangle$

$-\alpha \mid 111010100 \rangle + \alpha \mid 111011100 \rangle + \alpha \mid 110100101 \rangle - \alpha \mid 110101101 \rangle$

$+\alpha \mid 101000110 \rangle - \alpha \mid 101001110 \rangle - \alpha \mid 100110111 \rangle + \alpha \mid 100111111 \rangle$

$$
\begin{aligned}
&+\alpha \mid 110100000\rangle + \alpha \mid 110101000\rangle - \alpha \mid 111010001\rangle - \alpha \mid 111011001\rangle \\
&-\alpha \mid 100110010\rangle - \alpha \mid 100111010\rangle + \alpha \mid 101000011\rangle + \alpha \mid 101001011\rangle \\
&+\alpha \mid 110100100\rangle - \alpha \mid 110101100\rangle - \alpha \mid 111010101\rangle + \alpha \mid 111011101\rangle \\
&-\alpha \mid 100110110\rangle + \alpha \mid 100111110\rangle + \alpha \mid 101000111\rangle - \alpha \mid 101001111\rangle \\
&+\alpha \mid 011100000\rangle - \alpha \mid 011101000\rangle + \alpha \mid 010010001\rangle - \alpha \mid 010011001\rangle \\
&+\alpha \mid 001110010\rangle - \alpha \mid 001111010\rangle + \alpha \mid 000000011\rangle - \alpha \mid 000001011\rangle \\
&+\alpha \mid 011100100\rangle - \alpha \mid 011101100\rangle + \alpha \mid 010010101\rangle - \alpha \mid 010011101\rangle \\
&+\alpha \mid 001110110\rangle - \alpha \mid 001111110\rangle + \alpha \mid 000000111\rangle - \alpha \mid 000001111\rangle \\
&-\alpha \mid 100110000\rangle - \alpha \mid 100111000\rangle + \alpha \mid 101000001\rangle + \alpha \mid 101001001\rangle \\
&+\alpha \mid 110100010\rangle + \alpha \mid 110101010\rangle - \alpha \mid 111010011\rangle - \alpha \mid 111011011\rangle \\
&-\alpha \mid 100110100\rangle - \alpha \mid 100111100\rangle + \alpha \mid 101000101\rangle + \alpha \mid 101001101\rangle \\
&+\alpha \mid 110100110\rangle + \alpha \mid 110101110\rangle - \alpha \mid 111010111\rangle - \alpha \mid 111011111\rangle \\
&+\alpha \mid 001110000\rangle - \alpha \mid 001111000\rangle + \alpha \mid 000000001\rangle - \alpha \mid 000001001\rangle \\
&+\alpha \mid 011100010\rangle - \alpha \mid 011101010\rangle + \alpha \mid 010010011\rangle - \alpha \mid 010011011\rangle \\
&+\alpha \mid 001110100\rangle - \alpha \mid 001111100\rangle + \alpha \mid 000000101\rangle - \alpha \mid 000001101\rangle \\
&+\alpha \mid 011100110\rangle - \alpha \mid 011101110\rangle + \alpha \mid 010010111\rangle - \alpha \mid 010011111\rangle \\
&+\alpha \mid 011100000\rangle + \alpha \mid 011101000\rangle + \alpha \mid 010010001\rangle + \alpha \mid 010011001\rangle \\
&+\alpha \mid 001110010\rangle + \alpha \mid 001111010\rangle + \alpha \mid 000000011\rangle + \alpha \mid 000001011\rangle \\
&+\alpha \mid 011100100\rangle + \alpha \mid 011101100\rangle + \alpha \mid 010010101\rangle + \alpha \mid 010011101\rangle \\
&+\alpha \mid 001110110\rangle + \alpha \mid 001111110\rangle + \alpha \mid 000000111\rangle + \alpha \mid 000001111\rangle \\
&+\alpha \mid 110100000\rangle - \alpha \mid 110101000\rangle - \alpha \mid 111010001\rangle + \alpha \mid 111011001\rangle \\
&-\alpha \mid 100110010\rangle + \alpha \mid 100111010\rangle + \alpha \mid 101000011\rangle - \alpha \mid 101001011\rangle \\
&+\alpha \mid 110100100\rangle - \alpha \mid 110101100\rangle - \alpha \mid 111010101\rangle + \alpha \mid 111011101\rangle \\
&-\alpha \mid 100110110\rangle + \alpha \mid 100111110\rangle + \alpha \mid 101000111\rangle - \alpha \mid 101001111\rangle \\
&+\alpha \mid 001110000\rangle + \alpha \mid 001111000\rangle + \alpha \mid 000000001\rangle + \alpha \mid 000001001\rangle \\
&+\alpha \mid 011100010\rangle + \alpha \mid 011101010\rangle + \alpha \mid 010010011\rangle + \alpha \mid 010011011\rangle \\
&+\alpha \mid 001110100\rangle + \alpha \mid 001111100\rangle + \alpha \mid 000000101\rangle + \alpha \mid 000001101\rangle \\
&+\alpha \mid 011100110\rangle + \alpha \mid 011101110\rangle + \alpha \mid 010010111\rangle + \alpha \mid 010011111\rangle \\
&-\alpha \mid 100110000\rangle + \alpha \mid 100111000\rangle + \alpha \mid 101000001\rangle - \alpha \mid 101001001\rangle \\
&+\alpha \mid 110100010\rangle - \alpha \mid 110101010\rangle - \alpha \mid 111010011\rangle + \alpha \mid 111011011\rangle \\
&-\alpha \mid 100110100\rangle - \alpha \mid 100111100\rangle + \alpha \mid 101000101\rangle - \alpha \mid 101001101\rangle \\
&+\alpha \mid 110100110\rangle - \alpha \mid 110101110\rangle - \alpha \mid 111010111\rangle + \alpha \mid 111011111\rangle
\end{aligned}
$$

$+\alpha \mid 101000000\rangle + \alpha \mid 101001000 \rangle - \alpha \mid 100110001\rangle - \alpha \mid 100111001\rangle$

$-\alpha \mid 111010010 \rangle - \alpha \mid 111011010 \rangle + \alpha \mid 110100011 \rangle + \alpha \mid 110101011\rangle$

$+\alpha \mid 101000100 \rangle + \alpha \mid 101001100 \rangle - \alpha \mid 100110101 \rangle - \alpha \mid 100111101\rangle$

$-\alpha \mid 111010110 \rangle - \alpha \mid 111011110 \rangle + \alpha \mid 110100111 \rangle + \alpha \mid 110101111\rangle$

$+\alpha \mid 000000000\rangle - \alpha \mid 000001000 \rangle + \alpha \mid 001110001 \rangle - \alpha \mid 001111001\rangle$

$+\alpha \mid 010010010 \rangle - \alpha \mid 010011010 \rangle + \alpha \mid 011100011 \rangle - \alpha \mid 011101011\rangle$

$+\alpha \mid 000000100 \rangle - \alpha \mid 000001100 \rangle + \alpha \mid 001110101 \rangle - \alpha \mid 001111101\rangle$

$+\alpha \mid 010010110 \rangle - \alpha \mid 010011110 \rangle + \alpha \mid 011100111 \rangle - \alpha \mid 011101111\rangle$

$-\alpha \mid 111010000 \rangle - \alpha \mid 111011000 \rangle + \alpha \mid 110100001 \rangle + \alpha \mid 110101001 \rangle$

$+\alpha \mid 101000010 \rangle + \alpha \mid 101001010 \rangle - \alpha \mid 100110011 \rangle - \alpha \mid 100111011\rangle$

$-\alpha \mid 111010100 \rangle - \alpha \mid 111011100 \rangle + \alpha \mid 110100101 \rangle + \alpha \mid 110101101\rangle$

$+\alpha \mid 101000110 \rangle + \alpha \mid 101001110 \rangle - \alpha \mid 100110111 \rangle - \alpha \mid 100111111\rangle$

$+\alpha \mid 010010000 \rangle - \alpha \mid 010011000 \rangle + \alpha \mid 011100001\rangle - \alpha \mid 011101001\rangle$

$+\alpha \mid 000000010 \rangle - \alpha \mid 000001010 \rangle + \alpha \mid 001110011\rangle - \alpha \mid 001111011\rangle$

$+\alpha \mid 010010100 \rangle - \alpha \mid 010011100 \rangle + \alpha \mid 011100101\rangle - \alpha \mid 011101101\rangle$

$+\alpha \mid 000000110 \rangle - \alpha \mid 000001110 \rangle + \alpha \mid 001110111\rangle - \alpha \mid 001111111\rangle$

$+\beta \mid 110000000\rangle + \beta \mid 110001000 \rangle - \beta \mid 111110001 \rangle - \beta \mid 111111001 \rangle$

$+ \beta \mid 100010010\rangle + \beta \mid 100011010 \rangle - \beta \mid 101100011\rangle - \beta \mid 101101011\rangle$

$+\beta \mid 110000100\rangle + \beta \mid 110001100 \rangle - \beta \mid 111110101 \rangle - \beta \mid 111111101 \rangle$

$+\beta \mid 100010110 \rangle + \beta \mid 100011110 \rangle - \beta \mid 101100111 \rangle - \beta \mid 101101111 \rangle$

$-\beta \mid 011000000 \rangle + \beta \mid 011001000 \rangle - \beta \mid 010110001 \rangle + \beta \mid 010111001 \rangle$

$+\beta \mid 001010010 \rangle - \beta \mid 001011010 \rangle + \beta \mid 000100011 \rangle - \beta \mid 000101011 \rangle$

$-\beta \mid 011000100 \rangle + \beta \mid 011001100 \rangle - \beta \mid 010110101 \rangle + \beta \mid 010111101 \rangle$

$+\beta \mid 001010110 \rangle - \beta \mid 001011110 \rangle + \beta \mid 000100111 \rangle - \beta \mid 000101111 \rangle$

$+\beta \mid 100010000 \rangle + \beta \mid 100011000 \rangle - \beta \mid 101100001 \rangle - \beta \mid 101101001 \rangle$

$+\beta \mid 110000010 \rangle + \beta \mid 110001010 \rangle - \beta \mid 111110011 \rangle - \beta \mid 111111011 \rangle$

$+\beta \mid 100010100 \rangle + \beta \mid 100011100 \rangle - \beta \mid 101100101 \rangle - \beta \mid 101101101 \rangle$

$+\beta \mid 110000110 \rangle + \beta \mid 110001110 \rangle - \beta \mid 111110111 \rangle - \beta \mid 111111111 \rangle$

$+\beta \mid 001010000 \rangle - \beta \mid 001011000 \rangle + \beta \mid 000100001 \rangle - \beta \mid 000101001 \rangle$

$-\beta \mid 011000010 \rangle + \beta \mid 011001010 \rangle - \beta \mid 010110011 \rangle + \beta \mid 010111011 \rangle$

$+\beta \mid 001010100 \rangle - \beta \mid 001011100 \rangle + \beta \mid 000100101 \rangle - \beta \mid 000101101 \rangle$

$-\beta \mid 011000110 \rangle + \beta \mid 011001110 \rangle - \beta \mid 010110111 \rangle + \beta \mid 010111111 \rangle$

$+\beta \mid 000100000\,\rangle + \beta \mid 000101000\,\rangle + \beta \mid 001010001\,\rangle + \beta \mid 001011001\,\rangle$
$-\beta \mid 010110010\,\rangle - \beta \mid 010111010\,\rangle - \beta \mid 011000011\,\rangle - \beta \mid 011001011\,\rangle$
$+\beta \mid 000100100\,\rangle + \beta \mid 000101100\,\rangle + \beta \mid 001010101\,\rangle + \beta \mid 001011101\,\rangle$
$-\beta \mid 010110110\,\rangle - \beta \mid 010111110\,\rangle - \beta \mid 011000111\,\rangle - \beta \mid 011001111\,\rangle$
$-\beta \mid 101100000\,\rangle + \beta \mid 101101000\,\rangle + \beta \mid 100010001\,\rangle - \beta \mid 100011001\,\rangle$
$-\beta \mid 111110010\,\rangle + \beta \mid 111111010\,\rangle + \beta \mid 110000011\,\rangle - \beta \mid 110001011\,\rangle$
$-\beta \mid 101100100\,\rangle + \beta \mid 101101100\,\rangle + \beta \mid 100010101\,\rangle - \beta \mid 100011101\,\rangle$
$-\beta \mid 111110110\,\rangle + \beta \mid 111111110\,\rangle + \beta \mid 110000111\,\rangle - \beta \mid 110001111\,\rangle$
-$\beta \mid 010110000\,\rangle - \beta \mid 010111000\,\rangle - \beta \mid 011000001\,\rangle - \beta \mid 011001001\,\rangle$
$+\beta \mid 000100010\,\rangle + \beta \mid 000101010\,\rangle + \beta \mid 001010011\,\rangle + \beta \mid 001011011\,\rangle$
$-\beta \mid 010110100\,\rangle - \beta \mid 010111100\,\rangle - \beta \mid 011000101\,\rangle - \beta \mid 011001101\,\rangle$
$+\beta \mid 000100110\,\rangle + \beta \mid 000101110\,\rangle + \beta \mid 001010111\,\rangle + \beta \mid 001011111\,\rangle$
$-\beta \mid 111110000\,\rangle + \beta \mid 111111000\,\rangle + \beta \mid 110000001\,\rangle - \beta \mid 110001001\,\rangle$
$-\beta \mid 101100010\,\rangle + \beta \mid 101101010\,\rangle + \beta \mid 100010011\,\rangle - \beta \mid 100011011\,\rangle$
$-\beta \mid 111110100\,\rangle + \beta \mid 111111100\,\rangle + \beta \mid 110000101\,\rangle - \beta \mid 110001101\,\rangle$
$-\beta \mid 101100110\,\rangle + \beta \mid 101101110\,\rangle + \beta \mid 100010111\,\rangle - \beta \mid 100011111\,\rangle$
-$\beta \mid 101100000\,\rangle - \beta \mid 101101000\,\rangle + \beta \mid 100010001\,\rangle + \beta \mid 100011001\,\rangle$
$-\beta \mid 111110010\,\rangle - \beta \mid 111111010\,\rangle + \beta \mid 110000011\,\rangle + \beta \mid 110001011\,\rangle$
$-\beta \mid 101100100\,\rangle - \beta \mid 101101100\,\rangle + \beta \mid 100010101\,\rangle + \beta \mid 100011101\,\rangle$
$-\beta \mid 111110110\,\rangle - \beta \mid 111111110\,\rangle + \beta \mid 110000111\,\rangle + \beta \mid 110001111\,\rangle$
$+\beta \mid 000100000\,\rangle - \beta \mid 000101000\,\rangle + \beta \mid 001010001\,\rangle - \beta \mid 001011001\,\rangle$
$-\beta \mid 010110010\,\rangle + \beta \mid 010111010\,\rangle - \beta \mid 011000011\rangle + \beta \mid 011001011\rangle$
$+\beta \mid 000100100\,\rangle - \beta \mid 000101100\,\rangle + \beta \mid 001010101\,\rangle - \beta \mid 001011101\,\rangle$
$-\beta \mid 010110110\,\rangle + \beta \mid 010111110\,\rangle - \beta \mid 011000111\,\rangle + \beta \mid 011001111\,\rangle$
-$\beta \mid 111110000\,\rangle - \beta \mid 111111000\,\rangle + \beta \mid 110000001\,\rangle + \beta \mid 110001001\,\rangle$
$-\beta \mid 101100010\,\rangle - \beta \mid 101101010\,\rangle + \beta \mid 100010011\,\rangle + \beta \mid 100011011\,\rangle$
$-\beta \mid 111110100\,\rangle - \beta \mid 111111100\,\rangle + \beta \mid 110000101\,\rangle + \beta \mid 110001101\,\rangle$
$-\beta \mid 101100110\,\rangle - \beta \mid 101101110\,\rangle + \beta \mid 100010111\,\rangle + \beta \mid 100011111\,\rangle$
$-\beta \mid 010110000\,\rangle + \beta \mid 010111000\,\rangle - \beta \mid 011000001\,\rangle + \beta \mid 011001001\,\rangle$
$+\beta \mid 000100010\,\rangle - \beta \mid 000101010\,\rangle + \beta \mid 001010011\,\rangle - \beta \mid 001011011\,\rangle$
$-\beta \mid \ 010110100\rangle + \beta \mid \ 010111100\rangle - \beta \mid \ 011000101\rangle + \beta \mid \ 011001101\rangle$
$+\beta \mid \ 000100110\rangle - \beta \mid \ 000101110\rangle + \beta \mid \ 001010111\rangle - \beta \mid \ 001011111\rangle$

$-\beta \mid 011000000 \rangle - \beta \mid 011001000 \rangle - \beta \mid 010110001 \rangle - \beta \mid 010111001 \rangle$
$+\beta \mid 001010010 \rangle + \beta \mid 001011010 \rangle + \beta \mid 000100011 \rangle + \beta \mid 000101011 \rangle$
$-\beta \mid 011000100 \rangle - \beta \mid 011001100 \rangle - \beta \mid 010110101 \rangle - \beta \mid 010111101 \rangle$
$+\beta \mid 001010110 \rangle + \beta \mid 001011110 \rangle + \beta \mid 000100111 \rangle + \beta \mid 000101111 \rangle$
$+\beta \mid 110000000 \rangle - \beta \mid 110001000 \rangle - \beta \mid 111110001\rangle + \beta \mid 111111001\rangle$
$+\beta \mid 100010010 \rangle - \beta \mid 100011010 \rangle - \beta \mid 101100011\rangle + \beta \mid 101101011\rangle$
$+\beta \mid 110000100\rangle - \beta \mid 110001100\rangle - \beta \mid 111110101\rangle + \beta \mid 111111101\rangle$
$+\beta \mid 100010110\rangle - \beta \mid 100011110\rangle - \beta \mid 101100111\rangle + \beta \mid 101101111\rangle$
$+\beta \mid 001010000\rangle + \beta \mid 001011000\rangle + \beta \mid 000100001\rangle + \beta \mid 000101001\rangle$
$-\beta \mid 011000010\rangle - \beta \mid 011001010\rangle - \beta \mid 010110011\rangle - \beta \mid 010111011\rangle$
$+\beta \mid 001010100\rangle + \beta \mid 001011100\rangle + \beta \mid 000100101\rangle + \beta \mid 000101101\rangle$
$-\beta \mid 011000110\rangle - \beta \mid 011001110\rangle - \beta \mid 010110111\rangle - \beta \mid 010111111\rangle$
$+\beta \mid 100010000\rangle - \beta \mid 100011000\rangle - \beta \mid 101100001\rangle + \beta \mid 101101001\rangle$
$+\beta \mid 110000010\rangle - \beta \mid 110001010\rangle - \beta \mid 111110011\rangle + \beta \mid 111111011\rangle$
$+\beta \mid 100010100 \rangle - \beta \mid 100011100 \rangle - \beta \mid 101100101 \rangle + \beta \mid 101101101 \rangle$
$+\beta \mid 110000110 \rangle - \beta \mid 110001110 \rangle - \beta \mid 111110111 \rangle + \beta \mid 111111111 \rangle)$

· L'application de la porte H (7)

$\mid \Psi_1\rangle = \frac{1}{8\sqrt{2}}(\alpha \mid 000000000\rangle + \alpha \mid 000000100\rangle + \alpha \mid 001110001\rangle + \alpha \mid 001110101\rangle$
$+\alpha \mid 010010010\rangle + \alpha \mid 010010110\rangle + \alpha \mid 011100011\rangle + \alpha \mid 011100111\rangle$
$+\alpha \mid 000000000\rangle - \alpha \mid 000000100\rangle + \alpha \mid 001110001\rangle - \alpha \mid 001110101\rangle$
$+\alpha \mid 010010010\rangle - \alpha \mid 010010110\rangle + \alpha \mid 011100011\rangle - \alpha \mid 011100111\rangle$
$+\alpha \mid 101000000\rangle + \alpha \mid 101000100\rangle - \alpha \mid 100110001\rangle + \alpha \mid 100110101\rangle$
$-\alpha \mid 111010010\rangle - \alpha \mid 111010110\rangle + \alpha \mid 110100011\rangle - \alpha \mid 11010111\rangle$
$+\alpha \mid 101000000\rangle - \alpha \mid 101000100\rangle - \alpha \mid 100110001\rangle + \alpha \mid 100110101\rangle$
$-\alpha \mid 111010010\rangle + \alpha \mid 111010110\rangle + \alpha \mid 110100011\rangle - \alpha \mid 110100111\rangle$
$+\alpha \mid 010010000 \rangle + \alpha \mid 010010100 \rangle + \alpha \mid 011100001 \rangle + \alpha \mid 011100101 \rangle$
$+\alpha \mid 000000010 \rangle + \alpha \mid 000000110 \rangle + \alpha \mid 001110011 \rangle + \alpha \mid 001110111 \rangle$
$+\alpha \mid 010010000 \rangle - \alpha \mid 010010100 \rangle + \alpha \mid 011100001 \rangle - \alpha \mid 011100101 \rangle$
$+\alpha \mid 000000010 \rangle - \alpha \mid 000000110 \rangle + \alpha \mid 001110011 \rangle - \alpha \mid 001110111 \rangle$
$-\alpha \mid 111010000 \rangle - \alpha \mid 111010100 \rangle + \alpha \mid 110100001 \rangle - \alpha \mid 110100101 \rangle$
$+\alpha \mid 101000010 \rangle + \alpha \mid 101000110 \rangle - \alpha \mid 100110011 \rangle + \alpha \mid 100110111 \rangle$
$-\alpha \mid 111010000 \rangle + \alpha \mid 111010100 \rangle + \alpha \mid 110100001 \rangle - \alpha \mid 110100101 \rangle$

$+\alpha \mid 101000010 \rangle - \alpha \mid 101000110 \rangle - \alpha \mid 100110011 \rangle + \alpha \mid 100110111 \rangle$
$+\alpha \mid 110100000\rangle + \alpha \mid 110100100\rangle - \alpha \mid 111010001 \rangle - \alpha \mid 111010101 \rangle$
-$\alpha \mid 100110010 \rangle - \alpha \mid 100110110 \rangle + \alpha \mid 101000011 \rangle + \alpha \mid 101000111 \rangle$
$+\alpha \mid 110100000\rangle - \alpha \mid 110100100\rangle - \alpha \mid 111010001\rangle + \alpha \mid 111010101\rangle$
$-\alpha \mid 100110010 \rangle + \alpha \mid 100110110 \rangle + \alpha \mid 101000011 \rangle - \alpha \mid 101000111 \rangle$
$+\alpha \mid 011100000\rangle + \alpha \mid 011100100\rangle - \alpha \mid 010010001 \rangle - \alpha \mid 010010101 \rangle$
$+\alpha \mid 001110010\rangle + \alpha \mid 001110110\rangle + \alpha \mid 000000011\rangle - \alpha \mid 000000111 \rangle$
$+\alpha \mid 011100000\rangle - \alpha \mid 011100100\rangle + \alpha \mid 010010001 \rangle - \alpha \mid 010010101 \rangle$
$+\alpha \mid 001110010\rangle - \alpha \mid 001110110\rangle + \alpha \mid 000000011 \rangle - \alpha \mid 000000111 \rangle$
- $\alpha \mid 100110000\rangle - \alpha \mid 100110100\rangle + \alpha \mid 101000001 \rangle + \alpha \mid 101000101 \rangle$
$+\alpha \mid 110100010\rangle + \alpha \mid 110100110\rangle - \alpha \mid 111010011 \rangle - \alpha \mid 111010111 \rangle$
$-\alpha \mid 100110000\rangle + \alpha \mid 100110100\rangle + \alpha \mid 101000001 \rangle + \alpha \mid 101000101 \rangle$
$+\alpha \mid 110100010\rangle - \alpha \mid 110100110\rangle - \alpha \mid 111010011 \rangle - \alpha \mid 111010111 \rangle$
$+\alpha \mid 001110000\rangle + \alpha \mid 001110100\rangle + \alpha \mid 000000001 \rangle - \alpha \mid 000000101 \rangle$
$+\alpha \mid 011100010\rangle + \alpha \mid 011100110\rangle + \alpha \mid 010010011 \rangle - \alpha \mid 010010111 \rangle$
$+\alpha \mid 001110000\rangle - \alpha \mid 001110100\rangle + \alpha \mid 000000001 \rangle - \alpha \mid 000000101 \rangle$
$+\alpha \mid 011100010\rangle - \alpha \mid 011100110\rangle + \alpha \mid 010010011 \rangle - \alpha \mid 010010111 \rangle$
$+\beta \mid 110000000\rangle + \beta \mid 110000100\rangle - \beta \mid 111110001 \rangle - \beta \mid 111110101 \rangle$
$+ \beta \mid 100010010\rangle + \beta \mid 100010110\rangle - \beta \mid 101100011\rangle - \beta \mid 101100111\rangle$
$+\beta \mid 110000000\rangle - \beta \mid 110000100\rangle - \beta \mid 111110001 \rangle + \beta \mid 111110101 \rangle$
$+\beta \mid 100010010 \rangle - \beta \mid 100010110 \rangle - \beta \mid 101100011 \rangle + \beta \mid 101100111 \rangle$
$-\beta \mid 011000000 \rangle - \beta \mid 011000100 \rangle - \beta \mid 010110001 \rangle + \beta \mid 010110101 \rangle$
$+\beta \mid 001010010 \rangle + \beta \mid 001010110 \rangle + \beta \mid 000100011 \rangle - \beta \mid 000100111 \rangle$
$-\beta \mid 011000000 \rangle + \beta \mid 011000100 \rangle - \beta \mid 010110001 \rangle + \beta \mid 010110101 \rangle$
$+\beta \mid 001010010 \rangle - \beta \mid 001010110 \rangle + \beta \mid 000100011 \rangle - \beta \mid 000100111 \rangle$
$+\beta \mid 100010000 \rangle + \beta \mid 100010100 \rangle - \beta \mid 101100001 \rangle - \beta \mid 101100101 \rangle$
$+\beta \mid 110000010 \rangle + \beta \mid 110000110 \rangle - \beta \mid 111110011 \rangle - \beta \mid 11111111 \rangle$
$+\beta \mid 100010000 \rangle - \beta \mid 100010100 \rangle - \beta \mid 101100001 \rangle + \beta \mid 101100101 \rangle$
$+\beta \mid 110000010 \rangle - \beta \mid 110000110 \rangle - \beta \mid 111110011 \rangle + \beta \mid 111110111 \rangle$
$+\beta \mid 001010000 \rangle + \beta \mid 001010100 \rangle + \beta \mid 000100001 \rangle - \beta \mid 000100101 \rangle$
$-\beta \mid 011000010 \rangle - \beta \mid 011000110 \rangle - \beta \mid 010110011 \rangle + \beta \mid 010110111 \rangle$
$+\beta \mid 001010000 \rangle - \beta \mid 001010100 \rangle + \beta \mid 000100001 \rangle - \beta \mid 000100101 \rangle$

$-\beta \mid 011000010 \rangle + \beta \mid 011000110 \rangle \; - \beta \mid 010110011 \rangle + \beta \mid 010110111 \rangle$

$+\beta \mid 000100000 \rangle + \beta \mid 000100100 \rangle \; + \beta \mid 001010001 \rangle + \beta \mid 001010101 \rangle$

$-\beta \mid 010110010 \rangle \; - \beta \mid 010110110 \rangle \; - \beta \mid 011000011 \rangle - \beta \mid 011000111 \rangle$

$+\beta \mid 000100000 \rangle - \beta \mid 000100100 \rangle \; + \beta \mid 001010001 \rangle - \beta \mid 001010101 \rangle$

$-\beta \mid 010110010 \rangle \; + \beta \mid 010110110 \rangle \; - \beta \mid 011000011 \rangle + \beta \mid 011000111 \rangle$

$-\beta \mid 101100000 \rangle - \beta \mid 101100100 \rangle \; + \beta \mid 100010001 \rangle - \beta \mid 100010101 \rangle$

$-\beta \mid 111110010 \rangle \; - \beta \mid 111110110 \rangle \; + \beta \mid 110000011 \rangle - \beta \mid 110000111 \rangle$

$-\beta \mid 101100000 \rangle \; + \beta \mid 101100100 \rangle \; + \beta \mid 100010001 \rangle - \beta \mid 100010101 \rangle$

$-\beta \mid 111110010 \rangle + \beta \mid 111110110 \rangle \; + \beta \mid 110000011 \rangle - \beta \mid 110000111 \rangle$

$-\beta \mid 010110000 \rangle - \beta \mid 010110100 \rangle \; - \beta \mid 011000001 \rangle - \beta \mid 011000101 \rangle$

$+\beta \mid 000100010 \rangle + \beta \mid 000100110 \rangle \; + \beta \mid 001010011 \rangle + \beta \mid 001010111 \rangle$

$-\beta \mid 010110000 \rangle \; + \beta \mid 010110100 \rangle \; - \beta \mid 011000001 \rangle + \beta \mid 011000101 \rangle$

$+\beta \mid 000100010 \rangle \; - \beta \mid 000100110 \rangle \; + \beta \mid 001010011 \rangle - \beta \mid 001010111 \rangle$

$-\beta \mid 111110000 \rangle - \beta \mid 111110100 \rangle \; + \beta \mid 110000001 \rangle - \beta \mid 110000101 \rangle$

$-\beta \mid 101100010 \rangle \; - \beta \mid 101100110 \rangle \; + \beta \mid 100010011 \rangle \; \beta \mid 100010111 \rangle$

$-\beta \mid 111110000 \rangle + \beta \mid 111110100 \rangle \; + \beta \mid 110000001 \rangle \; - \beta \mid 110000101 \rangle$

$-\beta \mid 101100010 \rangle \; + \beta \mid 101100110 \rangle \; + \beta \mid 100010011 \rangle - \beta \mid 100010111 \rangle \;)$

· <u>L'application de la porte $H\,(8)$</u>

$\mid \Psi_1 \rangle = \frac{1}{8}(\; \alpha \mid 000000000 \rangle + \; \alpha \mid 000000010 \rangle \; + \alpha \mid 001110001 \rangle + \alpha \mid 001110011 \rangle$

$+\alpha \mid 010010000 \rangle - \alpha \mid 010010010 \rangle \; + \alpha \mid 011100001 \rangle - \alpha \mid 011100011 \rangle$

$+\alpha \mid 101000000 \rangle \; + \alpha \mid 101000010 \rangle \; - \alpha \mid 100110001 \rangle + \alpha \mid 100110011 \rangle$

$-\alpha \mid 111010000 \rangle + \alpha \mid 111010010 \rangle \; + \alpha \mid 110100001 \rangle - \alpha \mid 11010011 \rangle$

$+\alpha \mid 010010000 \rangle + \alpha \mid 010010010 \rangle \; + \alpha \mid 011100001 \rangle + \alpha \mid 011100011 \rangle$

$+\alpha \mid 000000000 \rangle \; - \alpha \mid 000000010 \rangle \; + \alpha \mid 001110001 \rangle - \alpha \mid 001110011 \rangle$

$-\alpha \mid 111010000 \rangle - \alpha \mid 111010010 \rangle \; + \alpha \mid 110100001 \rangle - \alpha \mid 110100011 \rangle$

$+\alpha \mid 101000000 \rangle - \alpha \mid 101000010 \rangle \; - \alpha \mid 100110001 \rangle + \alpha \mid 100110011 \rangle$

$+\alpha \mid 110100000 \rangle \; + \alpha \mid 110100010 \rangle \; - \alpha \mid \; 111010001 \rangle - \alpha \mid \; 111010011 \rangle$

$-\alpha \mid \; 100110000 \rangle \; + \alpha \mid \; 100110010 \rangle \; + \alpha \mid \; 101000001 \rangle - \alpha \mid \; 101000011 \rangle$

$+\alpha \mid 011100000 \rangle \; + \alpha \mid 011100010 \rangle \; + \alpha \mid 010010001 \; \rangle + \alpha \mid 010010011 \; \rangle$

$+\alpha \mid 001110000 \rangle - \alpha \mid 001110010 \rangle \; + \alpha \mid 000000001 \rangle - \alpha \mid 000000011 \; \rangle$

$- \; \alpha \mid 100110000 \rangle - \alpha \mid 100110010 \rangle \; + \alpha \mid 101000001 \; \rangle + \alpha \mid 101000011 \; \rangle$

$+\alpha \mid 110100000 \rangle \; - \alpha \mid 110100010 \rangle \; - \alpha \mid 111010001 \; \rangle + \alpha \mid 111010011 \; \rangle$

$+\alpha \mid 001110000\rangle + \alpha \mid 001110010\rangle + \alpha \mid 000000001 \rangle - \alpha \mid 000000011 \rangle$
$+\alpha \mid 011100000\rangle - \alpha \mid 011100010\rangle + \alpha \mid 010010001 \rangle - \alpha \mid 010010011 \rangle$
$+\beta \mid 110000000\rangle + \beta \mid 110000010\rangle - \beta \mid 111110001 \rangle - \beta \mid 111110011 \rangle$
$+ \beta \mid 100010000\rangle - \beta \mid 100010010\rangle - \beta \mid 101100001\rangle + \beta \mid 101100011\rangle$
$-\beta \mid 011000000 \rangle - \beta \mid 011000010 \rangle - \beta \mid 010110001 \rangle + \beta \mid 010110011 \rangle$
$+\beta \mid 001010000 \rangle - \beta \mid 001010010 \rangle + \beta \mid 000100001 \rangle - \beta \mid 000100011 \rangle$
$+\beta \mid 100010000 \rangle + \beta \mid 100010010 \rangle - \beta \mid 101100001 \rangle - \beta \mid 101100011 \rangle$
$+\beta \mid 110000000 \rangle - \beta \mid 110000010 \rangle - \beta \mid 111110001 \rangle + \beta \mid 11111011 \rangle$
$+\beta \mid 001010000 \rangle + \beta \mid 001010010 \rangle + \beta \mid 000100001 \rangle - \beta \mid 000100011 \rangle$
$-\beta \mid 011000000 \rangle + \beta \mid 011000010 \rangle - \beta \mid 010110001 \rangle + \beta \mid 010110011 \rangle$
$+\beta \mid 000100000 \rangle + \beta \mid 000100010 \rangle + \beta \mid 001010001 \rangle + \beta \mid 001010011 \rangle$
$-\beta \mid 010110000 \rangle + \beta \mid 010110010 \rangle - \beta \mid 011000001 \rangle + \beta \mid 011000011 \rangle$
$-\beta \mid 101100000 \rangle - \beta \mid 101100010 \rangle + \beta \mid 100010001 \rangle - \beta \mid 100010011 \rangle$
$-\beta \mid 111110000 \rangle + \beta \mid 111110010 \rangle + \beta \mid 110000001 \rangle - \beta \mid 110000011 \rangle$
-$\beta \mid 010110000 \rangle - \beta \mid 010110010 \rangle - \beta \mid 011000001 \rangle - \beta \mid 011000011 \rangle$
$+\beta \mid 000100000 \rangle - \beta \mid 000100010 \rangle + \beta \mid 001010001 \rangle - \beta \mid 001010011 \rangle$
$-\beta \mid 111110000 \rangle - \beta \mid 111110010 \rangle + \beta \mid 110000001 \rangle - \beta \mid 110000011 \rangle$
$-\beta \mid 101100000 \rangle + \beta \mid 101100010 \rangle + \beta \mid 100010001 \rangle - \beta \mid 100010011 \rangle)$

· L'application de la porte $H\,(9)$

$\mid \Psi_1\rangle = \frac{1}{4\sqrt{2}}(\alpha \mid 000000000\rangle + \alpha \mid 000000001\rangle + \alpha \mid 001110000\rangle - \alpha \mid 001110001\rangle$
$+\alpha \mid 010010000\rangle + \alpha \mid 010010001\rangle + \alpha \mid 011100000\rangle - \alpha \mid 011100001\rangle$
$+\alpha \mid 101000000\rangle + \alpha \mid 101000001\rangle - \alpha \mid 100110000\rangle + \alpha \mid 100110001\rangle$
$-\alpha \mid 111010000\rangle - \alpha \mid 111010001\rangle + \alpha \mid 110100000\rangle - \alpha \mid 11010001\rangle$
$+\alpha \mid 110100000\rangle + \alpha \mid 110100001\rangle - \alpha \mid 111010000 \rangle + \alpha \mid 111010001 \rangle$
-$\alpha \mid 100110000 \rangle - \alpha \mid 100110001 \rangle + \alpha \mid 101000000 \rangle - \alpha \mid 101000001 \rangle$
$+\alpha \mid 011100000\rangle + \alpha \mid 011100001\rangle + \alpha \mid 010010000 \rangle - \alpha \mid 010010001 \rangle$
$+\alpha \mid 001110000\rangle + \alpha \mid 001110001\rangle + \alpha \mid 000000000\rangle - \alpha \mid 000000001 \rangle$
$+\beta \mid 110000000\rangle + \beta \mid 110000001\rangle - \beta \mid 111110000 \rangle + \beta \mid 111110001 \rangle$
$+ \beta \mid 100010000\rangle + \beta \mid 100010001\rangle - \beta \mid 101100000\rangle + \beta \mid 101100001\rangle$
$-\beta \mid 011000000 \rangle - \beta \mid 011000001 \rangle - \beta \mid 010110000 \rangle + \beta \mid 010110001 \rangle$
$+\beta \mid 001010000 \rangle + \beta \mid 001010001 \rangle + \beta \mid 000100000 \rangle - \beta \mid 000100001 \rangle$
$+\beta \mid 000100000 \rangle + \beta \mid 000100001 \rangle + \beta \mid 001010000\rangle - \beta \mid 001010001 \rangle$

$-\beta \mid 010110000\ \rangle\ -\beta \mid 010110001\rangle\ -\beta \mid 011000000\ \rangle + \beta \mid 0110000001\rangle$

$-\beta \mid 101100000\ \rangle - \beta \mid 101100001\rangle\ +\beta \mid 100010000\rangle - \beta \mid 100010001\ \rangle$

$-\beta \mid 111110000\ \rangle\ -\beta \mid 111110001\rangle\ +\beta \mid 110000000\ \rangle - \beta \mid 110000001\ \rangle\ )$

Après toutes les simplifications, on aura :

$\mid \Psi_1\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 000000000\rangle + \alpha \mid 001110000\rangle + \alpha \mid 010010000\rangle + \alpha \mid 011100000\rangle$
$+\alpha \mid 101000000\rangle - \alpha \mid 100110000\rangle - \alpha \mid 111010000\rangle + \alpha \mid 110100000\rangle + \beta \mid 110000000\rangle$
$-\ \beta \mid 111110000\rangle + \beta \mid 100010000\rangle - \beta \mid 101100000\rangle - \beta \mid 011000000\rangle - \beta \mid 010110000\rangle$
$+\beta \mid 001010000\rangle + \beta \mid 000100000\rangle)$

*<u>**Correction d'Erreur :**</u>

Résultat de mesure=0000, donc sans Erreur

$\mid \Psi_1\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 000000000\rangle + \alpha \mid 001110000\rangle + \alpha \mid 010010000\rangle + \alpha \mid 011100000\rangle$
$+\alpha \mid 101000000\rangle - \alpha \mid 100110000\rangle - \alpha \mid 111010000\rangle + \alpha \mid 110100000\rangle$
$+\beta \mid 110000000\rangle - \ \beta \mid 111110000\rangle + \beta \mid 100010000\rangle - \ \beta \mid 101100000\rangle$
$-\beta \mid 011000000\rangle - \beta \mid 010110000\rangle + \beta \mid 001010000\rangle + \beta \mid 000100000\rangle)$

○ <u>**Suppression des quatre qubits du syndrome :**</u>

$$\mid \Psi_1\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 00111\rangle + \alpha \mid 01001\rangle + \alpha \mid 01110\rangle + \alpha \mid 10100\rangle - \alpha \mid 10011\rangle$$
$$-\alpha \mid 11101\rangle + \alpha \mid 11010\rangle + \beta \mid 11000\rangle - \ \beta \mid 11111\rangle + \ \beta \mid 10001\rangle - \ \beta \mid 10110\rangle - \beta \mid 01100\rangle$$
$$-\beta \mid 01011\rangle + \beta \mid 00101\rangle + \beta \mid 00010\rangle)$$

*<u>**Décodage :**</u>

Application de circuit de décodage :

· <u>L'application de la porte $CNOT\,(5,2)$</u>:

$$\mid \Psi_1\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 01111\rangle + \alpha \mid 00001\rangle + \alpha \mid 01110\rangle + \alpha \mid 10100\rangle - \alpha \mid 11011\rangle$$
$$-\alpha \mid 10101\rangle + \alpha \mid 11010\rangle + \beta \mid 11000\rangle - \ \beta \mid 10111\rangle + \ \beta \mid 11001\rangle - \ \beta \mid 10110\rangle - \beta \mid 01100\rangle$$
$$-\beta \mid 00011\rangle + \beta \mid 01101\rangle + \beta \mid 00010\rangle)$$

· <u>L'application de la porte $H\,(5)$</u>:

$\mid \Psi_1\rangle = \frac{1}{4}(\alpha \mid 00000\rangle + \alpha \mid 00001\rangle + \alpha \mid 01110\rangle - \alpha \mid 01111\rangle + \alpha \mid 00000\rangle - \alpha \mid 00001\rangle$
$+\alpha \mid 01110\rangle + \alpha \mid 01111\rangle + \alpha \mid 10100\rangle + \alpha \mid 10101\rangle - \alpha \mid 11010\rangle + \alpha \mid 11011\rangle - \alpha \mid 10100\rangle$

$+\alpha \mid 10101\rangle + \alpha \mid 11010\rangle + \alpha \mid 11011\rangle + \beta \mid 11000\rangle + \beta \mid 11001\rangle - \beta \mid 10110\rangle + \beta \mid 10111\rangle$
$+ \beta \mid 11000\rangle - \beta \mid 11001\rangle - \beta \mid 10110\rangle - \beta \mid 10111\rangle - \beta \mid 01100\rangle - \beta \mid 01101\rangle - \beta \mid 00010\rangle$
$+\beta \mid 00011\rangle + \beta \mid 01100\rangle - \beta \mid 01101\rangle + \beta \mid 00010\rangle + \beta \mid 00011\rangle)$

Aprés la simplification, on aura :

$$\mid \Psi_1\rangle = \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 01110\rangle + \alpha \mid 10101\rangle + \alpha \mid 11011\rangle + \beta \mid 11000\rangle - \beta \mid 10110\rangle - \beta \mid 01101\rangle + \beta \mid 00011\rangle)$$

· L'application de la porte $CNOT\,(1,2)$:

$$\mid \Psi_1\rangle = \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 01110\rangle + \alpha \mid 11101\rangle + \alpha \mid 10011\rangle + \beta \mid 10000\rangle - \beta \mid 11110\rangle - \beta \mid 01101\rangle + \beta \mid 00011\rangle)$$

· L'application de la porte $CNOT\,(3,4)$:

$$\mid \Psi_1\rangle = \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 01100\rangle + \alpha \mid 11111\rangle + \alpha \mid 10011\rangle + \beta \mid 10000\rangle - \beta \mid 11100\rangle - \beta \mid 01111\rangle + \beta \mid 00011\rangle)$$

· L'application de la porte $CNOT\,(4,5)$:

$$\mid \Psi_1\rangle = \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 01100\rangle + \alpha \mid 11110\rangle + \alpha \mid 10010\rangle + \beta \mid 10000\rangle - \beta \mid 11100\rangle - \beta \mid 01110\rangle + \beta \mid 00010\rangle)$$

· L'application de la porte $CNOT\,(4,1)$:

$$\mid \Psi_1\rangle = \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 01100\rangle + \alpha \mid 01110\rangle + \alpha \mid 00010\rangle + \beta \mid 10000\rangle - \beta \mid 11100\rangle - \beta \mid 11110\rangle + \beta \mid 10010\rangle)$$

· L'application de la porte $H\,(4)$:

$$\mid \Psi_1\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 00010\rangle + \alpha \mid 01100\rangle + \alpha \mid 01110\rangle + \alpha \mid 01100\rangle - \alpha \mid 01110\rangle + \alpha \mid 00000\rangle - \alpha \mid 00010\rangle + \beta \mid 10000\rangle + \beta \mid 10010\rangle - \beta \mid 11100\rangle - \beta \mid 11110\rangle - \beta \mid 11100\rangle + \beta \mid 11110\rangle + \beta \mid 10000\rangle - \beta \mid 10010\rangle)$$

Aprés la simplification :

$$\mid \Psi_1\rangle = \frac{1}{\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 01100\rangle + \beta \mid 10000\rangle - \beta \mid 11100\rangle)$$

· L'application de la porte $CNOT\,(2,3)$:

$$\mid \Psi_1\rangle = \frac{1}{\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 01000\rangle + \beta \mid 10000\rangle - \beta \mid 11000\rangle)$$

· L'application de la porte $H\,(2)$:

$$\mid \Psi_1\rangle = \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 01000\rangle + \alpha \mid 00000\rangle - \alpha \mid 01000\rangle + \beta \mid 10000\rangle + \beta \mid 11000\rangle - \beta \mid 10000\rangle + \beta \mid 11000\rangle)$$

Aprés la simplification, on aura :

$$\mid \Psi_1\rangle = \alpha \mid 00000\rangle + \beta \mid 11000\rangle$$

· L'application de la porte $CNOT\,(1,2)$:

$$\mid \Psi_1\rangle = \alpha \mid 00000\rangle + \beta \mid 10000\rangle$$

◦ **Suppression des qubits auxiliaires :**

$$\mid \Psi\rangle = \alpha \mid 0\rangle + \beta \mid 1\rangle$$

∗**Cas 02 : Erreur de type X**

• **Injecter une erreure X sur le premier qubit** :

$$\mid \Psi_2\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 10000\rangle + \alpha \mid 11001\rangle + \alpha \mid 01010\rangle - \alpha \mid 00011\rangle + \alpha \mid 11110\rangle + \alpha \mid 10111\rangle + \alpha \mid 00100\rangle - \alpha \mid 01101\rangle + \beta \mid 01000\rangle + \beta \mid 00001\rangle + \beta \mid 10010\rangle - \beta \mid 11011\rangle - \beta \mid 00110\rangle - \beta \mid 01111\rangle - \beta \mid 11100\rangle + \beta \mid 10101\rangle)$$

*__Mesure du syndrome :__

Ajout des quatre qubits du syndrome :

$|\ \Psi_2\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 100000000\rangle + \alpha \mid 110010000\rangle + \alpha \mid 010100000\rangle - \alpha \mid 000110000\rangle$
$+\alpha \mid 111100000\rangle + \alpha \mid 101110000\rangle + \alpha \mid 001000000\rangle - \alpha \mid 011010000\rangle + \beta \mid 010000000\rangle$
$+ \beta \mid 000010000\rangle + \beta \mid 100100000\rangle - \beta \mid 110110000\rangle - \beta \mid 001100000\rangle - \beta \mid 011110000\rangle$
$-\beta \mid 111000000\rangle + \beta \mid 101010000\rangle)$

·L'application de la porte $H(6)$:

·L'application de la porte $H(7)$:

·L'application de la porte $H(8)$:

·L'application de la porte $H(9)$:

Aprés ces applications, on obtient a :

$|\ \Psi_2\rangle = \frac{1}{8\sqrt{2}}(\alpha \mid 100000000\rangle + \alpha \mid 100000001\rangle + \alpha \mid 100000010\rangle + \alpha \mid 100000011\rangle$
$+\alpha \mid 100000100\rangle + \alpha \mid 100000101\rangle + \alpha \mid 100000110\rangle + \alpha \mid 100000111\rangle$
$+\alpha \mid 100001000\rangle + \alpha \mid 100001001\rangle + \alpha \mid 100001010\rangle + \alpha \mid 100001011\rangle$
$+\alpha \mid 100001100\rangle + \alpha \mid 100001101\rangle + \alpha \mid 100001110\rangle + \alpha \mid 100001111\rangle$
$+\alpha \mid 110010000\rangle + \alpha \mid 110010001\rangle + \alpha \mid 110010010\rangle + \alpha \mid 110010011\rangle$
$+\alpha \mid 110010100\rangle + \alpha \mid 110010101\rangle + \alpha \mid 110010110\rangle + \alpha \mid 110010111\rangle$
$+\alpha \mid 110011000\rangle + \alpha \mid 110011001\rangle + \alpha \mid 110011010\rangle + \alpha \mid 110011011\rangle$
$+\alpha \mid 110011100\rangle + \alpha \mid 110011101\rangle + \alpha \mid 110011110\rangle + \alpha \mid 110011111\rangle$
$+\alpha \mid 010100000\rangle + \alpha \mid 010100001\rangle + \alpha \mid 010100010\rangle + \alpha \mid 010100011\rangle$
$+\alpha \mid 010100100\rangle + \alpha \mid 010100101\rangle + \alpha \mid 010100110\rangle + \alpha \mid 010100111\rangle$
$+\alpha \mid 010101000\rangle + \alpha \mid 010101001\rangle + \alpha \mid 010101010\rangle + \alpha \mid 010101011\rangle$
$+\alpha \mid 010101100\rangle + \alpha \mid 010101101\rangle + \alpha \mid 010101110\rangle + \alpha \mid 010101111\rangle$
$-\alpha \mid 000110000\rangle - \alpha \mid 000110001\rangle - \alpha \mid 000110010\rangle - \alpha \mid 000110011\rangle$
$-\alpha \mid 000110100\rangle - \alpha \mid 000110101\rangle - \alpha \mid 000110110\rangle - \alpha \mid 000110111\rangle$
$-\alpha \mid 000111000\rangle - \alpha \mid 000111001\rangle - \alpha \mid 000111010\rangle - \alpha \mid 000111011\rangle$
$-\alpha \mid 000111100\rangle - \alpha \mid 000111101\rangle - \alpha \mid 000111110\rangle - \alpha \mid 000111111\rangle$
$+\alpha \mid 111100000\rangle + \alpha \mid 111100001\rangle + \alpha \mid 111100010\rangle + \alpha \mid 111100011\rangle$
$+\alpha \mid 111100100\rangle + \alpha \mid 111100101\rangle + \alpha \mid 111100110\rangle + \alpha \mid 111100111\rangle$
$+\alpha \mid 111101000\rangle + \alpha \mid 111101001\rangle + \alpha \mid 111101010\rangle + \alpha \mid 111101011\rangle$
$+\alpha \mid 111101100\rangle + \alpha \mid 111101101\rangle + \alpha \mid 111101110\rangle + \alpha \mid 111101111\rangle$
$+\alpha \mid 101110000\rangle + \alpha \mid 101110001\rangle + \alpha \mid 101110010\rangle + \alpha \mid 101110011\rangle$
$+\alpha \mid 101110100\rangle + \alpha \mid 101110101\rangle + \alpha \mid 101110110\rangle + \alpha \mid 101110111\rangle$
$+\alpha \mid 1011101000\rangle + \alpha \mid 101111001\rangle + \alpha \mid 101111010\rangle + \alpha \mid 101111011\rangle$

$$+\alpha \mid 101111100\rangle + \alpha \mid 101111101\rangle + \alpha \mid 101111110\rangle + \alpha \mid 101111111\rangle$$
$$+\alpha \mid 001000000\rangle + \alpha \mid 001000001\rangle + \alpha \mid 001000010\rangle + \alpha \mid 001000011\rangle$$
$$+\alpha \mid 001000100\rangle + \alpha \mid 001000101\rangle + \alpha \mid 001000110\rangle + \alpha \mid 001000111\rangle$$
$$+\alpha \mid 001001000\rangle + \alpha \mid 001001001\rangle + \alpha \mid 001001010\rangle + \alpha \mid 001001011\rangle$$
$$+\alpha \mid 001001100\rangle + \alpha \mid 001001101\rangle + \alpha \mid 001001100\rangle + \alpha \mid 001001111\rangle$$
$$-\alpha \mid 011010000\rangle - \alpha \mid 011010001\rangle - \alpha \mid 011010010\rangle - \alpha \mid 011010011\rangle$$
$$-\alpha \mid 011010100\rangle - \alpha \mid 011010101\rangle - \alpha \mid 011010110\rangle - \alpha \mid 011010111\rangle$$
$$-\alpha \mid 011011000\rangle - \alpha \mid 011011001\rangle - \alpha \mid 011011010\rangle - \alpha \mid 011011011\rangle$$
$$-\alpha \mid 011011100\rangle - \alpha \mid 011011101\rangle - \alpha \mid 011011110\rangle - \alpha \mid 011011111\rangle$$
$$+\ \beta \mid 010000000\rangle +\ \beta \mid 010000001\rangle +\ \beta \mid 010000010\rangle +\ \beta \mid 010000011\rangle$$
$$+\ \beta \mid 010000100\rangle +\ \beta \mid 010000101\rangle +\ \beta \mid 010000110\rangle +\ \beta \mid 010000111\rangle$$
$$+\ \beta \mid 010001000\rangle +\ \beta \mid 010001001\rangle +\ \beta \mid 010001010\rangle +\ \beta \mid 010001011\rangle$$
$$+\ \beta \mid 010001100\rangle +\ \beta \mid 010001101\rangle +\ \beta \mid 010001110\rangle +\ \beta \mid 010001111\rangle$$
$$+\ \beta \mid 000010000\rangle +\ \beta \mid 000010001\rangle +\ \beta \mid 000010010\rangle +\ \beta \mid 000010011\rangle$$
$$+\ \beta \mid 000010100\rangle +\ \beta \mid 000010101\rangle +\ \beta \mid 000010110\rangle +\ \beta \mid 000010111\rangle$$
$$+\ \beta \mid 000011000\rangle +\ \beta \mid 000011001\rangle +\ \beta \mid 000011010\rangle +\ \beta \mid 000011011\rangle$$
$$+\ \beta \mid 000011100\rangle +\ \beta \mid 000011101\rangle +\ \beta \mid 000011110\rangle +\ \beta \mid 000011111\rangle$$
$$+\ \beta \mid 100100000\rangle +\ \beta \mid 100100001\rangle +\ \beta \mid 100100010\rangle +\ \beta \mid 100100011\rangle$$
$$+\ \beta \mid 100100100\rangle +\ \beta \mid 100100101\rangle +\ \beta \mid 100100110\rangle +\ \beta \mid 100100111\rangle$$
$$+\ \beta \mid 100101000\rangle +\ \beta \mid 100101001\rangle +\ \beta \mid 100101010\rangle +\ \beta \mid 100101011\rangle$$
$$+\ \beta \mid 100101100\rangle +\ \beta \mid 100101101\rangle +\ \beta \mid 100101110\rangle +\ \beta \mid 100101111\rangle$$
$$-\ \beta \mid 110110000\rangle -\ \beta \mid 110110001\rangle -\ \beta \mid 110110010\rangle -\ \beta \mid 110110011\rangle$$
$$-\ \beta \mid 110110100\rangle -\ \beta \mid 110110101\rangle -\ \beta \mid 110110110\rangle -\ \beta \mid 110110111\rangle$$
$$-\ \beta \mid 110111000\rangle -\ \beta \mid 110111001\rangle -\ \beta \mid 110111010\rangle -\ \beta \mid 110111011\rangle$$
$$-\ \beta \mid 110111100\rangle -\ \beta \mid 110111101\rangle -\ \beta \mid 110111110\rangle -\ \beta \mid 110111111\rangle$$
$$-\ \beta \mid 001100000\rangle -\ \beta \mid 001100001\rangle -\ \beta \mid 001100010\rangle -\ \beta \mid 001100011\rangle$$
$$-\ \beta \mid 001100100\rangle -\ \beta \mid 001100101\rangle -\ \beta \mid 001100110\rangle -\ \beta \mid 001100111\rangle$$
$$-\ \beta \mid 001101000\rangle -\ \beta \mid 001101001\rangle -\ \beta \mid 001101010\rangle -\ \beta \mid 001101011\rangle$$
$$-\ \beta \mid 001101100\rangle -\ \beta \mid 001101101\rangle -\ \beta \mid 001101110\rangle -\ \beta \mid 001101111\rangle$$
$$-\ \beta \mid 011110000\rangle\ -\ \beta \mid 011110001\rangle\ -\ \beta \mid 011110010\rangle\ -\ \beta \mid 011110011\rangle$$
$$-\ \beta \mid 011110100\rangle\ -\ \beta \mid 011110101\rangle\ -\ \beta \mid 011110110\rangle\ -\ \beta \mid 011110111\rangle$$
$$-\ \beta \mid 011111000\rangle\ -\ \beta \mid 011111001\rangle\ -\ \beta \mid 011111010\rangle\ -\ \beta \mid 011111011\rangle$$

$- \beta \mid 011111100\rangle - \beta \mid 011111101\rangle - \beta \mid 011111110\rangle - \beta \mid 011111111\rangle$

$-\beta \mid 111000000\rangle - \beta \mid 111000001\rangle - \beta \mid 111000010\rangle - \beta \mid 111000011\rangle$

$-\beta \mid 111000100\rangle - \beta \mid 111000101\rangle - \beta \mid 111000110\rangle - \beta \mid 111000111\rangle$

$-\beta \mid 111001000\rangle - \beta \mid 111001001\rangle - \beta \mid 111001010\rangle - \beta \mid 111001011\rangle$

$-\beta \mid 111001100\rangle - \beta \mid 111001101\rangle - \beta \mid 111001110\rangle - \beta \mid 111001111\rangle$

$+\beta \mid 101010000\rangle + \beta \mid 101010001\rangle + \beta \mid 101010010\rangle + \beta \mid 101010011\rangle$

$+\beta \mid 101010100\rangle + \beta \mid 101010101\rangle + \beta \mid 101010110\rangle + \beta \mid 101010111\rangle$

$+\beta \mid 101011000\rangle + \beta \mid 101011001\rangle + \beta \mid 101011010\rangle + \beta \mid 101011011\rangle$

$+\beta \mid 101011100\rangle + \beta \mid 101011100\rangle + \beta \mid 101011110\rangle + \beta \mid 101011111\rangle)$

◦ **Application controlée des stabilisateurs :**

·Pour $M_0 = X(1)Z(2)X(3)Z(4)I(5)$

·Pour$M_1 = Z(1)Z(2)Z(3)I(4)Z(5)$

·Pour$M_2 = I(1)X(2)Z(3)Z(4)X(5)$

·Pour$M_3 = Z(1)I(2)X(3)X(4)X(5)$

Par conséquent, on trouve :

$\mid \Psi_2\rangle = \frac{1}{8\sqrt{2}}(\alpha \mid 100000000\rangle - \alpha \mid 101110001\rangle + \alpha \mid 110010010\rangle - \alpha \mid 111100011\rangle$

$-\alpha \mid 100000100\rangle + \alpha \mid 101110101\rangle - \alpha \mid 110010110\rangle + \alpha \mid 111100111\rangle$

$+\alpha \mid 001001000\rangle + \alpha \mid 000111001\rangle - \alpha \mid 011011010\rangle - \alpha \mid 010101011\rangle$

$-\alpha \mid 001001100\rangle - \alpha \mid 000111101\rangle + \alpha \mid 011011110\rangle + \alpha \mid 010101111\rangle$

$+ \alpha \mid 110010000\rangle - \alpha \mid 111100001\rangle + \alpha \mid 100000010\rangle - \alpha \mid 101110011\rangle$

$-\alpha \mid 110010100\rangle + \alpha \mid 111100101\rangle - \alpha \mid 100000110\rangle + \alpha \mid 101110111\rangle$

$-\alpha \mid 011011000\rangle - \alpha \mid 010101001\rangle + \alpha \mid 001001010\rangle + \alpha \mid 000111011\rangle$

$+\alpha \mid 011011100\rangle + \alpha \mid 010101101\rangle - \alpha \mid 001001110\rangle - \alpha \mid 000111111\rangle$

$+\alpha \mid 010100000\rangle + \alpha \mid 011010001\rangle$ -$\alpha \mid 000110010\rangle - \alpha \mid 001000011\rangle$

$-\alpha \mid 010100100\rangle - \alpha \mid 011010101\rangle + \alpha \mid 000110110\rangle + \alpha \mid 001000111\rangle$

$+\alpha \mid 111101000\rangle - \alpha \mid 110011001\rangle + \alpha \mid 101111010\rangle - \alpha \mid 100001011\rangle$

$-\alpha \mid 111101100\rangle + \alpha \mid 110011101\rangle - \alpha \mid 101111110\rangle + \alpha \mid 100001111\rangle$

-$\alpha \mid 000110000\rangle - \alpha \mid 001000001\rangle + \alpha \mid 010100010\rangle + \alpha \mid 011010011\rangle$

$+\alpha \mid 000110100\rangle + \alpha \mid 001000101\rangle - \alpha \mid 010100110\rangle - \alpha \mid 011010111\rangle$

$+\alpha \mid 101111000\rangle - \alpha \mid 100001001\rangle + \alpha \mid 111101010\rangle - \alpha \mid 110011011\rangle$

$-\alpha \mid 101111100\rangle + \alpha \mid 100001101\rangle - \alpha \mid 111101110\rangle + \alpha \mid 110011111\rangle$

$+\alpha \mid 111100000\rangle - \alpha \mid 110010001\rangle + \alpha \mid 101110010\rangle - \alpha \mid 100000011\rangle$

$-\alpha \mid 111100100 \rangle + \alpha \mid 110010101 \rangle - \alpha \mid 101110110 \rangle + \alpha \mid 100000111 \rangle$

$+\alpha \mid 010101000 \rangle + \alpha \mid 011011001 \rangle - \alpha \mid 000111010 \rangle - \alpha \mid 001001011 \rangle$

$-\alpha \mid 010101100 \rangle - \alpha \mid 011011101 \rangle + \alpha \mid 000111110 \rangle + \alpha \mid 001001111 \rangle$

$+\alpha \mid 101110000 \rangle - \alpha \mid 100000001 \rangle + \alpha \mid 111100010 \rangle - \alpha \mid 110010011 \rangle$

$-\alpha \mid 101110100 \rangle + \alpha \mid 100000101 \rangle - \alpha \mid 111100110 \rangle + \alpha \mid 110010111 \rangle$

$-\alpha \mid 000111000 \rangle - \alpha \mid 001001001 \rangle + \alpha \mid 010101010 \rangle + \alpha \mid 011011011 \rangle$

$+\alpha \mid 000111100 \rangle + \alpha \mid 001001101 \rangle - \alpha \mid 010101110 \rangle - \alpha \mid 011011111 \rangle$

$+\alpha \mid 001000000 \rangle + \alpha \mid 000110001 \rangle - \alpha \mid 011010010 \rangle - \alpha \mid 010100011 \rangle$

$-\alpha \mid 001000100 \rangle - \alpha \mid 000110101 \rangle + \alpha \mid 011010110 \rangle + \alpha \mid 010100111 \rangle$

$+\alpha \mid 100001000 \rangle - \alpha \mid 101111001 \rangle + \alpha \mid 110011010 \rangle - \alpha \mid 111101011 \rangle$

$-\alpha \mid 100001100 \rangle + \alpha \mid 101111101 \rangle - \alpha \mid 110011110 \rangle + \alpha \mid 111101111 \rangle$

$-\alpha \mid 011010000 \rangle - \alpha \mid 010100001 \rangle + \alpha \mid 001000010 \rangle + \alpha \mid 000110011 \rangle$

$+\alpha \mid 011010100 \rangle + \alpha \mid 010100101 \rangle - \alpha \mid 001000110 \rangle - \alpha \mid 000110111 \rangle$

$+\alpha \mid 110011000 \rangle - \alpha \mid 111101001 \rangle + \alpha \mid 100001010 \rangle - \alpha \mid 101111011 \rangle$

$-\alpha \mid 110011100 \rangle + \alpha \mid 111101101 \rangle - \alpha \mid 100001110 \rangle + \alpha \mid 101111111 \rangle$

$+ \beta \mid 010000000 \rangle + \beta \mid 011110001 \rangle + \beta \mid 000010010 \rangle + \beta \mid 001100011 \rangle$

$-\beta \mid 010000100 \rangle - \beta \mid 011110101 \rangle - \beta \mid 000010110 \rangle - \beta \mid 001100111 \rangle$

$-\beta \mid 111001000 \rangle + \beta \mid 110111001 \rangle + \beta \mid 101011010 \rangle - \beta \mid 100101011 \rangle$

$+\beta \mid 111001100 \rangle - \beta \mid 110111101 \rangle - \beta \mid 101011110 \rangle + \beta \mid 100101111 \rangle$

$+\beta \mid 000010000 \rangle + \beta \mid 001100001 \rangle + \beta \mid 010000010 \rangle + \beta \mid 011110011 \rangle$

$-\beta \mid 000010100 \rangle - \beta \mid 001100101 \rangle - \beta \mid 010000110 \rangle - \beta \mid 011110111 \rangle$

$+\beta \mid 101011000 \rangle - \beta \mid 100101001 \rangle - \beta \mid 111001010 \rangle + \beta \mid 110111011 \rangle$

$-\beta \mid 101011100 \rangle + \beta \mid 100101101 \rangle + \beta \mid 111001110 \rangle - \beta \mid 110111111 \rangle$

$+\beta \mid 100100000 \rangle - \beta \mid 101010001 \rangle - \beta \mid 110110010 \rangle + \beta \mid 111000011 \rangle$

$-\beta \mid 100100100 \rangle + \beta \mid 101010101 \rangle + \beta \mid 110110110 \rangle - \beta \mid 111000111 \rangle$

$-\beta \mid 001101000 \rangle - \beta \mid 000011001 \rangle - \beta \mid 011111010 \rangle - \beta \mid 010001011 \rangle$

$+\beta \mid 001101100 \rangle + \beta \mid 000011101 \rangle + \beta \mid 011111110 \rangle + \beta \mid 010001111 \rangle$

$-\beta \mid 110110000 \rangle + \beta \mid 111000001 \rangle + \beta \mid 100100010 \rangle - \beta \mid 101010011 \rangle$

$+\beta \mid 110110100 \rangle - \beta \mid 111000101 \rangle - \beta \mid 100100110 \rangle + \beta \mid 101010111 \rangle$

$-\beta \mid 011111000 \rangle - \beta \mid 010001001 \rangle - \beta \mid 001101010 \rangle - \beta \mid 000011011 \rangle$

$+\beta \mid 011111100 \rangle + \beta \mid 010001101 \rangle + \beta \mid 001101110 \rangle + \beta \mid 000011111 \rangle$

$-\beta \mid 001100000 \rangle - \beta \mid 000010001 \rangle - \beta \mid 011110010 \rangle - \beta \mid 010000011 \rangle$

$+\beta \mid 001100100 \rangle + \beta \mid 000010101 \rangle + \beta \mid 011110110 \rangle + \beta \mid 010000111 \rangle$

$+\beta \mid 100101000 \rangle - \beta \mid 101011001 \rangle - \beta \mid 110111010 \rangle + \beta \mid 111001011\rangle$

$-\beta \mid 100101100 \rangle + \beta \mid 101011101 \rangle + \beta \mid 110111110 \rangle - \beta \mid 111001111 \rangle$

-$\beta \mid 011110000 \rangle - \beta \mid 010000001 \rangle - \beta \mid 001100010 \rangle - \beta \mid 000010011 \rangle$

$+\beta \mid 011110100 \rangle + \beta \mid 010000101 \rangle + \beta \mid 001100110 \rangle + \beta \mid 000010111 \rangle$

$-\beta \mid 110111000 \rangle + \beta \mid 111001001 \rangle + \beta \mid 100101010 \rangle - \beta \mid 101011011 \rangle$

$+\beta \mid 110111100\rangle - \beta \mid 111001101\rangle - \beta \mid 100101110\rangle + \beta \mid 101011111\rangle$

-$\beta \mid 111000000 \rangle + \beta \mid 110110001 \rangle + \beta \mid 101010010 \rangle - \beta \mid 100100011 \rangle$

$+\beta \mid 111000100 \rangle - \beta \mid 110110101 \rangle - \beta \mid 101010110 \rangle + \beta \mid 100100111 \rangle$

$+\beta \mid 010001000 \rangle + \beta \mid 011111001 \rangle + \beta \mid 000011010 \rangle + \beta \mid 001101011\rangle$

$-\beta \mid 010001100\rangle - \beta \mid 011111101\rangle - \beta \mid 000011110\rangle - \beta \mid 001101111\rangle$

$+\beta \mid 101010000\rangle - \beta \mid 100100001\rangle - \beta \mid 111000010\rangle + \beta \mid 110110011\rangle$

$-\beta \mid 101010100\rangle + \beta \mid 100100101\rangle + \beta \mid 111000110\rangle - \beta \mid 110110111\rangle$

$+\beta \mid 000011000\rangle + \beta \mid 001101001\rangle + \beta \mid 010001010\rangle + \beta \mid 011111011\rangle$

$-\beta \mid 000011100 \rangle - \beta \mid 001101101 \rangle - \beta \mid 010001110 \rangle - \beta \mid 011111111 \rangle)$

·L'application de la porte $H(6)$:

·L'application de la porte $H(7)$:

·L'application de la porte $H(8)$:

·L'application de la porte $H(9)$:

Et après toutes les simplifications, on aura

$\mid \Psi_2\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 100000101\rangle + \alpha \mid 101110101\rangle + \alpha \mid 110010101\rangle + \alpha \mid 111100101\rangle$

$+\alpha \mid 001000101\rangle - \alpha \mid 000110101\rangle - \alpha \mid 011010101\rangle + \alpha \mid 010100101\rangle + \beta \mid 010000101\rangle$

$-\beta \mid 011110101\rangle + \beta \mid 000010101\rangle - \beta \mid 001100101\rangle - \beta \mid 111000101\rangle - \beta \mid 110110101\rangle$

$+\beta \mid 101010101\rangle + \beta \mid 100100101\rangle)$

* **Correction d'Erreur :**

Résultat de mesure=0101, donc Erreur de type X_1.

La correction : appliquer la porte X sur le premier qubit.

$\mid \Psi_2\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 000000101\rangle + \alpha \mid 001110101\rangle + \alpha \mid 010010101\rangle + \alpha \mid 011100101\rangle$

$+\alpha \mid 101000101\rangle - \alpha \mid 100110101\rangle - \alpha \mid 111010101\rangle + \alpha \mid 110100101\rangle + \beta \mid 110000101 \rangle$

$-\beta \mid 111110101 \rangle + \beta \mid 100010101 \rangle - \beta \mid 101100101 \rangle - \beta \mid 011000101\rangle - \beta \mid 010110101\rangle$

$+\beta \mid 001010101\rangle + \beta \mid 000100101\rangle)$

Suppression des quatre qubits du syndrome :

$$| \quad \Psi_2\rangle = \frac{1}{2\sqrt{2}}(\ \alpha \mid 00000\rangle + \alpha \mid 00111 \ \rangle + \alpha \mid 01001 \ \rangle + \alpha \mid 01110 \ \rangle + \alpha \mid 10100 \ \rangle - \alpha \mid 10011 \ \rangle$$
$$-\alpha \quad | \quad 11101 \ \rangle + \alpha \mid 11010 \ \rangle + \beta \mid 11000 \ \rangle - \beta \mid 11111 \ \rangle + \beta \mid 10001 \ \rangle - \beta \mid 10110 \ \rangle - \beta \mid 01100 \ \rangle$$
$$-\beta \quad | \quad 01011 \ \rangle + \beta \mid 00101 \ \rangle + \beta \mid 00010 \ \rangle)$$

*<u>**Décodage :**</u>

Application de circuit de décodage :

· <u>L'application de la porte $CNOT\,(5,2)$:</u>

$$| \quad \Psi_2\rangle = \frac{1}{2\sqrt{2}}(\ \alpha \mid 00000\rangle + \alpha \mid 01111 \ \rangle + \alpha \mid 00001 \ \rangle + \alpha \mid 01110 \ \rangle + \alpha \mid 10100 \ \rangle - \alpha \mid 11011 \ \rangle$$
$$-\alpha \quad | \quad 10101 \ \rangle + \alpha \mid 11010 \ \rangle + \beta \mid 11000 \ \rangle - \beta \mid 10111 \ \rangle + \beta \mid 11001 \ \rangle - \beta \mid 10110 \ \rangle - \beta \mid 01100 \ \rangle$$
$$-\beta \quad | \quad 00011 \ \rangle + \beta \mid 01101 \ \rangle + \beta \mid 00010 \ \rangle)$$

· <u>L'application de la porte $H\,(5)$:</u>

$| \ \Psi_1\rangle = \frac{1}{4}(\alpha \mid 00000\rangle + \alpha \mid 00001\rangle + \alpha \mid 01110\rangle - \alpha \mid 01111\rangle$
$+\alpha \mid 00000\rangle - \alpha \mid 00001\rangle + \alpha \mid 01110\rangle + \alpha \mid 01111\rangle$
$+\alpha \mid 10100\rangle + \alpha \mid 10101\rangle - \alpha \mid 11010\rangle + \alpha \mid 11011\rangle$
$-\alpha \mid 10100\rangle + \alpha \mid 10101\rangle + \alpha \mid 11010\rangle + \alpha \mid 11011\rangle$
$+\beta \mid 11000\rangle + \beta \mid 11001\rangle - \ \beta \mid 10110\rangle + \beta \mid 10111\rangle$
$+ \ \beta \mid 11000\rangle \ - \beta \mid 11001\rangle - \ \beta \mid 10110\rangle \ - \beta \mid 10111\rangle$
$-\beta \mid 01100\rangle - \beta \mid 01101\rangle - \beta \mid 00010\rangle + \beta \mid 00011\rangle$
$+\beta \mid 01100\rangle - \beta \mid 01101\rangle + \beta \mid 00010\rangle + \beta \mid 00011\rangle)$

Aprés la simplification, on aura :

$$| \quad \Psi_1\rangle = \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 01110\rangle + \alpha \mid 10101\rangle + \alpha \mid 11011\rangle + \beta \mid 11000\rangle - \ \beta \mid 10110\rangle$$
$$-\beta \quad | \quad 01101\rangle + \beta \mid 00011\rangle)$$

· <u>L'application de la porte $CNOT\,(1,2)$:</u>

$$| \quad \Psi_1\rangle = \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 01110\rangle + \alpha \mid 11101\rangle + \alpha \mid 10011\rangle + \beta \mid 10000\rangle - \ \beta \mid 11110\rangle$$
$$-\beta \quad | \quad 01101\rangle + \beta \mid 00011\rangle)$$

·L'application de la porte $CNOT\,(3,4)$:

$$\mid \Psi_1\rangle = \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 01100\rangle + \alpha \mid 11111\rangle + \alpha \mid 10011\rangle + \beta \mid 10000\rangle - \beta \mid 11100\rangle$$
$$-\beta \mid 01111\rangle + \beta \mid 00011\rangle)$$

·L'application de la Port $CNOT\,(4,5)$:

$$\mid \Psi_1\rangle = \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 01100\rangle + \alpha \mid 11110\rangle + \alpha \mid 10010\rangle + \beta \mid 10000\rangle - \beta \mid 11100\rangle$$
$$-\beta \mid 01110\rangle + \beta \mid 00010\rangle)$$

· L'application de la Port $CNOT\,(4,1)$:

$$\mid \Psi_1\rangle = \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 01100\rangle + \alpha \mid 01110\rangle + \alpha \mid 00010\rangle + \beta \mid 10000\rangle - \beta \mid 11100\rangle$$
$$-\beta \mid 11110\rangle + \beta \mid 10010\rangle)$$

· L'application de la Port $H\,(4)$:

$$\mid \Psi_1\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 00010\rangle + \alpha \mid 01100\rangle + \alpha \mid 01110\rangle + \alpha \mid 01100\rangle - \alpha \mid 01110\rangle$$
$$+\alpha \mid 00000\rangle - \alpha \mid 00010\rangle + \beta \mid 10000\rangle + \beta \mid 10010\rangle - \beta \mid 11100\rangle - \beta \mid 11110\rangle - \beta \mid 11100\rangle$$
$$+\beta \mid 11110\rangle + \beta \mid 10000\rangle - \beta \mid 10010\rangle)$$

Aprés la simplification :

$$\mid \Psi_1\rangle = \frac{1}{\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 01100\rangle + \beta \mid 10000\rangle - \beta \mid 11100\rangle)$$

· L'application de la Port $CNOT\,(2,3)$:

$$\mid \Psi_1\rangle = \frac{1}{\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 01000\rangle + \beta \mid 10000\rangle - \beta \mid 11000\rangle)$$

· L'application de la Port $H\,(2)$:

$$\mid \Psi_1\rangle = \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 01000\rangle + \alpha \mid 00000\rangle - \alpha \mid 01000\rangle + \beta \mid 10000\rangle + \beta \mid 11000\rangle$$
$$-\beta \mid 10000\rangle + \beta \mid 11000\rangle)$$

Aprés la simplification :

$$| \Psi_1\rangle = \alpha \mid 00000\rangle + \beta \mid 11000\rangle$$

· L'application de la Port $CNOT\,(1,2)$:

$$| \Psi_1\rangle = \alpha \mid 00000\rangle + \beta \mid 10000\rangle$$

○ **Suppression des qubits auxiliaires** :

$$| \Psi\rangle = \alpha \mid 0\rangle + \beta \mid 1\rangle$$

• **Injecter une erreure X sur le 2**^{ème} **qubit** :

$$| \Psi_2\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 01000\rangle + \alpha \mid 00001\rangle + \alpha \mid 10010\rangle - \alpha \mid 11011\rangle + \alpha \mid 00110\rangle + \alpha \mid 01111\rangle$$
$$+\alpha \mid 11100\rangle - \alpha \mid 10101\rangle + \beta \mid 10000\rangle + \beta \mid 11001\rangle + \beta \mid 01010\rangle - \beta \mid 00011\rangle - \beta \mid 11110\rangle$$
$$- \beta \mid 10111\rangle - \beta \mid 00100\rangle + \beta \mid 01101\rangle)$$

*__Mesure du syndrome__ :

Ajout des quatre qubits du syndrome :

$$| \Psi_2\rangle = \tfrac{1}{2\sqrt{2}}(\alpha \mid 010000000\rangle + \alpha \mid 000010000\rangle + \alpha \mid 100100000\rangle - \alpha \mid 110110000\rangle$$
$$+\alpha \mid 001100000\rangle + \alpha \mid 011110000\rangle + \alpha \mid 111000000\rangle - \alpha \mid 101010000\rangle$$
$$+ \beta \mid 100000000\rangle + \beta \mid 110010000\rangle + \beta \mid 010100000\rangle - \beta \mid 000110000\rangle$$
$$- \beta \mid 111100000\rangle - \beta \mid 101110000\rangle - \beta \mid 001000000\rangle + \beta \mid 011010000\rangle)$$

·L'application du $H(6)$:

·L'application du $H(7)$:

·L'application du$H(8)$:

·L'application du $H(9)$:

Aprés ces applications, on obtient a :

$$| \Psi_1\rangle = \tfrac{1}{8\sqrt{2}}(\alpha \mid 010000000\rangle + \alpha \mid 010000001\rangle + \alpha \mid 010000010\rangle + \alpha \mid 010000011\rangle$$
$$+\alpha \mid 010000100\rangle + \alpha \mid 010000101\rangle + \alpha \mid 010000110\rangle + \alpha \mid 010000111\rangle$$
$$+\alpha \mid 010001000\rangle + \alpha \mid 010001001\rangle + \alpha \mid 010001010\rangle + \alpha \mid 010001011\rangle$$
$$+\alpha \mid 010001100\rangle + \alpha \mid 010001101\rangle + \alpha \mid 010001110\rangle + \alpha \mid 010001111\rangle$$

$$+ \alpha \mid 000010000 \rangle + \alpha \mid 000010001 \rangle + \alpha \mid 000010010 \rangle + \alpha \mid 000010011 \rangle$$
$$+\alpha \mid 000010100 \rangle + \alpha \mid 000010101 \rangle + \alpha \mid 000010110 \rangle + \alpha \mid 000010111 \rangle$$
$$+\alpha \mid 000011000 \rangle + \alpha \mid 000011001 \rangle + \alpha \mid 000011010 \rangle + \alpha \mid 000011011 \rangle$$
$$+\alpha \mid 000011100 \rangle + \alpha \mid 000011101 \rangle + \alpha \mid 000011110 \rangle + \alpha \mid 000011111 \rangle$$
$$+\alpha \mid 100100000 \rangle + \alpha \mid 100100001 \rangle + \alpha \mid 100100010 \rangle + \alpha \mid 100100011 \rangle$$
$$+\alpha \mid 100100100 \rangle + \alpha \mid 100100101 \rangle + \alpha \mid 100100110 \rangle + \alpha \mid 100100111 \rangle$$
$$+\alpha \mid 100101000 \rangle + \alpha \mid 100101001 \rangle + \alpha \mid 100101010 \rangle + \alpha \mid 100101011 \rangle$$
$$+\alpha \mid 100101100 \rangle + \alpha \mid 100101101 \rangle + \alpha \mid 100101110 \rangle + \alpha \mid 100101111 \rangle$$
$$-\alpha \mid 110110000 \rangle - \alpha \mid 110110001 \rangle - \alpha \mid 110110010 \rangle - \alpha \mid 110110011 \rangle$$
$$-\alpha \mid 110110100 \rangle - \alpha \mid 110110101 \rangle - \alpha \mid 110110110 \rangle - \alpha \mid 110110111 \rangle$$
$$-\alpha \mid 110111000 \rangle - \alpha \mid 110111001 \rangle - \alpha \mid 110111010 \rangle - \alpha \mid 110111011 \rangle$$
$$-\alpha \mid 110111100 \rangle - \alpha \mid 110111101 \rangle - \alpha \mid 110111110 \rangle - \alpha \mid 110111111 \rangle$$
$$+\alpha \mid 001100000 \rangle + \alpha \mid 001100001 \rangle + \alpha \mid 001100010 \rangle + \alpha \mid 001100011 \rangle$$
$$+\alpha \mid 001100100 \rangle + \alpha \mid 001100101 \rangle + \alpha \mid 001100110 \rangle + \alpha \mid 001100111 \rangle$$
$$+\alpha \mid 001101000 \rangle + \alpha \mid 001101001 \rangle + \alpha \mid 001101010 \rangle + \alpha \mid 001101011 \rangle$$
$$+\alpha \mid 001101100 \rangle + \alpha \mid 001101101 \rangle + \alpha \mid 001101110 \rangle + \alpha \mid 001101111 \rangle$$
$$+\alpha \mid 011110000 \rangle + \alpha \mid 011110001 \rangle + \alpha \mid 011110010 \rangle + \alpha \mid 011110011 \rangle$$
$$+\alpha \mid 011110100 \rangle + \alpha \mid 011110101 \rangle + \alpha \mid 011110110 \rangle + \alpha \mid 011110111 \rangle$$
$$+\alpha \mid 011111000 \rangle + \alpha \mid 011111001 \rangle + \alpha \mid 011111010 \rangle + \alpha \mid 011111011 \rangle$$
$$+\alpha \mid 011111100 \rangle + \alpha \mid 011111101 \rangle + \alpha \mid 011111110 \rangle + \alpha \mid 011111111 \rangle$$
$$+\alpha \mid 111000000 \rangle + \alpha \mid 111000001 \rangle + \alpha \mid 111000010 \rangle + \alpha \mid 111000011 \rangle$$
$$+\alpha \mid 111000100 \rangle + \alpha \mid 111000101 \rangle + \alpha \mid 111000110 \rangle + \alpha \mid 111000111 \rangle$$
$$+\alpha \mid 111001000 \rangle + \alpha \mid 111001001 \rangle + \alpha \mid 111001010 \rangle + \alpha \mid 111001011 \rangle$$
$$+\alpha \mid 111001100 \rangle + \alpha \mid 111001101 \rangle + \alpha \mid 111001110 \rangle + \alpha \mid 111001111 \rangle$$
$$-\alpha \mid 101010000 \rangle - \alpha \mid 101010001 \rangle - \alpha \mid 101010010 \rangle - \alpha \mid 101010011 \rangle$$
$$-\alpha \mid 101010100 \rangle - \alpha \mid 101010101 \rangle - \alpha \mid 101010110 \rangle - \alpha \mid 101010111 \rangle$$
$$-\alpha \mid 101011000 \rangle - \alpha \mid 101011001 \rangle - \alpha \mid 101011010 \rangle - \alpha \mid 101011011 \rangle$$
$$-\alpha \mid 101011100 \rangle - \alpha \mid 101011101 \rangle - \alpha \mid 101011110 \rangle - \alpha \mid 101011111 \rangle$$
$$+ \beta \mid 100000000 \rangle + \beta \mid 100000001 \rangle + \beta \mid 100000010 \rangle + \beta \mid 100000011 \rangle$$
$$+\beta \mid 100000100 \rangle + \beta \mid 100000101 \rangle + \beta \mid 100000110 \rangle + \beta \mid 100000111 \rangle$$
$$+\beta \mid 100001000 \rangle + \beta \mid 100001001 \rangle + \beta \mid 100001010 \rangle + \beta \mid 100001011 \rangle$$
$$+\beta \mid 100001100 \rangle + \beta \mid 100001101 \rangle + \beta \mid 100001110 \rangle + \beta \mid 100001111 \rangle$$

$+\beta \mid 110010000 \rangle + \beta \mid 110010001 \rangle + \beta \mid 110010010 \rangle + \beta \mid 110010011 \rangle$

$+\beta \mid 110010100 \rangle + \beta \mid 110010101 \rangle + \beta \mid 110010110 \rangle + \beta \mid 110010111 \rangle$

$+\beta \mid 110011000 \rangle + \beta \mid 110011001 \rangle + \beta \mid 110011010 \rangle + \beta \mid 110011011 \rangle$

$+\beta \mid 110011100 \rangle + \beta \mid 110011101 \rangle + \beta \mid 110011110 \rangle + \beta \mid 110011111 \rangle$

$+\beta \mid 010100000 \rangle + \beta \mid 010100001 \rangle + \beta \mid 010100010 \rangle + \beta \mid 010100011 \rangle$

$+\beta \mid 010100100 \rangle + \beta \mid 010100101 \rangle + \beta \mid 010100110 \rangle + \beta \mid 010100111 \rangle$

$+\beta \mid 010101000 \rangle + \beta \mid 010101001 \rangle + \beta \mid 010101010 \rangle + \beta \mid 010101011 \rangle$

$+\beta \mid 010101100 \rangle + \beta \mid 010101101 \rangle + \beta \mid 010101110 \rangle + \beta \mid 010101111 \rangle$

-$\beta \mid 000110000 \rangle \beta \mid 000110001 \rangle - \beta \mid 000110010 \rangle - \beta \mid 000110011 \rangle$

$-\beta \mid 000110100 \rangle - \beta \mid 000110101 \rangle - \beta \mid 000110110 \rangle - \beta \mid 000110111 \rangle$

$-\beta \mid 000111000 \rangle - \beta \mid 000111001 \rangle - \beta \mid 000111010 \rangle - \beta \mid 000111011 \rangle$

$-\beta \mid 000111100 \rangle - \beta \mid 000111101 \rangle - \beta \mid 000111110 \rangle - \beta \mid 000111111 \rangle$

-$\beta \mid 111100000 \rangle - \beta \mid 111100001 \rangle - \beta \mid 111100010 \rangle - \beta \mid 111100011 \rangle$

$-\beta \mid 111100100 \rangle - \beta \mid 111100101 \rangle - \beta \mid 111100110 \rangle - \beta \mid 111100111 \rangle$

$-\beta \mid 111101000 \rangle - \beta \mid 111101001 \rangle - \beta \mid 111101010 \rangle - \beta \mid 111101011\rangle$

$-\beta \mid 111101100 \rangle - \beta \mid 111101101 \rangle - \beta \mid 111101110 \rangle - \beta \mid 111101111 \rangle$

-$\beta \mid 101110000 \rangle - \beta \mid 101110001 \rangle - \beta \mid 101110010 \rangle - \beta \mid 101110011 \rangle$

$-\beta \mid 101110100 \rangle - \beta \mid 101110101 \rangle - \beta \mid 101110110 \rangle - \beta \mid 101110111 \rangle$

$-\beta \mid 101111000 \rangle - \beta \mid 101111001 \rangle - \beta \mid 101111010 \rangle - \beta \mid 101111011 \rangle$

$-\beta \mid \ 101111100\rangle - \beta \mid \ 101111101\rangle - \beta \mid \ 101111110\rangle - \beta \mid \ 101111111\rangle$

-$\beta \mid 001000000 \rangle - \beta \mid 001000001 \rangle - \beta \mid 001000010 \rangle - \beta \mid 001000011 \rangle$

$-\beta \mid 001000100 \rangle - \beta \mid 001000101 \rangle - \beta \mid 001000110 \rangle - \beta \mid 001000111 \rangle$

$-\beta \mid 001001000 \rangle - \beta \mid 001001001 \rangle - \beta \mid 001001010 \rangle - \beta \mid 001001011\rangle$

$-\beta \mid 001001100\rangle - \beta \mid 001001101\rangle - \beta \mid 001001110\rangle - \beta \mid 001001111\rangle$

$+\beta \mid 011010000\rangle + \beta \mid 011010001\rangle + \beta \mid 011010010\rangle + \beta \mid 011010011\rangle$

$+\beta \mid 011010100\rangle + \beta \mid 011010101\rangle + \beta \mid 011010110\rangle + \beta \mid 011010111\rangle$

$+\beta \mid 011011000\rangle + \beta \mid 011011001\rangle + \beta \mid 011011010\rangle + \beta \mid 011011011\rangle$

$+\beta \mid 011011100 \rangle + \beta \mid 011011101 \rangle + \beta \mid 011011110 \rangle + \beta \mid 011011111 \rangle)$

○ **Application controlée des stabilisateurs** :

·Pour$M_0 = X(1)Z(2)\,X(3)\,Z(4)\,I(5)$

·Pour$M_1 = Z(1)\,Z(2)\,Z(3)\,I(4)\,Z(5)$

·Pour$M_2 = I(1)\,X(2)\,Z(3)\,Z(4)\,X(5)$

·Pour$M_3 = Z\left(1\right)I\left(2\right)X\left(3\right)X\left(4\right)X\left(5\right)$

Par conséquent, on trouve :

$\mid\Psi_1\rangle = \frac{1}{2\sqrt{2}}(\alpha\mid 010000000\rangle + \alpha\mid 011110001\rangle + \alpha\mid 000010010\rangle + \alpha\mid 001100011\rangle$

$- \alpha\mid 010000100\rangle - \alpha\mid 011\ 110101\rangle - \alpha\mid 000010110\rangle\ \alpha\mid 001100111\rangle$

$-\alpha\mid 111001000\rangle + \alpha\mid 110111001\rangle + \alpha\mid 101011010\rangle - \alpha\mid 100101011\rangle$

$+\alpha\mid 111001100\rangle - \alpha\mid 110111101\rangle - \alpha\mid 10101110\rangle + \alpha\mid 100101111\rangle$

$+ \alpha\mid 000010000\rangle + \alpha\mid 001100001\rangle + \alpha\mid 010000010\rangle + \alpha\mid 011110011\rangle$

$-\alpha\mid 000010100\rangle - \alpha\mid 001100101\rangle - \alpha\mid 010000110\rangle - \alpha\mid 011110111\rangle$

$+\alpha\mid 101011000\rangle - \alpha\mid 100101001\rangle - \alpha\mid 111001010\rangle + \alpha\mid 110111011\rangle$

$-\alpha\mid 101011100\rangle + \alpha\mid 100101101\rangle + \alpha\mid 111001110\rangle - \alpha\mid 110111111\rangle$

$+\alpha\mid 100100000\rangle - \alpha\mid 1001010001\rangle - \alpha\mid 110110010\rangle + \alpha\mid 111000011\rangle$

$-\alpha\mid 100100100\rangle + \alpha\mid 101010101\rangle + \alpha\mid 110110110\rangle - \alpha\mid 111000111\rangle$

$-\alpha\mid 001101000\rangle - \alpha\mid 000011001\rangle - \alpha\mid 011111010\rangle - \alpha\mid 010001011\rangle$

$+\alpha\mid 001101100\rangle + \alpha\mid 000011101\rangle + \alpha\mid 011111110\rangle + \alpha\mid 010001111\rangle$

$-\alpha\mid 110110000\rangle + \alpha\mid 111000001\rangle + \alpha\mid 100100010\rangle - \alpha\mid 101010011\rangle$

$+\alpha\mid 110110100\rangle - \alpha\mid 111000101\rangle - \alpha\mid 100100110\rangle + \alpha\mid 101010111\rangle$

$-\alpha\mid 011111000\rangle - \alpha\mid 010001001\rangle - \alpha\mid 001101010\rangle - \alpha\mid 000011011\rangle$

$+\alpha\mid 011111100\rangle + \alpha\mid 010001101\rangle + \alpha\mid 001101110\rangle + \alpha\mid 000011111\rangle$

$+\alpha\mid 001100000\rangle + \alpha\mid 000010001\rangle + \alpha\mid 011110010\rangle + \alpha\mid 010000011\rangle$

$-\alpha\mid 001100100\rangle - \alpha\mid 000010101\rangle - \alpha\mid 011110110\rangle - \alpha\mid 010000111\rangle$

$-\alpha\mid 100101000\rangle + \alpha\mid 101011001\rangle + \alpha\mid 110111010\rangle - \alpha\mid 111001011\rangle$

$+\alpha\mid 100101100\rangle - \alpha\mid 101011101\rangle - \alpha\mid 110111110\rangle + \alpha\mid 111001111\rangle$

$+\alpha\mid 011110000\rangle + \alpha\mid 010000001\rangle + \alpha\mid 001100010\rangle + \alpha\mid 000010011\rangle$

$-\alpha\mid 011110100\rangle - \alpha\mid 010000101\rangle - \alpha\mid 001100110\rangle - \alpha\mid 001100111\rangle$

$+\alpha\mid 110111000\rangle - \alpha\mid 111001001\rangle - \alpha\mid 100101010\rangle + \alpha\mid 101011011\rangle$

$-\alpha\mid 110111100\rangle + \alpha\mid 111001101\rangle + \alpha\mid 100101110\rangle - \alpha\mid 101011111\rangle$

$+\alpha\mid 111000000\rangle - \alpha\mid 110110001\rangle - \alpha\mid 101010010\rangle + \alpha\mid 100100011\rangle$

$-\alpha\mid 111000100\rangle + \alpha\mid 110110101\rangle + \alpha\mid 101010110\rangle - \alpha\mid 100100111\rangle$

$-\alpha\mid 010001000\rangle - \alpha\mid 011111001\rangle - \alpha\mid 000011010\rangle - \alpha\mid 001101011\rangle$

$+\alpha\mid 010001100\rangle + \alpha\mid 011111101\rangle + \alpha\mid 000011110\rangle + \alpha\mid 001101111\rangle$

$-\alpha\mid 101010000\rangle + \alpha\mid 100100001\rangle + \alpha\mid 111000010\rangle - \alpha\mid 110110011\rangle$

$+\alpha\mid 101010100\rangle - \alpha\mid 100100101\rangle - \alpha\mid 111000110\rangle + \alpha\mid 110110111\rangle$

$-\alpha \mid 000011000\rangle - \alpha \mid 001101001\rangle - \alpha \mid 010001010\rangle - \alpha \mid 011111011\rangle$
$+\alpha \mid 000011100\rangle + \alpha \mid 001101101\rangle + \alpha \mid 010001110\rangle + \alpha \mid 011111111\rangle$
$+\beta \mid 100000000\rangle - \beta \mid 101110001\rangle + \beta \mid 110010010\rangle - \beta \mid 111100011\rangle$
$-\beta \mid 100000100\rangle + \beta \mid 101110101\rangle - \beta \mid 100010110\rangle + \beta \mid 111100111\rangle$
$+\beta \mid 001001000\rangle + \beta \mid 000111001\rangle - \beta \mid 011011010\rangle - \beta \mid 010101011\rangle$
$-\beta \mid 001001100\rangle - \beta \mid 000111101\rangle + \beta \mid 011011110\rangle + \beta \mid 010101111\rangle$
$+\beta \mid 110010000\rangle - \beta \mid 111100001\rangle + \beta \mid 100000010\rangle - \beta \mid 101110011\rangle$
$-\beta \mid 110010100\rangle + \beta \mid 111100101\rangle - \beta \mid 100000110\rangle + \beta \mid 101110111\rangle$
$-\beta \mid 011011000\rangle - \beta \mid 010101001\rangle + \beta \mid 001001010\rangle + \beta \mid 000111011\rangle$
$+\beta \mid 011011100\rangle + \beta \mid 010101101\rangle - \beta \mid 001001110\rangle - \beta \mid 000111111\rangle$
$+\beta \mid 010100000\rangle + \beta \mid 011010001\rangle - \beta \mid 000110010\rangle - \beta \mid 001000011\rangle$
$-\beta \mid 010100100\rangle - \beta \mid 011010101\rangle + \beta \mid 000110110\rangle + \beta \mid 001000111\rangle$
$+\beta \mid 111101000\rangle - \beta \mid 110011001\rangle + \beta \mid 101111010\rangle - \beta \mid 100001011\rangle$
$-\beta \mid 111101100\rangle + \beta \mid 110011101\rangle - \beta \mid 101111110\rangle + \beta \mid 100001111\rangle$
$-\beta \mid 000110000\rangle - \beta \mid 001000001\rangle + \beta \mid 010100010\rangle + \beta \mid 011010011\rangle$
$+\beta \mid 000110100\rangle + \beta \mid 001000101\rangle - \beta \mid 010100110\rangle - \beta \mid 011010111\rangle$
$+\beta \mid 101111000\rangle - \beta \mid 100001001\rangle + \beta \mid 111101010\rangle - \beta \mid 110011011\rangle$
$-\beta \mid 101111100\rangle + \beta \mid 100001101\rangle - \beta \mid 111101110\rangle + \beta \mid 110011111\rangle$
$-\beta \mid 111100000\rangle + \beta \mid 110000001\rangle - \beta \mid 101110010\rangle + \beta \mid 100000011\rangle$
$+\beta \mid 111100100\rangle - \beta \mid 110010101\rangle + \beta \mid 101110110\rangle - \beta \mid 100000111\rangle$
$-\beta \mid 010101000\rangle - \beta \mid 011011001\rangle + \beta \mid 000111010\rangle + \beta \mid 001001011\rangle$
$+\beta \mid 010101100\rangle + \beta \mid 011011101\rangle - \beta \mid 000111110\rangle - \beta \mid 001001111\rangle$
$-\beta \mid 101110000\rangle + \beta \mid 100000001\rangle - \beta \mid 111100010\rangle + \beta \mid 110010011\rangle$
$+\beta \mid 101110100\rangle - \beta \mid 100000101\rangle + \beta \mid 101100110\rangle - \beta \mid 110010111\rangle$
$+\beta \mid 000111000\rangle + \beta \mid 001001001\rangle - \beta \mid 010101010\rangle - \beta \mid 011011011\rangle$
$-\beta \mid 000111100\rangle - \beta \mid 001001101\rangle + \beta \mid 010101110\rangle + \beta \mid 011011111\rangle$
$-\beta \mid 001000000\rangle - \beta \mid 000110001\rangle + \beta \mid 011010010\rangle + \beta \mid 010100011\rangle$
$+\beta \mid 001000100\rangle + \beta \mid 000110101\rangle - \beta \mid 011010110\rangle - \beta \mid 010100111\rangle$
$-\beta \mid 100001000\rangle + \beta \mid 101111001\rangle - \beta \mid 110011010\rangle + \beta \mid 111101011\rangle$
$+\beta \mid 100001100\rangle - \beta \mid 101111101\rangle + \beta \mid 110011110\rangle - \beta \mid 111101111\rangle$
$+\beta \mid 011010000\rangle + \beta \mid 010100001\rangle - \beta \mid 001000010\rangle - \beta \mid 000110011\rangle$
$-\beta \mid 011010100\rangle - \beta \mid 010100101\rangle + \beta \mid 001000110\rangle + \beta \mid 000110111\rangle$

$-\beta \mid 110011000\rangle + \beta \mid 111011001\rangle - \beta \mid 100001010\rangle + \beta \mid 101111011\rangle$

$+\beta \mid 110011100\ \rangle - \beta \mid 111101101\ \rangle + \beta \mid 100001110\ \rangle - \beta \mid 101111111\ \rangle)$

·<u>L'application du $H(6)$</u> :

·<u>L'application du $H(7)$</u> :

·<u>L'application du $H(8)$</u> :

·<u>L'application du$H(9)$</u> :

Et après toutes les simplifications,onaura

$$\begin{aligned}\mid \Psi_1\rangle = \tfrac{1}{2\sqrt{2}}(&\alpha \mid 010001100\rangle + \alpha \mid 011111100\rangle + \alpha \mid 000011100\rangle + \alpha \mid 001101100\rangle \\ &+\alpha \mid 111001100\rangle - \alpha \mid 110111100\rangle - \alpha \mid 101011100\rangle + \alpha \mid 100101100\rangle \\ &+\beta \mid 100001100\rangle - \beta \mid 101111100\ \rangle + \beta \mid 110011100\rangle - \beta \mid 111101100\rangle \\ &-\beta \mid 001001100\ \rangle - \beta \mid 000111100\ \rangle + \beta \mid 011011100\rangle + \beta \mid 010101100\ \rangle)\end{aligned}$$

*__Correction d'erreur__ :

Résultat de mesure= 1100,donc erreur de type X_2

$$\begin{aligned}\mid \Psi_1\rangle = \tfrac{1}{2\sqrt{2}}(&\alpha \mid 000000000\rangle + \alpha \mid 001110000\rangle + \alpha \mid 010010000\rangle + \alpha \mid 011100000\rangle \\ &+\alpha \mid 101000000\rangle - \alpha \mid 100110000\rangle - \alpha \mid 111010000\rangle + \alpha \mid 110100000\rangle \\ &+\beta \mid 110000000\rangle - \beta \mid 111110000\ \rangle + \beta \mid 100010000\rangle - \beta \mid 101100000\rangle \\ &-\beta \mid 011000000\ \rangle - \beta \mid 010110000\ \rangle + \beta \mid 001010000\ \rangle + \beta \mid 000100000\ \rangle)\end{aligned}$$

∘ Suppression des quatre qubits du syndrome :

$$\mid \Psi_1\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 00111\rangle + \alpha \mid 01001\rangle + \alpha \mid 01110\rangle + \alpha \mid 10100\rangle - \alpha \mid 10011\rangle$$
$$-\alpha \mid 11101\rangle + \alpha \mid 11010\rangle + \beta \mid 11000\rangle - \beta \mid 11111\rangle + \beta \mid 10001\rangle - \beta \mid 10110\rangle - \beta \mid 01100\rangle$$
$$-\beta \mid 01011\rangle + \beta \mid 00101\rangle + \beta \mid 00010\rangle)$$

*__Décodage__ :

Application de circuit de décodage :

·<u>L'application de la Port $CNOT\,(5,2)$</u>:

$$\mid \Psi_1\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 01111\rangle + \alpha \mid 00001\rangle + \alpha \mid 01110\rangle + \alpha \mid 10100\rangle - \alpha \mid 11011\rangle$$
$$-\alpha \mid 10101\rangle + \alpha \mid 11010\rangle + \beta \mid 11000\rangle - \beta \mid 10111\rangle + \beta \mid 11001\rangle - \beta \mid 10110\rangle - \beta \mid 01100\rangle$$
$$-\beta \mid 00011\rangle + \beta \mid 01101\rangle + \beta \mid 00010\rangle)$$

·<u>L'application de la Port $H\,(5)$</u>:

$$| \quad \Psi_1\rangle = \frac{1}{4}(\alpha \mid 00000\rangle + \alpha \mid 00001\rangle + \alpha \mid 01110\rangle - \alpha \mid 01111\rangle + \alpha \mid 00000\rangle - \alpha \mid 00001\rangle$$
$$+\alpha \quad | \quad 01110\rangle + \alpha \mid 01111\rangle + \alpha \mid 10100\rangle + \alpha \mid 10101\rangle - \alpha \mid 11010\rangle + \alpha \mid 11011\rangle - \alpha \mid 10100\rangle$$
$$+\alpha \quad | \quad 10101\rangle + \alpha \mid 11010\rangle + \alpha \mid 11011\rangle + \beta \mid 11000\rangle + \beta \mid 11001\rangle - \beta \mid 10110\rangle + \beta \mid 10111\rangle$$
$$+\beta \quad | \quad 11000\rangle - \beta \mid 11001\rangle - \beta \mid 10110\rangle - \beta \mid 10111\rangle - \beta \mid 01100\rangle - \beta \mid 01101\rangle - \beta \mid 00010\rangle$$
$$+\beta \quad | \quad 00011\rangle + \beta \mid 01100\rangle - \beta \mid 01101\rangle + \beta \mid 00010\rangle + \beta \mid 00011\rangle)$$

Aprés la simplification :

$$| \quad \Psi_1\rangle = \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 01110\rangle + \alpha \mid 10101\rangle + \alpha \mid 11011\rangle$$
$$+\beta \quad | \quad 11000\rangle - \beta \mid 10110\rangle - \beta \mid 01101\rangle + \beta \mid 00011\rangle)$$

· L'application de laPort $CNOT\,(1,2)$:

$$| \quad \Psi_1\rangle = \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 01110\rangle + \alpha \mid 11101\rangle + \alpha \mid 10011\rangle$$
$$+\beta \quad | \quad 10000\rangle - \beta \mid 11110\rangle - \beta \mid 01101\rangle + \beta \mid 00011\rangle)$$

·L'application de laPort $CNOT\,(3,4)$:

$$| \quad \Psi_1\rangle = \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 01100\rangle + \alpha \mid 11111\rangle + \alpha \mid 10011\rangle$$
$$+\beta \quad | \quad 10000\rangle - \beta \mid 11100\rangle - \beta \mid 01111\rangle + \beta \mid 00011\rangle)$$

·L'application de laPort $CNOT\,(4,5)$:

$$| \quad \Psi_1\rangle = \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 01100\rangle + \alpha \mid 11110\rangle + \alpha \mid 10010\rangle$$
$$+\beta \quad | \quad 10000\rangle - \beta \mid 11100\rangle - \beta \mid 01110\rangle + \beta \mid 00010\rangle)$$

·L'application de la Port $CNOT\,(4,1)$:

$$| \quad \Psi_1\rangle = \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 01100\rangle + \alpha \mid 01110\rangle + \alpha \mid 00010\rangle$$
$$+\beta \quad | \quad 10000\rangle - \beta \mid 11100\rangle - \beta \mid 11110\rangle + \beta \mid 10010\rangle)$$

·<u>L'application de la Port $H\,(4)$</u>:

$$|\ \ \Psi_1\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 00010\rangle + \alpha \mid 01100\rangle + \alpha \mid 01110\rangle + \alpha \mid 01100\rangle - \alpha \mid 01110\rangle$$
$$+\alpha \ \mid \ \ 00000\rangle - \alpha \mid 00010\rangle + \beta \mid 10000\rangle + \beta \mid 10010\rangle - \ \beta \mid 11100\rangle - \beta \mid 11110\rangle - \beta \mid 11100\rangle$$
$$+\beta \ \mid \ \ 11110\rangle + \beta \mid 10000\rangle - \beta \mid 10010\rangle)$$

Apres la simplification :

$$\mid \Psi_1\rangle = \frac{1}{\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 01100\rangle + \beta \mid 10000\rangle - \ \beta \mid 11100\rangle)$$

· <u>L'application de laPort $CNOT\,(2,3)$</u>:

$$\mid \Psi_1\rangle = \frac{1}{\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 01000\rangle + \beta \mid 10000\rangle - \ \beta \mid 11000\rangle)$$

· <u>L'application de laPort $H\,(2)$</u>:

$$|\ \ \Psi_1\rangle = \frac{1}{2}(\alpha \mid 00000\rangle + \alpha \mid 01000\rangle + \alpha \mid 00000\rangle - \alpha \mid 01000\rangle$$
$$+\beta \ \mid \ \ 10000\rangle + \beta \mid 11000\rangle - \beta \mid 10000\rangle + \beta \mid 11000\rangle)$$

Aprés la simplification :

$$\mid \Psi_1\rangle = \alpha \mid 00000\rangle + \beta \mid 11000\rangle$$

·<u>L'application de la Port $CNOT\,(1,2)$</u>:

$$\mid \Psi_1\rangle = \alpha \mid 00000\rangle + \beta \mid 10000\rangle$$

○ **Suppression des qubits auxiliaires** :

$$\mid \Psi\rangle = \alpha \mid 0\rangle + \beta \mid 1\rangle$$

•Le calcul du autres cas des erreure de type $X(X_3, X_4, X_5)$ est de la même méthode.

∗<u>**Cas 03**</u> :Erreur de type Y

· **Injecter une erreure Y sur le premier qubit** :

$$| \ \Psi_1\rangle = \frac{1}{2\sqrt{2}}(-\alpha \mid 10000\rangle - \alpha \mid 11001\rangle + \alpha \mid 01010\rangle - \alpha \mid 00011\rangle - \alpha \mid 11110\rangle - \alpha \mid 10111\rangle$$
$$+\alpha \mid 00100\rangle - \alpha \mid 01101\rangle + \beta \mid 01000\rangle + \beta \mid 00001\rangle - \beta \mid 10010\rangle + \beta \mid 11011\rangle - \beta \mid 00110\rangle$$
$$- \beta \mid 01111\rangle + \beta \mid 11100\rangle - \beta \mid 10101\rangle)$$

***Mesure du syndrome** :

Ajout des quatre qubits du syndrome :

$| \Psi_1\rangle = \frac{1}{2\sqrt{2}}(-\alpha \mid 100000000\rangle - \alpha \mid 110010000\rangle + \alpha \mid 010100000\rangle - \alpha \mid 000110000\rangle$
$-\alpha \mid 111100000\rangle - \alpha \mid 101110000\rangle + \alpha \mid 001000000\rangle - \alpha \mid 011010000\rangle + \beta \mid 010000000\rangle$
$+ \beta \mid 000010000\rangle - \beta \mid 100100000\rangle + \beta \mid 110110000\rangle - \beta \mid 001100000\rangle - \beta \mid 011110000\rangle$
$+\beta \mid 111000000\rangle - \beta \mid 101010000\rangle)$

·L'application du $H(6)$:

·L'application du $H(7)$:

·L'application du $H(8)$:

·L'application du $H(9)$:

Aprés ces applications, on obtient a :

$| \Psi_1\rangle = \frac{1}{8\sqrt{2}}(-\alpha \mid 100000000\rangle - \alpha \mid 100000001\rangle - \alpha \mid 100000010\rangle - \alpha \mid 100000011\rangle$
-$\alpha \mid 100000100\rangle - \alpha \mid 100000101\rangle - \alpha \mid 100000110\rangle - \alpha \mid 100000111\rangle$
-$\alpha \mid 100001000\rangle - \alpha \mid 100001001\rangle - \alpha \mid 100001010\rangle - \alpha \mid 100001011\rangle$
-$\alpha \mid 100001100\rangle - \alpha \mid 100001101\rangle - \alpha \mid 100001110\rangle - \alpha \mid 100001111\rangle$
$-\alpha \mid 110010000\rangle - \alpha \mid 110010001\rangle - \alpha \mid 110010010\rangle - \alpha \mid 110010011 \rangle$
$-\alpha \mid 110010100\rangle - \alpha \mid 110010101\rangle - \alpha \mid 110010110\rangle - \alpha \mid 110010111\rangle$
$-\alpha \mid 110011000\rangle - \alpha \mid 110011001\rangle - \alpha \mid 110011010\rangle - \alpha \mid 110011011\rangle$
$-\alpha \mid 110011100\rangle - \alpha \mid 110011101\rangle - \alpha \mid 110011110\rangle - \alpha \mid 110011111\rangle$
$+\alpha \mid 010100000\rangle + \alpha \mid 010100001\rangle + \alpha \mid 010100010\rangle + \alpha \mid 010100011 \rangle$
$+\alpha \mid 010100100\rangle + \alpha \mid 010100101\rangle + \alpha \mid 010100110\rangle + \alpha \mid 010100111\rangle$
$+\alpha \mid 010101000\rangle + \alpha \mid 010101001\rangle + \alpha \mid 010101010\rangle + \alpha \mid 010101011\rangle$
$+\alpha \mid 010101100\rangle + \alpha \mid 010101101\rangle + \alpha \mid 010101110\rangle + \alpha \mid 010101111 \rangle$
$-\alpha \mid 000110000\rangle - \alpha \mid 000110001\rangle - \alpha \mid 000110010\rangle - \alpha \mid 000110011 \rangle$
$-\alpha \mid 000110100\rangle - \alpha \mid 000110101\rangle - \alpha \mid 000110110\rangle - \alpha \mid 000110111 \rangle$
$-\alpha \mid 000111000\rangle - \alpha \mid 000111001\rangle - \alpha \mid 000111010\rangle - \alpha \mid 000111011\rangle$

$$-\alpha \mid 000111100\rangle - \alpha \mid 000111101\rangle - \alpha \mid 000111110\rangle - \alpha \mid 000111111\rangle$$
$$-\alpha \mid 111100000\rangle - \alpha \mid 111100001\rangle - \alpha \mid 111100010\rangle - \alpha \mid 111100011\rangle$$
$$-\alpha \mid 111100100\rangle - \alpha \mid 111100101\rangle - \alpha \mid 111100110\rangle - \alpha \mid 111100111\rangle$$
$$-\alpha \mid 111101000\rangle - \alpha \mid 111101001\rangle - \alpha \mid 111101010\rangle - \alpha \mid 111101011\rangle$$
$$-\alpha \mid 111101100\rangle - \alpha \mid 111101101\rangle - \alpha \mid 111101110\rangle - \alpha \mid 111101111\rangle$$
$$-\alpha \mid 101110000\rangle - \alpha \mid 101110001\rangle - \alpha \mid 101110010\rangle - \alpha \mid 101110011\rangle$$
$$-\alpha \mid 101110100\rangle - \alpha \mid 101110101\rangle - \alpha \mid 101110110\rangle - \alpha \mid 101110111\rangle$$
$$-\alpha \mid 101111000\rangle - \alpha \mid 101111001\rangle - \alpha \mid 101111010\rangle - \alpha \mid 101111011\rangle$$
$$-\alpha \mid 101111100\rangle - \alpha \mid 101111101\rangle - \alpha \mid 101111110\rangle - \alpha \mid 101111111\rangle$$
$$+\alpha \mid 001000000\rangle + \alpha \mid 001000001\rangle + \alpha \mid 001000010\rangle + \alpha \mid 001000011\rangle$$
$$+\alpha \mid 001000100\rangle + \alpha \mid 001000101\rangle + \alpha \mid 001000110\rangle + \alpha \mid 001000111\rangle$$
$$+\alpha \mid 001001000\rangle + \alpha \mid 001001001\rangle + \alpha \mid 001001010\rangle + \alpha \mid 001001011\rangle$$
$$+\alpha \mid 001001100\rangle + \alpha \mid 001001101\rangle + \alpha \mid 001001100\rangle + \alpha \mid 001001111\rangle$$
$$-\alpha \mid 011010000\rangle - \alpha \mid 011010001\rangle - \alpha \mid 011010010\rangle - \alpha \mid 011010011\rangle$$
$$-\alpha \mid 011010100\rangle - \alpha \mid 011010101\rangle - \alpha \mid 011010110\rangle - \alpha \mid 011010111\rangle$$
$$-\alpha \mid 011011000\rangle - \alpha \mid 011011001\rangle - \alpha \mid 011011010\rangle - \alpha \mid 011011011\rangle$$
$$-\alpha \mid 011011100\rangle - \alpha \mid 011011101\rangle - \alpha \mid 011011110\rangle - \alpha \mid 011011111\rangle$$
$$+\ \beta \mid 010000000\rangle +\ \beta \mid 010000001\rangle +\ \beta \mid 010000010\rangle +\ \beta \mid 010000011\rangle$$
$$+\ \beta \mid 010000100\rangle +\ \beta \mid 010000101\rangle +\ \beta \mid 010000110\rangle +\ \beta \mid 010000111\rangle$$
$$+\ \beta \mid 010001000\rangle +\ \beta \mid 010001001\rangle +\ \beta \mid 010001010\rangle +\ \beta \mid 010001011\rangle$$
$$+\ \beta \mid 010001100\rangle +\ \beta \mid 010001101\rangle +\ \beta \mid 010001110\rangle +\ \beta \mid 010001111\rangle$$
$$+\ \beta \mid 000010000\rangle +\ \beta \mid 000010001\rangle +\ \beta \mid 000010010\rangle +\ \beta \mid 000010011\rangle$$
$$+\ \beta \mid 000010100\rangle +\ \beta \mid 000010101\rangle +\ \beta \mid 000010110\rangle +\ \beta \mid 000010111\rangle$$
$$+\ \beta \mid 000011000\rangle +\ \beta \mid 000011001\rangle +\ \beta \mid 000011010\rangle +\ \beta \mid 000011011\rangle$$
$$+\ \beta \mid 000011100\rangle +\ \beta \mid 000011101\rangle +\ \beta \mid 000011110\rangle +\ \beta \mid 000011111\rangle$$
$$+\ \beta \mid 100100000\rangle +\ \beta \mid 100100001\rangle +\ \beta \mid 100100010\rangle +\ \beta \mid 100100011\rangle$$
$$+\ \beta \mid 100100100\rangle +\ \beta \mid 100100101\rangle +\ \beta \mid 100100110\rangle +\ \beta \mid 100100111\rangle$$
$$+\ \beta \mid 100101000\rangle +\ \beta \mid 100101001\rangle +\ \beta \mid 100101010\rangle +\ \beta \mid 100101011\rangle$$
$$+\ \beta \mid 100101100\rangle +\ \beta \mid 100101101\rangle +\ \beta \mid 100101110\rangle +\ \beta \mid 100101111\rangle$$
$$+\beta \mid 110110000\rangle + \beta \mid 110110001\rangle +\ \beta \mid 110110010\rangle + \beta \mid 110110011\rangle$$
$$+\ \beta \mid 110110100\rangle +\ \beta \mid 110110101\rangle +\ \beta \mid 110110110\rangle +\ \beta \mid 110110111\rangle$$
$$+\ \beta \mid 110111000\rangle +\ \beta \mid 110111001\rangle +\ \beta \mid 110111010\rangle +\ \beta \mid 110111011\rangle$$

$+ \beta \mid 110111100\rangle + \beta \mid 110111101\rangle + \beta \mid 110111110\rangle + \beta \mid 110111111\rangle$
$- \beta \mid 001100000\rangle - \beta \mid 001100001\rangle - \beta \mid 001100010\rangle - \beta \mid 001100011\rangle$
$- \beta \mid 001100100\rangle - \beta \mid 001100101\rangle - \beta \mid 001100110\rangle - \beta \mid 001100111\rangle$
$- \beta \mid 001101000\rangle - \beta \mid 001101001\rangle - \beta \mid 001101010\rangle - \beta \mid 001101011\rangle$
$- \beta \mid 001101100\rangle - \beta \mid 001101101\rangle - \beta \mid 001101110\rangle - \beta \mid 001101111\rangle$
$- \beta \mid 011110000\rangle - \beta \mid 011110001\rangle - \beta \mid 011110010\rangle - \beta \mid 011110011\rangle$
$- \beta \mid 011110100\rangle - \beta \mid 011110101\rangle - \beta \mid 011110110\rangle - \beta \mid 011110111\rangle$
$- \beta \mid 011111000\rangle - \beta \mid 011111001\rangle - \beta \mid 011111010\rangle - \beta \mid 011111011\rangle$
$- \beta \mid 011111100\rangle - \beta \mid 011111101\rangle - \beta \mid 011111110\rangle - \beta \mid 011111111\rangle$
$+\beta \mid 111000000\rangle + \beta \mid 111000001\rangle + \beta \mid 111000010\rangle + \beta \mid 111000011\rangle$
$+\beta \mid 111000100\rangle + \beta \mid 111000101\rangle + \beta \mid 111000110\rangle + \beta \mid 111000111\rangle$
$+\beta \mid 111001000\rangle + \beta \mid 111001001\rangle + \beta \mid 111001010\rangle + \beta \mid 111001011\rangle$
$+\beta \mid 111001100\rangle + \beta \mid 111001101\rangle + \beta \mid 111001110\rangle + \beta \mid 111001111\rangle$
$-\beta \mid 101010000\rangle - \beta \mid 101010001\rangle - \beta \mid 101010010\rangle - \beta \mid 101010011\rangle$
$-\beta \mid 101010100\rangle - \beta \mid 101010101\rangle - \beta \mid 101010110\rangle - \beta \mid 101010111\rangle$
$-\beta \mid 101011000\rangle - \beta \mid 101011001\rangle - \beta \mid 101011010\rangle - \beta \mid 101011011\rangle$
$-\beta \mid 101011100\rangle - \beta \mid 101011100\rangle - \beta \mid 101011110\rangle - \beta \mid 101011111\rangle)$

∘ **Application controlée des stabilisateurs** :

·Pour$M_0 = X(1)Z(2)X(3)Z(4)I(5)$

·Pour$M_1 = Z(1)Z(2)Z(3)I(4)Z(5)$

·Pour$M_2 = I(1)X(2)Z(3)Z(4)X(5)$

·Pour$M_3 = Z(1)I(2)X(3)X(4)X(5)$

Par conséquent, on trouve :

$\mid \Psi_1\rangle = \frac{1}{8\sqrt{2}}(-\alpha \mid 100000000\rangle + \alpha \mid 101110001\rangle - \alpha \mid 110010010\rangle + \alpha \mid 111100011\rangle$
$+\alpha \mid 100000100\rangle - \alpha \mid 101110101\rangle + \alpha \mid 110010110\rangle - \alpha \mid 111100111\rangle$
$-\alpha \mid 001001000\rangle - \alpha \mid 000111001\rangle + \alpha \mid 011011010\rangle + \alpha \mid 010101011\rangle$
$+\alpha \mid 001001100\rangle + \alpha \mid 000111101\rangle - \alpha \mid 011011110\rangle - \alpha \mid 010101111\rangle$
$- \alpha \mid 110010000\rangle + \alpha \mid 111100001\rangle - \alpha \mid 100000010\rangle + \alpha \mid 101110011\rangle$
$+\alpha \mid 110010100\rangle - \alpha \mid 111100101\rangle + \alpha \mid 100000110\rangle - \alpha \mid 101110111\rangle$
$+\alpha \mid 011011000\rangle + \alpha \mid 010101001\rangle - \alpha \mid 001001010\rangle - \alpha \mid 000111011\rangle$
$-\alpha \mid 011011100\rangle - \alpha \mid 010101101\rangle + \alpha \mid 001001110\rangle + \alpha \mid 000111111\rangle$
$+\alpha \mid 010100000\rangle + \alpha \mid 011010001\rangle - \alpha \mid 000110010\rangle - \alpha \mid 001000011\rangle$

$-\alpha \mid 010100100\rangle - \alpha \mid 011010101\rangle + \alpha \mid 000110110\rangle + \alpha \mid 001000111\rangle$

$+\alpha \mid 111101000\rangle - \alpha \mid 110011001\rangle + \alpha \mid 101111010\rangle - \alpha \mid 100001011\rangle$

$-\alpha \mid 111101100\rangle + \alpha \mid 110011101\rangle - \alpha \mid 101111110\rangle + \alpha \mid 100001111\rangle$

$-\alpha \mid 000110000\rangle - \alpha \mid 001000001\rangle + \alpha \mid 010100010\rangle + \alpha \mid 011010011\rangle$

$+\alpha \mid 000110100\rangle + \alpha \mid 001000101\rangle - \alpha \mid 010100110\rangle - \alpha \mid 011010111\rangle$

$+\alpha \mid 101111000\rangle - \alpha \mid 100001001\rangle + \alpha \mid 111101010\rangle - \alpha \mid 110011011\rangle$

$-\alpha \mid 101111100\rangle + \alpha \mid 100001101\rangle - \alpha \mid 111101110\rangle + \alpha \mid 110011111\rangle$

$-\alpha \mid 111100000\rangle + \alpha \mid 110010001\rangle - \alpha \mid 101110010\rangle + \alpha \mid 100000011\rangle$

$+\alpha \mid 111100100\rangle - \alpha \mid 110010101\rangle + \alpha \mid 101110110\rangle - \alpha \mid 100000111\rangle$

$-\alpha \mid 010101000\rangle - \alpha \mid 011011001\rangle + \alpha \mid 000111010\rangle + \alpha \mid 001001011\rangle$

$+\alpha \mid 010101100\rangle + \alpha \mid 011011101\rangle - \alpha \mid 000111110\rangle - \alpha \mid 001001111\rangle$

$-\alpha \mid 101110000\rangle + \alpha \mid 100000001\rangle - \alpha \mid 111100010\rangle + \alpha \mid 110010011\rangle$

$+\alpha \mid 101110100\rangle - \alpha \mid 100000101\rangle + \alpha \mid 111100110\rangle - \alpha \mid 110010111\rangle$

$+\alpha \mid 000111000\rangle + \alpha \mid 001001001\rangle - \alpha \mid 010101010\rangle - \alpha \mid 011011011\rangle$

$-\alpha \mid 000111100\rangle - \alpha \mid 001001101\rangle + \alpha \mid 010101110\rangle + \alpha \mid 011011111\rangle$

$+\alpha \mid 001000000\rangle + \alpha \mid 000110001\rangle - \alpha \mid 011010010\rangle - \alpha \mid 010100011\rangle$

$-\alpha \mid 001000100\rangle - \alpha \mid 000110101\rangle + \alpha \mid 011010110\rangle + \alpha \mid 010100111\rangle$

$+\alpha \mid 100001000\rangle - \alpha \mid 101111001\rangle + \alpha \mid 110011010\rangle - \alpha \mid 111101011\rangle$

$-\alpha \mid 100001100\rangle + \alpha \mid 101111101\rangle - \alpha \mid 110011110\rangle + \alpha \mid 111101111\rangle$

$-\alpha \mid 011010000\rangle - \alpha \mid 010100001\rangle + \alpha \mid 001000010\rangle + \alpha \mid 000110011\rangle$

$+\alpha \mid 011010100\rangle + \alpha \mid 010100101\rangle - \alpha \mid 001000110\rangle - \alpha \mid 000110111\rangle$

$+\alpha \mid 110011000\rangle - \alpha \mid 111101001\rangle + \alpha \mid 100001010\rangle - \alpha \mid 101111011\rangle$

$-\alpha \mid 110011100\rangle + \alpha \mid 111101101\rangle - \alpha \mid 100001110\rangle + \alpha \mid 101111111\rangle$

$+ \beta \mid 010000000\rangle + \beta \mid 011110001\rangle + \beta \mid 000010010\rangle + \beta \mid 001100011\rangle$

$-\beta \mid 010000100\rangle - \beta \mid 011110101\rangle - \beta \mid 000010110\rangle - \beta \mid 001100111\rangle$

$-\beta \mid 111001000\rangle + \beta \mid 110111001\rangle + \beta \mid 101011010\rangle - \beta \mid 100101011\rangle$

$+\beta \mid 111001100\rangle - \beta \mid 110111101\rangle - \beta \mid 101011110\rangle + \beta \mid 100101111\rangle$

$+\beta \mid 000010000\rangle + \beta \mid 001100001\rangle + \beta \mid 010000010\rangle + \beta \mid 011110011\rangle$

$-\beta \mid 000010100\rangle - \beta \mid 001100101\rangle - \beta \mid 010000110\rangle - \beta \mid 011110111\rangle$

$+\beta \mid 101011000\rangle - \beta \mid 100101001\rangle - \beta \mid 111001010\rangle + \beta \mid 110111011\rangle$

$-\beta \mid 101011100\rangle + \beta \mid 100101101\rangle + \beta \mid 111001110\rangle - \beta \mid 110111111\rangle$

$-\beta \mid 100100000\rangle + \beta \mid 101010001\rangle + \beta \mid 110110010\rangle - \beta \mid 111000011\rangle$

$+\beta \mid 100100100\rangle - \beta \mid 101010101\rangle - \beta \mid 110110110\rangle + \beta \mid 111000111\rangle$
$+\beta \mid 001101000\rangle + \beta \mid 000011001\rangle + \beta \mid 011111010\rangle + \beta \mid 010001011\rangle$
$-\beta \mid 001101100\rangle - \beta \mid 000011101\rangle - \beta \mid 011111110\rangle - \beta \mid 010001111\rangle$
$+\beta \mid 110110000\rangle - \beta \mid 111000001\rangle - \beta \mid 100100010\rangle + \beta \mid 101010011\rangle$
$-\beta \mid 110110100\rangle + \beta \mid 111000101\rangle + \beta \mid 100100110\rangle - \beta \mid 101010111\rangle$
$+\beta \mid 011111000\rangle + \beta \mid 010001001\rangle + \beta \mid 001101010\rangle + \beta \mid 000011011\rangle$
$-\beta \mid 011111100\rangle - \beta \mid 010001101\rangle - \beta \mid 001101110\rangle - \beta \mid 000011111\rangle$
-$\beta \mid 001100000\rangle - \beta \mid 000010001\rangle - \beta \mid 011110010\rangle - \beta \mid 010000011\rangle$
$+\beta \mid 001100100\rangle + \beta \mid 000010101\rangle + \beta \mid 011110110\rangle + \beta \mid 010000111\rangle$
$+\beta \mid 100101000\rangle - \beta \mid 101011001\rangle - \beta \mid 110111010\rangle + \beta \mid 111001011\rangle$
$-\beta \mid 100101100\rangle + \beta \mid 101011101\rangle + \beta \mid 110111110\rangle - \beta \mid 111001111\rangle$
-$\beta \mid 011110000\rangle - \beta \mid 010000001\rangle - \beta \mid 001100010\rangle - \beta \mid 000010011\rangle$
$+\beta \mid 011110100\rangle + \beta \mid 010000101\rangle + \beta \mid 001100110\rangle + \beta \mid 000010111\rangle$
$-\beta \mid 110111000\rangle + \beta \mid 111001001\rangle + \beta \mid 100101010\rangle - \beta \mid 101011011\rangle$
$+\beta \mid 110111100\rangle - \beta \mid 111001101\rangle - \beta \mid 100101110\rangle + \beta \mid 101011111\rangle$
$+\beta \mid 111000000\rangle - \beta \mid 110110001\rangle - \beta \mid 101010010\rangle + \beta \mid 100100011\rangle$
$-\beta \mid 111000100\rangle + \beta \mid 110110101\rangle + \beta \mid 101010110\rangle - \beta \mid 100100111\rangle$
$-\beta \mid 010001000\rangle - \beta \mid 011111001\rangle - \beta \mid 000011010\rangle - \beta \mid 001101011\rangle$
$+\beta \mid 010001100\rangle + \beta \mid 011111101\rangle + \beta \mid 000011110\rangle + \beta \mid 001101111\rangle$
-$\beta \mid 101010000\rangle + \beta \mid 100100001\rangle + \beta \mid 111000010\rangle - \beta \mid 110110011\rangle$
$+\beta \mid 101010100\rangle - \beta \mid 100100101\rangle - \beta \mid 111000110\rangle + \beta \mid 110110111\rangle$
$-\beta \mid 000011000\rangle - \beta \mid 001101001\rangle - \beta \mid 010001010\rangle - \beta \mid 011111011\rangle$
$+\beta \mid 000011100\rangle + \beta \mid 001101101\rangle + \beta \mid 010001110\rangle + \beta \mid 011111111\rangle\ )$

·L'application du $H(6)$:

·L'application du $H(7)$:

·L'application du $H(8)$:

·L'application du $H(9)$:

Et après toutes les simplifications,onaura

$$\begin{aligned}\mid \Psi_1\rangle = \tfrac{1}{2\sqrt{2}}(& -\alpha \mid 100001101\rangle - \alpha \mid 101111101\rangle - \alpha \mid 110011101\rangle - \alpha \mid 111101101\rangle \\ & +\alpha \mid 001001101\rangle - \alpha \mid 000111101\rangle - \alpha \mid 011011101\rangle + \alpha \mid 010101101\rangle \\ & +\beta \mid 010001101\rangle - \beta \mid 011111101\rangle + \beta \mid 000011101\rangle - \beta \mid 001101101\rangle \\ & +\beta \mid 111001101\rangle + \beta \mid 110111101\rangle - \beta \mid 101011101\rangle - \beta \mid 100101101\rangle)\end{aligned}$$

* **Correction d'erreure** :

Résultat de mesure= 1101,donc erreur de type Y_1

La correction :appliquer la port Ysur le premier qubit

$\mid \Psi_1\rangle = \frac{1}{2\sqrt{2}}(-\alpha \mid 000000101\rangle - \alpha \mid 001110101\rangle - \alpha \mid 010010101\rangle - \alpha \mid 011100101\rangle$
$-\alpha \mid 101000101\rangle + \alpha \mid 100110101\rangle + \alpha \mid 111010101\rangle - \alpha \mid 110100101\rangle - \beta \mid 110000101\rangle$
$+\beta \mid 111110101\rangle - \beta \mid 100010101\rangle + \beta \mid 101100101\rangle + \beta \mid 011000101\rangle + \beta \mid 010110101\rangle$
$-\beta \mid 001010101\rangle - \beta \mid 000100101\rangle)$

Suppression des quatre qubits du syndrome :

$$\mid \Psi_1\rangle = \frac{1}{2\sqrt{2}}(-\alpha \mid 00000\rangle - \alpha \mid 00111 \rangle - \alpha \mid 01001 \rangle - \alpha \mid 01110 \rangle - \alpha \mid 10100 \rangle + \alpha \mid 10011 \rangle$$
$$+\alpha \mid 11101 \rangle - \alpha \mid 11010 \rangle - \beta \mid 11000 \rangle + \beta \mid 11111 \rangle - \beta \mid 10001 \rangle + \beta \mid 10110 \rangle + \beta \mid 01100 \rangle$$
$$+\beta \mid 01011 \rangle - \beta \mid 00101 \rangle - \beta \mid 00010 \rangle)$$

·Le calcul du autres cas des erreure de type $Y(Y_2, Y_3, Y_4, Y_5)$est de la même méthode.

∗**Cas 04** :Erreur de type Z

- **Injecter une erreure Z sur le premier qubit :**

$$\mid \Psi_1\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 01001\rangle - \alpha \mid 11010\rangle + \alpha \mid 10011\rangle + \alpha \mid 01110\rangle + \alpha \mid 00111\rangle$$
$$-\alpha \mid 10100\rangle + \alpha \mid 11101\rangle - \beta \mid 11000\rangle - \beta \mid 10001\rangle + \beta \mid 00010\rangle - \beta \mid 01011\rangle + \beta \mid 10110\rangle$$
$$+\beta \mid 11111\rangle - \beta \mid 01100\rangle + \beta \mid 00101\rangle)$$

***Mesure du syndrome** :

Ajout des quatre qubits du syndrome :

$\mid \Psi_1\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 000000000\rangle + \alpha \mid 010010000\rangle - \alpha \mid 110100000\rangle + \alpha \mid 100110000\rangle$
$+\alpha \mid 011100000\rangle + \alpha \mid 001110000\rangle - \alpha \mid 101000000\rangle + \alpha \mid 111010000\rangle - \beta \mid 110000000\rangle$
$- \beta \mid 100010000\rangle + \beta \mid 000100000\rangle - \beta \mid 010110000\rangle + \beta \mid 101100000\rangle + \beta \mid 111110000\rangle$
$-\beta \mid 011000000\rangle + \beta \mid 001010000\rangle)$

·L'application du $H(6)$:

·L'application du $H(7)$:

·L'application du $H(8)$:

·<u>L'application du $H(9)$</u> :

Aprés ces applications, on obtient a :

$|\ \Psi_1\rangle = \frac{1}{8\sqrt{2}}(\alpha \mid 000000000\rangle + \alpha \mid 000000001\rangle + \alpha \mid 000000010\rangle + \alpha \mid 000000011\rangle$

$+\alpha \mid 000000100\rangle + \alpha \mid 000000101\rangle + \alpha \mid 000000110\rangle + \alpha \mid 000000111\rangle$

$+\alpha \mid 000001000\rangle + \alpha \mid 000001001\rangle + \alpha \mid 000001010\rangle + \alpha \mid 000001011\rangle$

$+\alpha \mid 000001100\rangle + \alpha \mid 000001101\rangle + \alpha \mid 000001110\rangle + \alpha \mid 000001111\rangle$

$+ \alpha \mid 010010000 \rangle + \alpha \mid 010010001 \rangle + \alpha \mid 010010010 \rangle + \alpha \mid 010010011 \rangle$

$+\alpha \mid 010010100 \rangle + \alpha \mid 010010101 \rangle + \alpha \mid 010010110 \rangle + \alpha \mid 010010111 \rangle$

$+\alpha \mid 010011000 \rangle + \alpha \mid 010011001 \rangle + \alpha \mid 010011010 \rangle + \alpha \mid 010011011 \rangle$

$+\alpha \mid 010011100 \rangle + \alpha \mid 010011101 \rangle + \alpha \mid 010011110 \rangle + \alpha \mid 010011111 \rangle$

-$\alpha \mid 110100000 \rangle - \alpha \mid 110100001 \rangle$ -$\alpha \mid 110100010 \rangle - \alpha \mid 110100011 \rangle$

$-\alpha \mid 110100100 \rangle - \alpha \mid 110100101\rangle - \alpha \mid 110100110 \rangle - \alpha \mid 110100111 \rangle$

$-\alpha \mid 110101000 \rangle - \alpha \mid 110101001 \rangle - \alpha \mid 110101010 \rangle - \alpha \mid 110101011 \rangle$

$-\alpha \mid 110101100 \rangle - \alpha \mid 110101101 \rangle - \alpha \mid 110101110 \rangle - \alpha \mid 110101111 \rangle$

$+\alpha \mid 100110000 \rangle + \alpha \mid 100110001 \rangle+\alpha \mid 100110010 \rangle + \alpha \mid 100110011 \rangle$

$+\alpha \mid 100110100 \rangle + \alpha \mid 100110101 \rangle + \alpha \mid 100110110 \rangle + \alpha \mid 100110111 \rangle$

$+\alpha \mid 100111000 \rangle + \alpha \mid 100111001 \rangle + \alpha \mid 100111010 \rangle + \alpha \mid 100111011 \rangle$

$+\alpha \mid 100111100 \rangle + \alpha \mid 100111101 \rangle + \alpha \mid 100111110 \rangle + \alpha \mid 100111111 \rangle$

$+\alpha \mid 011100000 \rangle + \alpha \mid 011100001 \rangle+\alpha \mid 011100010 \rangle + \alpha \mid 011100011 \rangle$

$+\alpha \mid 011100100 \rangle + \alpha \mid 011100101 \rangle+\alpha \mid 011100110 \rangle + \alpha \mid 011100111 \rangle$

$+\alpha \mid 011101000 \rangle + \alpha \mid 011101001 \rangle + \alpha \mid 011101010 \rangle + \alpha \mid 011101011 \rangle$

$+\alpha \mid 011101100 \rangle + \alpha \mid 011101101 \rangle + \alpha \mid 011101110 \rangle + \alpha \mid 011101111 \rangle$

$+\alpha \mid 001110000 \rangle + \alpha \mid 001110001 \rangle+\alpha \mid 001110010 \rangle + \alpha \mid 001110011 \rangle$

$+\alpha \mid 001110100 \rangle + \alpha \mid 001110101 \rangle+\alpha \mid 001110110 \rangle + \alpha \mid 001110111 \rangle$

$+\alpha \mid 001111000 \rangle + \alpha \mid 001111001 \rangle + \alpha \mid 001111010 \rangle + \alpha \mid 001111011 \rangle$

$+\alpha \mid 001111100 \rangle + \alpha \mid 001111101 \rangle + \alpha \mid 001111110 \rangle + \alpha \mid 001111111 \rangle$

-$\alpha \mid 101000000 \rangle - \alpha \mid 101000001\rangle$-$\alpha \mid 101000010 \rangle - \alpha \mid 101000011 \rangle$

-$\alpha \mid 101000100 \rangle - \alpha \mid 101000101 \rangle$-$\alpha \mid 101000110 \rangle - \alpha \mid 101000111 \rangle$

$-\alpha \mid 101001000 \rangle - \alpha \mid 101001001 \rangle - \alpha \mid 101001010 \rangle - \alpha \mid 101001011 \rangle$

$-\alpha \mid 101001100 \rangle - \alpha \mid 101001101 \rangle - \alpha \mid 101001110 \rangle - \alpha \mid 101001111 \rangle$

$+\alpha \mid 111010000 \rangle + \alpha \mid 111010001 \rangle+\alpha \mid 111010010 \rangle + \alpha \mid 111010011 \rangle$

$+\alpha \mid 111010100 \rangle + \alpha \mid 111010101 \rangle+\alpha \mid 111010110 \rangle + \alpha \mid 111010111 \rangle$

$+\alpha \mid 111011000 \rangle + \alpha \mid 111011001\rangle + \alpha \mid 111011010 \rangle + \alpha \mid 111011011\rangle$
$+\alpha \mid 111011100 \rangle + \alpha \mid 111011101\rangle + \alpha \mid 111011110 \rangle + \alpha \mid 111011111\rangle$
- $\beta \mid 110000000 \rangle - \beta \mid 110000001 \rangle - \beta \mid 110000010 \rangle - \beta \mid 110000011\rangle$
$-\beta \mid 110000100 \rangle - \beta \mid 110000101 \rangle - \beta \mid 110000110 \rangle - \beta \mid 110000111 \rangle$
$-\beta \mid 110001000 \rangle - \beta \mid 110001001 \rangle - \beta \mid 110001010 \rangle - \beta \mid 110001011 \rangle$
$-\beta \mid 110001100 \rangle - \beta \mid 110001101 \rangle - \beta \mid 110001110 \rangle - \beta \mid 110001111 \rangle$
-$\beta \mid 100010000 \rangle - \beta \mid 100010001 \rangle - \beta \mid 100010010 \rangle - \beta \mid 100010011 \rangle$
$-\beta \mid 100010100 \rangle - \beta \mid 100010101 \rangle - \beta \mid 100010110 \rangle - \beta \mid 100010111 \rangle$
$-\beta \mid 100011000 \rangle - \beta \mid 100011001 \rangle - \beta \mid 100011010 \rangle - \beta \mid 100011011 \rangle$
$-\beta \mid 100011100 \rangle - \beta \mid 100011101 \rangle - \beta \mid 100011110 \rangle - \beta \mid 100011111 \rangle$
$+\beta \mid 000100000 \rangle + \beta \mid 000100001 \rangle + \beta \mid 000100010 \rangle + \beta \mid 000100011 \rangle$
$+\beta \mid 000100100 \rangle + \beta \mid 000100101 \rangle + \beta \mid 000100110 \rangle + \beta \mid 000100111 \rangle$
$+\beta \mid 000101000 \rangle + \beta \mid 000101001 \rangle + \beta \mid 000101010 \rangle + \beta \mid 000101011 \rangle$
$+\beta \mid 000101100 \rangle + \beta \mid 000101101 \rangle + \beta \mid 000101110 \rangle + \beta \mid 000101111 \rangle$
-$\beta \mid 010110000 \rangle \beta \mid 010110001 \rangle - \beta \mid 010110010 \rangle - \beta \mid 010110011 \rangle$
$-\beta \mid 010110100 \rangle - \beta \mid 010110101 \rangle - \beta \mid 010110110 \rangle - \beta \mid 010110111 \rangle$
$-\beta \mid 010111000 \rangle - \beta \mid 010111001 \rangle - \beta \mid 010111010 \rangle - \beta \mid 010111011 \rangle$
$-\beta \mid 010111100 \rangle - \beta \mid 010111101 \rangle - \beta \mid 010111110 \rangle - \beta \mid 010111111 \rangle$
$+\beta \mid 101100000 \rangle + \beta \mid 101100001 \rangle + \beta \mid 101100010 \rangle + \beta \mid 101100011 \rangle$
$+\beta \mid 101100100 \rangle + \beta \mid 101100101 \rangle + \beta \mid 101100110 \rangle + \beta \mid 101100111 \rangle$
$+\beta \mid 101101000 \rangle + \beta \mid 101101001 \rangle + \beta \mid 101101010 \rangle + \beta \mid 101101011\rangle$
$+\beta \mid 101101100 \rangle + \beta \mid 101101101 \rangle + \beta \mid 101101110 \rangle + \beta \mid 101101111 \rangle$
$+\beta \mid 111110000 \rangle + \beta \mid 111110001 \rangle + \beta \mid 111110010 \rangle + \beta \mid 111110011 \rangle$
$+\beta \mid 111110100 \rangle + \beta \mid 111110101 \rangle + \beta \mid 111110110 \rangle + \beta \mid 111110111 \rangle$
$+\beta \mid 111111000 \rangle + \beta \mid 111111001 \rangle + \beta \mid 111111010 \rangle + \beta \mid 111111011 \rangle$
$+\beta \mid 111111100\rangle + \beta \mid 111111101\rangle + \beta \mid 111111110\rangle + \beta \mid 111111111\rangle$
-$\beta \mid 011000000 \rangle - \beta \mid 011000001 \rangle - \beta \mid 011000010 \rangle - \beta \mid 011000011 \rangle$
$-\beta \mid 011000100 \rangle - \beta \mid 011000101 \rangle - \beta \mid 011000110 \rangle - \beta \mid 011000111 \rangle$
$-\beta \mid 011001000 \rangle - \beta \mid 011001001 \rangle - \beta \mid 011001010 \rangle - \beta \mid 011001011\rangle$
$-\beta \mid 011001100\rangle - \beta \mid 011001101\rangle - \beta \mid 011001110\rangle - \beta \mid 011001111\rangle$
$+\beta \mid 001010000\rangle + \beta \mid 001010001\rangle + \beta \mid 001010010\rangle + \beta \mid 001010011\rangle$
$+\beta \mid 001010100\rangle + \beta \mid 001010101\rangle + \beta \mid 001010110\rangle + \beta \mid 001010111\rangle$

$+\beta \mid 001011000\rangle + \beta \mid 001011001\rangle + \beta \mid 001011010\rangle + \beta \mid 001011011\rangle$

$+\beta \mid 001011100\rangle + \beta \mid 001011101\rangle + \beta \mid 001011110\rangle + \beta \mid 001011111\rangle)$

○ Application controlée des stabilisateurs :

·Pour $M_0 = X(1)Z(2)X(3)Z(4)I(5)$

·Pour$M_1 = Z(1)Z(2)Z(3)I(4)Z(5)$

·Pour$M_2 = I(1)X(2)Z(3)Z(4)X(5)$

·Pour$M_3 = Z(1)I(2)X(3)X(4)X(5)$

Par conséquent, on trouve :

$\mid \Psi_1\rangle = \frac{1}{8\sqrt{2}}(\ \alpha \mid 000000000\rangle + \alpha \mid 001110001\rangle + \alpha \mid 010010010\rangle + \alpha \mid 011100011\rangle$

$+\alpha \mid 000000100\rangle + \alpha \mid 001110101\rangle + \alpha \mid 010010110\rangle + \alpha \mid 011100111\rangle$

$+\alpha \mid 101001000\rangle - \alpha \mid 100111001\rangle - \alpha \mid 111011010\rangle + \alpha \mid 110101011\rangle$

$+\alpha \mid 101001100\rangle - \alpha \mid 100111101\rangle - \alpha \mid 111011110\rangle + \alpha \mid 110101111\rangle$

$+\alpha \mid 010010000\rangle + \alpha \mid 011100001\rangle + \alpha \mid 000000010\rangle + \alpha \mid 001110011\rangle$

$+\alpha \mid 010010100\rangle + \alpha \mid 011100101\rangle + \alpha \mid 000000110\rangle + \alpha \mid 001110111\rangle$

$-\alpha \mid 111011000\rangle + \alpha \mid 110101001\rangle + \alpha \mid 101001010\rangle - \alpha \mid 100111011\rangle$

$-\alpha \mid 111011100\rangle + \alpha \mid 110101101\rangle + \alpha \mid 101001110\rangle - \alpha \mid 100111111\rangle$

$-\alpha \mid 110100000\rangle + \alpha \mid 111010001\rangle + \alpha \mid 100110010\rangle - \alpha \mid 101000011\rangle$

$-\alpha \mid 110100100\rangle + \alpha \mid 111010101\rangle + \alpha \mid 100110110\rangle - \alpha \mid 101000111\rangle$

$-\alpha \mid 011101000\rangle - \alpha \mid 010011001\rangle - \alpha \mid 001111010\rangle - \alpha \mid 000001011\rangle$

$-\alpha \mid 011101100\rangle - \alpha \mid 010011101\rangle - \alpha \mid 001111110\rangle - \alpha \mid 000001111\rangle$

$+\alpha \mid 100110000\rangle - \alpha \mid 101000001\rangle - \alpha \mid 110100010\rangle + \alpha \mid 111010011\rangle$

$+\alpha \mid 100110100\rangle - \alpha \mid 101000101\rangle - \alpha \mid 110100110\rangle + \alpha \mid 111010111\rangle$

$-\alpha \mid 001111000\rangle - \alpha \mid 000001001\rangle - \alpha \mid 011101010\rangle - \alpha \mid 010011011\rangle$

$-\alpha \mid 001111100\rangle - \alpha \mid 000001101\rangle - \alpha \mid 011101110\rangle - \alpha \mid 010011111\rangle$

$+\alpha \mid 011100000\rangle + \alpha \mid 010010001\rangle + \alpha \mid 001110010\rangle + \alpha \mid 000000011\rangle$

$+\alpha \mid 011100100\rangle + \alpha \mid 010010101\rangle + \alpha \mid 001110110\rangle + \alpha \mid 000000111\rangle$

$+\alpha \mid 110101000\rangle - \alpha \mid 111011001\rangle - \alpha \mid 100111010\rangle + \alpha \mid 101001011\rangle$

$+\alpha \mid 110101100\rangle - \alpha \mid 111011101\rangle - \alpha \mid 100111110\rangle + \alpha \mid 101001111\rangle$

$+\alpha \mid 001110000\rangle + \alpha \mid 000000001\rangle + \alpha \mid 011100010\rangle + \alpha \mid 010010011\rangle$

$+\alpha \mid 001110100\rangle + \alpha \mid 000000101\rangle + \alpha \mid 011100110\rangle + \alpha \mid 010010111\rangle$

$-\alpha \mid 100111000\rangle + \alpha \mid 101001001\rangle + \alpha \mid 110101010\rangle - \alpha \mid 111011011\rangle$

$-\alpha \mid 100111100\rangle + \alpha \mid 101001101\rangle + \alpha \mid 110101110\rangle - \alpha \mid 111011111\rangle$

-$\alpha \mid 101000000 \rangle + \alpha \mid 100110001\rangle + \alpha \mid 111010010 \rangle - \alpha \mid 110100011 \rangle$

-$\alpha \mid 101000100 \rangle + \alpha \mid 100110101 \rangle + \alpha \mid 111010110 \rangle - \alpha \mid 110100111 \rangle$

$-\alpha \mid 000001000 \rangle - \alpha \mid 001111001 \rangle - \alpha \mid 010011010 \rangle - \alpha \mid 011101011 \rangle$

$-\alpha \mid 000001100 \rangle - \alpha \mid 001111101 \rangle - \alpha \mid 010011110 \rangle - \alpha \mid 011101111 \rangle$

$+\alpha \mid 111010000 \rangle - \alpha \mid 110100001 \rangle$-$\alpha \mid 101000010 \rangle + \alpha \mid 100110011 \rangle$

$+\alpha \mid 111010100 \rangle - \alpha \mid 110100101 \rangle$-$\alpha \mid 101000110 \rangle + \alpha \mid 100110111 \rangle$

$-\alpha \mid 010011000 \rangle - \alpha \mid 011101001\rangle - \alpha \mid 000001010 \rangle - \alpha \mid 001111011\rangle$

$-\alpha \mid 010011100 \rangle - \alpha \mid 011101101\rangle - \alpha \mid 000001110 \rangle - \alpha \mid 001111111\rangle$

-$\beta \mid 110000000 \rangle + \beta \mid 111110001 \rangle - \beta \mid 100010010 \rangle + \beta \mid 101100011\rangle$

$-\beta \mid 110000100 \rangle + \beta \mid 111110101 \rangle - \beta \mid 100010110 \rangle + \beta \mid 101100111 \rangle$

$+\beta \mid 011001000 \rangle + \beta \mid 010111001 \rangle - \beta \mid 001011010 \rangle - \beta \mid 000101011 \rangle$

$+\beta \mid 011001100 \rangle + \beta \mid 010111101 \rangle - \beta \mid 001011110 \rangle - \beta \mid 000101111 \rangle$

-$\beta \mid 100010000 \rangle + \beta \mid 101100001 \rangle - \beta \mid 110000010 \rangle + \beta \mid 111110011 \rangle$

$-\beta \mid 100010100 \rangle + \beta \mid 101100101 \rangle - \beta \mid 110000110 \rangle + \beta \mid 111110111 \rangle$

$-\beta \mid 001011000 \rangle - \beta \mid 000101001 \rangle + \beta \mid 011001010 \rangle + \beta \mid 010111011 \rangle$

$-\beta \mid 001011100 \rangle - \beta \mid 000101101 \rangle + \beta \mid 011001110 \rangle + \beta \mid 010111111 \rangle$

$+\beta \mid 000100000 \rangle + \beta \mid 001010001 \rangle - \beta \mid 010110010 \rangle - \beta \mid 011000011 \rangle$

$+\beta \mid 000100100 \rangle + \beta \mid 001010101 \rangle - \beta \mid 010110110 \rangle - \beta \mid 011000111 \rangle$

$-\beta \mid 101101000 \rangle + \beta \mid 100011001 \rangle - \beta \mid 111111010 \rangle + \beta \mid 110001011 \rangle$

$-\beta \mid 101101100 \rangle + \beta \mid 100011101 \rangle - \beta \mid 111111110 \rangle + \beta \mid 110001111 \rangle$

-$\beta \mid 010110000 \rangle \beta \mid 011000001 \rangle + \beta \mid 000100010 \rangle + \beta \mid 001010011 \rangle$

$-\beta \mid 010110100 \rangle - \beta \mid 011000101 \rangle + \beta \mid 000100110 \rangle + \beta \mid 001010111 \rangle$

$-\beta \mid 111111000 \rangle + \beta \mid 110001001 \rangle - \beta \mid 101101010 \rangle + \beta \mid 100011011 \rangle$

$-\beta \mid 111111100 \rangle + \beta \mid 110001101 \rangle - \beta \mid 101101110 \rangle + \beta \mid 100011111 \rangle$

$+\beta \mid 101100000 \rangle - \beta \mid 100010001 \rangle + \beta \mid 111110010 \rangle - \beta \mid 110000011 \rangle$

$+\beta \mid 101100100 \rangle - \beta \mid 100010101 \rangle + \beta \mid 111110110 \rangle - \beta \mid 110000111 \rangle$

$-\beta \mid 000101000 \rangle - \beta \mid 001011001 \rangle + \beta \mid 010111010 \rangle + \beta \mid 011001011\rangle$

$-\beta \mid 000101100 \rangle - \beta \mid 001011101 \rangle + \beta \mid 010111110 \rangle + \beta \mid 011001111 \rangle$

$+\beta \mid 111110000 \rangle - \beta \mid 110000001 \rangle + \beta \mid 101100010 \rangle - \beta \mid 100010011 \rangle$

$+\beta \mid 111110100 \rangle - \beta \mid 110000101 \rangle + \beta \mid 101100110 \rangle - \beta \mid 100010111 \rangle$

$+\beta \mid 010111000 \rangle + \beta \mid 011001001 \rangle - \beta \mid 000101010 \rangle - \beta \mid 001011011 \rangle$

$+\beta \mid 010111100\rangle + \beta \mid 011001101\rangle - \beta \mid 000101110\rangle - \beta \mid 001011111\rangle$

$-\beta \mid 011000000\rangle - \beta \mid 010110001\rangle + \beta \mid 001010010\rangle + \beta \mid 000100011\rangle$

$-\beta \mid 011000100\rangle - \beta \mid 010110101\rangle + \beta \mid 001010110\rangle + \beta \mid 000100111\rangle$

$+\beta \mid 110001000\rangle - \beta \mid 111111001\rangle + \beta \mid 100011010\rangle - \beta \mid 101101011\rangle$

$+\beta \mid 110001100\rangle - \beta \mid 111111101\rangle + \beta \mid 100011110\rangle - \beta \mid 101101111\rangle$

$+\beta \mid 001010000\rangle + \beta \mid 000100001\rangle - \beta \mid 011000010\rangle - \beta \mid 010110011\rangle$

$+\beta \mid 001010100\rangle + \beta \mid 000100101\rangle - \beta \mid 011000110\rangle - \beta \mid 010110111\rangle$

$+\beta \mid 100011000\rangle - \beta \mid 101101001\rangle + \beta \mid 110001010\rangle - \beta \mid 111111011\rangle$

$+\beta \mid 100011100\rangle - \beta \mid 101101101\rangle + \beta \mid 110001110\rangle - \beta \mid 111111111\rangle\ )$

·L'application du $H(6)$:

·L'application du $H(7)$:

·L'application du $H(8)$:

·L'application du $H(9)$:

Et après toutes les simplifications,onaura

$\mid \Psi_1\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 000001000\rangle + \alpha \mid 001111000\rangle + \alpha \mid 010011000\rangle + \alpha \mid 011101000\rangle$

$-\alpha \mid 101001000\rangle + \alpha \mid 100111000\rangle + \alpha \mid 111011000\rangle - \alpha \mid 110101000\rangle - \beta \mid 110001000\rangle$

$+\beta \mid 111111000\rangle - \beta \mid 100011000\rangle + \beta \mid 101101000\rangle - \beta \mid 011001000\rangle - \beta \mid 010111000\rangle$

$+\beta \mid 001011000\rangle + \beta \mid 000101000\rangle)$

***Correction d'erreur** :

Résultat de mesure =1000 ,donc erreur de type Z_1

La correction :appliquer la port Zsur le premier qubit

$\mid \Psi_1\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 000001000\rangle + \alpha \mid 001111000\rangle + \alpha \mid 010011000\rangle + \alpha \mid 011101000\rangle$

$-\alpha \mid 101001000\rangle + \alpha \mid 100111000\rangle + \alpha \mid 111011000\rangle - \alpha \mid 110101000\rangle - \beta \mid 110001000\rangle$

$+ \beta \mid 111111000\rangle - \beta \mid 100011000\rangle + \beta \mid 101101000\rangle - \beta \mid 011001000\rangle - \beta \mid 010111000\rangle$

$+\beta \mid 001011000\rangle + \beta \mid 000101000\rangle)$

∘ Suppression des quatre qubits du syndrome :

$$\mid \Psi_1\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 00000\rangle + \alpha \mid 00111\rangle + \alpha \mid 01001\rangle + \alpha \mid 01110\rangle - \alpha \mid 10100\rangle + \alpha \mid 10011\rangle$$
$$+\alpha \mid 11101\rangle - \alpha \mid 11010\rangle - \beta \mid 11000\rangle + \beta \mid 11111\rangle - \beta \mid 10001\rangle + \beta \mid 10110\rangle - \beta \mid 01100\rangle$$
$$-\beta \mid 01011\rangle + \beta \mid 00101\rangle + \beta \mid 00010\rangle)$$

·Le calcul du autres cas des erreure de type $Z(Z_2, Z_3, Z_4, Z_5)$est de la même méthode.

Les résultat de mesure sont résumés dans le tableau qui suit :

0000	0101	1100	0110	1010	0100	1101
sans erreur	X_1	X_2	X_3	X_4	X_5	Y_1

1110	1111	1011	0111	1000	0010	1001	0001	0011
Y_2	Y_3	Y_4	Y_5	Z_1	Z_2	Z_3	Z_4	Z_5

Remarques :

★ Le code à cinq qubits est le code quantique permettant la correction d'erreurs le plus optimal.C'est le code le plus court et donc il est d´un intérêt immense.

★ $M_i^2 = \mathbb{I}$ pour tous car: $X^2 = Y^2 = Z^2 = \mathbb{I}$.

★ $M_i M_j = M_j M_i$donc tout couple M_i, M_j commute et $M_i(1 + M_i) = (1 + M_i)$.

$$\begin{aligned} M_0 M_1 &= X(1)\,Z(2)\,X(3)\,Z(4)\,I(5) \times Z(1)\,Z(2)\,Z(3)\,I(4)\,Z(5) \\ &= X(1)\,Z(1)\,Z(2)\,Z(2)\,X(3)\,Z(3)\,Z(4)\,I(4)\,I(5)\,Z(5) \end{aligned}$$

$$\begin{aligned} M_1 M_0 &= Z(1)\,Z(2)\,Z(3)\,I(4)\,Z(5) \times X(1)\,Z(2)\,X(3)\,Z(4)\,I(5) \\ &= Z(1)\,X(1)\,Z(2)\,Z(2)\,Z(3)\,X(3)\,I(4)\,Z(4)\,Z(5)\,I(5) \end{aligned}$$

Puisque :

$$\begin{cases} X(1)\,Z(1) = -Z(1)\,X(1) \\ X(3)\,Z(3) = -Z(3)\,X(3) \end{cases} \quad \text{, Donc : } M_0 M_1 = M_1 M_0.$$

★ $\mid 0\rangle_c$ et$\mid\ 1\rangle_c$ est un vecteur propre de tous les M_i.

$$\begin{aligned} M_0 \quad \mid \quad 0\rangle c &= X(1)\,Z(2)\,X(3)\,Z(4)\,I(5)\,(\frac{1}{2\sqrt{2}} \mid 00000\rangle + \frac{1}{2\sqrt{2}} \mid 01001\rangle + \frac{1}{2\sqrt{2}} \mid 11010\rangle \\ -\frac{1}{2\sqrt{2}} \quad \mid \quad & 10011\rangle + \frac{1}{2\sqrt{2}} \mid 01110\rangle + \frac{1}{2\sqrt{2}} \mid 00111\rangle + \frac{1}{2\sqrt{2}} \mid 10100\rangle - \frac{1}{2\sqrt{2}} \mid 11101\rangle) \\ &= \frac{1}{2\sqrt{2}}(\mid 00000\rangle + \mid 01001\rangle + \mid 11010\rangle - \mid 10011\rangle + \mid 01110\rangle + \mid 00111\rangle + \mid 10100\rangle - \mid 11101\rangle) \end{aligned}$$

$M_0 \mid 0\rangle c = \mid 0\rangle c$ Donc$\mid 0\rangle c$ est un vecteur propre de M_0.

$$M_0 \mid 1\rangle c = X\,(1)\,Z\,(2)\,X\,(3)\,Z\,(4)\,I\,(5)\,(\frac{1}{2\sqrt{2}} \mid 11000\rangle + \frac{1}{2\sqrt{2}} \mid 10001\rangle + \frac{1}{2\sqrt{2}} \mid 00010\rangle$$

$$-\frac{1}{2\sqrt{2}} \mid 01011\rangle - \frac{1}{2\sqrt{2}} \mid 10110\rangle - \frac{1}{2\sqrt{2}} \mid 11111\rangle - \frac{1}{2\sqrt{2}} \mid 01100\rangle + \frac{1}{2\sqrt{2}} \mid 00101\rangle$$

$$= \frac{1}{2\sqrt{2}}(\mid 11000\rangle + \mid 10001\rangle + \mid 00010\rangle - \mid 01011\rangle - \mid 10110\rangle - \mid 11111\rangle - \mid 01100\rangle + \mid 00101\rangle)$$

$M_0 \mid 1\rangle c = \mid 1\rangle c$ Donc$\mid 1\rangle c$ est un vecteur propre de M_0

★ On peut vérifier que l'application des erreur $(X, Y$ ou $Z)$,les$X_k \mid \Psi\rangle$, $Y_k \mid \Psi\rangle, Z_k \mid \Psi\rangle$ est égale des vecteurs propres de tous les M,mais avec des ensembles différents d'états propres .

★ La mesure de tous les M$_i$ ne modifie pas l'état traité.

★ Le résultat de la mesure signe sans ambiguité la position d'erreur.

Chapitre 4

L'intrication quantique

L'intrication est une ressource essentielle dans le calcul quantique et les divers protocoles de communication quantique, et est l'un des aspects les plus surprenants de la physique quantique. Il s'agit d'un ‹‹état étrange›› dans lequel deux particules (ou plus) sont si profondément liées entre elles au niveau quantique (état quantique) qu'elles partagent la même "existence" même à grande distance. Si une mesure est faite sur l'un, l'état de l'autre est changé aussi instantanément afin d'être compatible avec la mesure de la première.

4.1 Matrice densité et entropie

4.1.1 Matrice densité :

La mécanique quantique peut être formulée dans un formalisme dit formalisme de la matrice de densité qui est plus convenable et plus compatible avec presque tons les scénarios qu'on puisse rencontrés. Donnons quelques définitions et propriétés de cette matrice densité.

Pour un **Etat pur** et qui n'est qu'un cas particulier, elle est définie par

$$\boxed{\rho = \mid \Psi_i\rangle\langle\Psi_i \mid}, \ \mathrm{p}_i = 1 \tag{4.1}$$

Pour un **Etat mélange** (état statistique) et qui est le cas le plus fréquent, elle est définie par

$$\boxed{\rho = \textstyle\sum_i \mathrm{p}_i \mid \Psi_i\rangle\langle\Psi_i \mid}, \sum_i \mathrm{p}_i = 1 \tag{4.2}$$

La matrice densité réduite :

Soit le système composé $\{AB\}$ décrit par la matrice densité ρ^{AB} , on peut décrire chacune des parties par une matrice densité comme suit

$$\begin{cases} \rho^{A} = tr_{B}\left(\rho^{AB}\right) \\ \rho^{B} = tr_{A}\left(\rho^{AB}\right) \end{cases} \tag{4.3}$$

où les traces tr_A, tr_B sont dites traces partielles respectivement sur la partie A et B et seront définies par la suite.

Propriété de la matrice densité :

1) ρ est hermitien $\rho^{+} = \rho$

2) $tr(\rho) = 1$

3) ρ est un opérateur defini positif($\rho \geq 0$)

4) $tr\ (\rho^2) \leq 1$

5) Pour état pur : $\rho^2 = \rho \;\; tr(\rho) = tr(\rho^2) = 1$

4.1.2 Espace des matrice densité :

L'ensemble des matrices densité $(N \times N)$ est noté $M^{(N)}$. Dans le cas pur on tr(ρ^2)= 1 ou bien $\rho^2 = \rho$. La propriété (3) veut dire que les valeurs propres de la matrice densité sont non-negatives(≥ 0)

Soit H un espace de Hilbert (complexe) de dimension N ($\dim_{\mathbb{C}} H = N$) son dual $H^*(H^* \simeq H$ en dimension finie). Definissons l'espace de Hilbert des opérateurs agissants sur$H$ $(H \to H)$ muni du produit Hermitien suivant : pour A : $H \quad \to \quad H$ et B :$H \quad \to \quad H$

$$\langle A, B\rangle \quad = cTr\ A^{+}B \text{ (ou bien } cTr\ (AB^{+})) \tag{4.4}$$

où c une constante réelle. Cet espace est un espace de Hilbert dit espace de Hilbert-Schmidt noté HS (ou bien $B(H)$ espace des opérateurs Bornés à trace en dimension quelconque)

La distance entre deux opérateurs dans cet espace est alors : ($c = \frac{1}{2}$)

$$D^2\left(A, B\right) \equiv \langle A - B, A - B\rangle = \frac{1}{2} tr\left[\left(A - B\right)\left(A^{+} - B^{+}\right)\right] \tag{4.5}$$

Cet espace HS peut être vu comme $H \otimes H^*$ $(HS \simeq H \otimes H^*)$; $\dim_{\mathbb{C}} HS = N^2$ qui veut dire que chaque opérateur s'écrit comme

$$A = \sum_{ij=1}^{N} a_{ij} \mid i\rangle\langle j \mid \;\text{ pour }\; \{\mid i\rangle\} \text{ base de } H$$

Un opérateur est diagonalisable que si et seulement si il est normal $[A, A^{+}] = 0$. En particulier si $A^{+} = A$ (Hermitien) et $A^{+} = A^{-1}$(unitaire)

Cet opérateur normal prend la forme

$$A = \sum_{i=1}^{n} \lambda_i \mid e_i\rangle\langle e_i \mid \tag{4.6}$$

où les λ_i sont valeurs propres non nulles (n valeurs)

$$A \mid e_i\rangle = \lambda_i \mid e_i\rangle \qquad (\lambda_i \neq 0) \tag{4.7}$$

Les $\{\mid e_i\rangle\}$ génére un sous espace de H dit support de A noté Supp(A) . L'ensemble des $\mid \Psi\rangle \in H$ / $A \mid \Psi\rangle = 0$ forme un sous espace de H dit le noyau de A, noté $\ker(A)$ qui est orthogonal à Supp(A), ie, $\ker(A) \perp$ Supp(A)) et bien sûr, $\ker(A) \oplus$Supp$(A) = H$, $\ker(A)$ est généré par l'ensemble des vecteurs propres de A à valeur propre nulle.

L'espace des matrices Hermitiennes (opérateurs Hermitiens) est réel comme espace vectoriel, noté HM et est de dimension N^2 (sur $\mathbb{R}$) $\dim_{\mathbb{R}} HM = N^2$; il n'est rien d'autre que l'adL de $U(N)$. Les matrices de trace nulle constitue un sous espace de dimension $(N^2 - 1)$ qui est l'adL de $SU(N)$ et toute matrice de $U(N)$ s'écrit alors comme

$$A \in HM \quad \exists r \equiv (r_0, r_1, ..., r_{N^2-1}) \in \mathbb{R}^{N^2}$$

$$A = r_0 \mathbb{I} + \sum_{i=1}^{N^2-1} r_i \sigma_i \quad \mathbb{I} = \mathbb{I}_N \quad , \sigma_i \in SU(N) \tag{4.8}$$

avec

$$r_0 = \frac{trA}{N} \quad ; \quad r_j = \frac{1}{2} tr(\sigma_j A)$$

Les $\sigma_j \in SU(N)$ verifient

$$\sigma_i \sigma_j = \frac{2}{N} \delta_{ij} + d_{ijk} \sigma_k + i f_{ijk} \sigma_k$$

avec les coefficients d_{ijk} totalement symétrique en (i, j, k) et nuls pour $N = 2$ et les coefficients f_{ijk} totalement anti-symétrique en (i, j, k)

Un opérateur P (matrice) est positif si et seulement si

$$\forall \mid \Psi\rangle \in H \qquad \langle \Psi \mid P \mid \Psi\rangle \geq 0$$

Et c'est équivalent aussi à :

$$\exists \ A \in HS \ / \ P = AA^{+}$$

L'ensemble de ces opérateurs positifs $\mathcal{P}$ est inclu dans HM ; $\mathcal{P} \subset HM$ c'est espace vectoriel de $\dim_{\mathbb{R}} [\mathcal{P}] = N^2$. Un opérateur positif possède une racine carré (radical) positive $\sqrt{P}$

$$\left(\sqrt{P}\right)^2 = P \tag{4.9}$$

En général, pour un opérateur $A \in HS$ on définit un opérateur positif AA^+ et une valeur absolue $|A|$ définie par

$$|A| \equiv \sqrt{AA^+} \tag{4.10}$$

En outre, on lui associe une forme polaire

$$A = |A|\, v = \sqrt{AA^+} v \tag{4.11}$$

avec v un opérateur unitaire. Cette forme polaire est unique si A est inversible. L'espace des matrices densité s'identifie alors , sachant que $\mathcal{P} \subset HM$, par tous les opérateurs positifs $\rho \geq 0$ de trace égale à 1, $tr\rho = 1$, Noté $M^{(N)}$ (ou M tout court). Ses états purs sont des projecteurs sur des sous -espaces de dim -1 dans H

$$\rho \;=\; \mid \Psi\rangle\langle\Psi \mid \text{ ou bien } \rho^2 = \rho$$
$$(\langle\Psi \mid \Psi\rangle = 1)$$

Il est clair alors que, $\rho \in U(N)$ par le choix

$$\rho = \frac{1}{N}\, \mathbb{I}_N + \sum_{i=1}^{N^2-1} r_i \sigma_i \tag{4.12}$$

et $\{\sigma_i\}$ générateurs de $SU(N)$.

Pour $r_i = 0$, on a

$$\rho = \frac{1}{N}\, \mathbb{I}_N$$

dite état (maximalement mélangé) mixte maximal (et aussi matrice d'ignorance).

les r_i sont appelés les coordonnées de mélange du vecteur de Bloch et on a (analogue de la distance Euclidienne)

$$D^2(\rho, \rho') = \sum_{i=1}^{N^2-1} (r_i - r_i')^2 \tag{4.13}$$

Mélange de Schrodinger :

Une matrice densité ayant la forme diagonale suivante

$$\rho = \sum_{i=1}^{N} \lambda_i \mid e_i\rangle\langle e_i \mid \tag{4.14}$$

peut être écrite sous la forme

$$\rho = \sum_{i=1}^{N} P_i \mid \Psi_i\rangle\langle\Psi_i \mid \; ; \; \sum_{i=1}^{N} P_i = 1 \qquad P_i \geq 0 \tag{4.15}$$

si et seulement si, il existe une matrice unitaire $(M \times M)$

$$\mid \Psi\rangle = \frac{1}{\sqrt{P_i}} \sum_{j=1}^{N} v_{ij} \sqrt{\lambda_j} \mid e_j\rangle \qquad (M > N) \tag{4.16}$$

Ces états sont normalisé mais on n'a pas besoin qu'ils soient orthogonaux.

<u>Purification des états mixte</u> :

"*Théoréme de décomposition de Schmidt* " : Pour tout $\mid \Psi\rangle \in H_1 \otimes H_2$ on a la décomposition suivante

$$\mid \Psi\rangle = \sum_{i=1}^{N} \sqrt{\lambda_i} \mid e_i\rangle \otimes \mid f_i \rangle \tag{4.17}$$

où $\{\mid e_i\rangle\}_{i=1-N_1}$ base de H_1 ,$\{\mid f_i\rangle\}_{i=1-N_2}$ base de H_2 et $N \leq mini\,\{N_1, N_2\}$. Les coefficients λ_i sont réel , dits coefficients de Schmidt et vérifient

$$\lambda_i \geq 0 \;\; ; \;\; \sum_{i=1}^{N} \lambda_i = 1$$

Le nombre de λ_i non nuls , est appele le rang de Schmidt $(r = \sharp\ \{\lambda_i \neq 0\}\ )$ de l'état $\mid \Psi\rangle$.

- Si $r > 1$; $\mid \Psi\rangle$ est un état intriqué de deux sous-systèmes (1 et 2).

-Cette décomposition concerne essentiellement le cas d'un espace de Hilbert formé par deux facteurs $H_{12} = H_1 \otimes H_2$.

Matrice densité réduite :

Soit un système composé de deux sous-systèmes décrit par la matrice densité ρ_{12} définie sur $H_{12} = H_1 \otimes H_2$. On décrit le sous-système (1) (respectivement (2)) par la matrice densité réduite ρ_1(respectivement ρ_2) définie par la trace partielle sur (2) (respectivement (1)) suivante

$$\rho_1 = Tr_2 \rho_{12} = \sum_{i=1}^{N_2} (\ \mathbb{I}_{N_1} \otimes \langle f_i \mid) \, \rho_{12} \, (\mathbb{I}_{N_1} \otimes \mid f_i \ \rangle) \tag{4.18}$$

respectivement,

$$\rho_2 = Tr_1 \rho_{12} = \sum_{i=1}^{N_1} \left(\langle e_i \mid \otimes \mathbb{I}_{N_2} \right) \rho_{12} \left(\mid e_i \rangle \otimes \mathbb{I}_{N_2} \right) \tag{4.19}$$

Il est facile de vérifier que ρ_1 et ρ_2 sont des bonnes matrices densité et si $A = A \otimes \mathbb{I}_{N_2}$ on a alors

$$\langle A \rangle = Tr\left(\rho_{12} A\right) = Tr_1\left(\rho_1 A_1\right) \tag{4.20}$$

et respectivement si $A = \mathbb{I}_{N_1} \otimes A$

$$\langle A \rangle = Tr\left(\rho_{12} A\right) = Tr_2\left(\rho_2 A_2\right) \tag{4.21}$$

Même si ρ_{12} est un état pur , en gènèral $\rho_1 = Tr_2 \rho_{12}$ (ou $\rho_2 = Tr_1 \rho_{12}$) est un état mixte.

Lemme de réduction :

Soit ρ_{12} un état pur sur $H_{12} = H_1 \otimes H_2$, alors le spectre des matrices densité réduites ρ_1 et ρ_2 est identique (sauf possible pour une dégénérescence différente des valeurs propres nulles).

Lemme de purification :

Pour une matrice densité ρ_1 définie sur H_1, il existe un espace de Hilbert H_2 et un état pur ρ_{12} defini sur $H_1 \otimes H_2$, tels que

$$\rho_1 = Tr_2 \rho_{12}$$

Définition1 :(Intrication d'états purs)

Soit $\mid \Psi_{AB} \rangle \in H_A \otimes H_B$ un état pur. On appeelle $\mid \Psi_{AB} \rangle$ intriqué (ou non séparable) si son nombre de Schmidt est supérieur à 1.

Propriété :

Soit $\mid \Psi_{AB} \rangle \in H_A \otimes H_B$ un état pur. Alors :

$$\mid \Psi_{AB} \rangle \text{ intriqué } \Leftrightarrow \nexists \mid a_A \rangle \in H_A \ , \ \mid b_B \rangle \in H_B \text{ tel que } \mid \Psi_{AB} \rangle = \mid a_A \rangle \otimes \mid b_B \rangle$$

Définition2 :

Soit $\mid \Psi_{AB} \rangle \in H_A \otimes H_B$ un état pur, $\dim\left(H_A\right) \leq \dim\left(H_B\right)$. Alors

$$\mid \Psi_{AB} \rangle \text{ maximalement intriqué : } \Leftrightarrow \ \rho_A = \frac{\mathbb{I}}{\dim\left(H_A\right)}$$

Définition3 :(Définition générale de l'intrication)

Un état ρ sur $H_A \otimes H_B$ est dit non-intriqué ou séparable s'il y a d'abord des opérateurs de densité $\rho_A^{(j)}$, $\rho_B^{(j)}$ et $p_j \geq 0$ avec $\sum_j p_j = 1$ tels que :

$$\rho = \sum_j p_j \rho_A^{(j)} \otimes \rho_B^{(j)} \tag{4.22}$$

Propriété :

Pour un état pur $\mid \Psi_{AB}\rangle \in H_A \otimes H_B$, la définition (3) se réduit à la définition (1)

Propriété :.

ρ_{AB} ensembles séparables $\Leftrightarrow \exists$ d'états normalisés $\left\{\mid a_A^{(j)}\rangle\right\}$, $\left\{\mid b_B^{(j)}\rangle\right\}$, $p_j \geq 0$ avec

$$\sum_j p_j = 1 \text{ tels que } \rho_{AB} = \sum_j p_j \mid a_A^{(j)}\rangle\langle a_A^{(j)} \mid \otimes \mid b_B^{(j)}\rangle\langle b_B^{(j)} \mid$$

4.1.3 Entropie de von Neumann :

Jusqu'à présent, nous n'avons défini que l'intrication. Maintenant nous voulons trouver une notion plus forte avec laquelle on mesure, pour un état pur, l'intrication, et cela est fait au moyen l'entropie de von Neumann.

Définition(4) :

Soit ρ un opérateur densité arbitraire. Alors l'entropie de von Neumann est défini par

$$S(\rho) = -tr\left[\rho. \log_2(\rho)\right] \tag{4.23}$$

Si p_j sont les valeurs propres de ρ, alors nous pouvons récrire l'entropie comme

$$S(\rho) = -\sum_j p_j. \log_2(P_j) \tag{4.24}$$

L'entropie de von Neumann est l'analogue quantique de l'entropie classique de Shannon

$$H(x) = -\sum_x p(x) \log p(x) \tag{4.25}$$

où $p(x) \equiv p(X = x)$ est une distribution de probabilité pour une variable aléatoire X.

L'entropie d'intrication : On a :

$$\rho_A = tr_B\left[\rho_{AB}\right] \tag{4.26}$$

$$\rho_B = tr_A\left[\rho_{AB}\right] \tag{4.27}$$

L'entropie de von Neumann réduite de ρ_{AB} par rapport au système A est $S(\rho_A)$ qui sous forme explicite est

$$S(\rho_A) = -tr\left\{tr_B\left[\rho_{AB}\right] \log tr_B\left[\rho_{AB}\right]\right\} \tag{4.28}$$

L'entropie de von Neumann réduite de ρ_{AB} par rapport au système B est $S(\rho_B)$ qui sous forme explicite est

$$S(\rho_B) = -tr\left\{tr_A\left[\rho_{AB}\right]\log tr_A\left[\rho_{AB}\right]\right\} \tag{4.29}$$

On les appelle aussi l'entropie d'intrication. Pour $S(\rho_A) = S(\rho_B)$ l'état est maximalement intriqué

Exemple :

1.$\rho = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ a pour valeurs propres 0 et 1 $\Rightarrow S(\rho) = 0$

2. $\mid \Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$. Alors $\rho = \mid \Psi\rangle\langle\Psi \mid = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$. Comme ρ a pour valeurs propres $0, 1$, $S(\rho) = 0$

3.$\rho = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \Rightarrow S(\rho) = \log_2(2)$.

Propriété de l'entropie :

1.$S(\rho) \geq 0$ et $S(\rho) = 0 \Leftrightarrow \rho$ état pur.

2.$\dim(H) = d \;\Rightarrow S(\rho) \leq \log_2(d)$ et $S(\rho) = \log_2(d) \Leftrightarrow \rho = \dfrac{\mathbb{I}_d}{d}$.

3.ρ_{AB} sur $H_A \otimes H_B$ pure $\Rightarrow S(\rho_A) = S(\rho_B)$.

4.Soit $p_j \geq 0$, $\sum_j p_j = 1$, ρ_j des opérateurs densité qui ont un support sur des sous-espace orthogonaux. Alors

$$S\left(\sum_j p_j\ \rho_j\right) = H\left(\{p_j\}\right) + \sum_j p_j S\left(\rho_j\right)\ ,\ H\left(\{p_j\}\right) = -\sum_j p_j \log_2(p_j) \tag{4.30}$$

($H(\{p_j\})$ L'entropie de Shannon)

5.*Théorème d'entropie conjointe* : Soit $p_j \geq 0$, $\sum_j p_j = 1$,$\{\mid j_A\rangle\} \subset H_A$ orthonormé , ρ_j des opérateurs densité sur H_B. Alors

$$S\left(\sum_j p_j\ \mid j_A\rangle\langle j_A \mid \otimes\rho_j\right) = H\left(\{p_j\}\right) + \sum_j p_j S\left(\rho_j\right) \tag{4.31}$$

6. ρ_A , ρ_B des opérateurs densité sur H_A et H_B. Alors

$$S\left(\rho_A \otimes \rho_B\right) = S\left(\rho_A\right) + S\left(\rho_B\right)$$

7.Soit $\rho = \mid \Psi_{AB}\rangle\langle\Psi_{AB} \mid$ un état pur sur $H_A \otimes H_B$, $\dim(H_A) \leq \dim(H_B)$. Alors

1)$S(\rho_A) = 0 \Leftrightarrow \mid \Psi_{AB}\rangle$ séparable ,$S(\rho_A) \succ 0 \Leftrightarrow \mid \Psi_{AB}\rangle$ intriqué.

2)$S(\rho_A) = \log(\dim H_A)$(=maximal)$\Leftrightarrow \mid \Psi_{AB}\rangle$ maximalement intriqué.

8.Les opérateurs densité forment un ensemble convexe:

$$D := \left\{\rho \mid \langle\Psi \mid \rho \mid \Psi\rangle \geq 0\forall \mid \Psi\rangle, \rho^+ = \rho, Tr(\rho) = 1\right\}$$

Les opérateurs densité séparables forment un ensemble convexe :

$$S := \{\rho \in D \mid \rho \text{ séparable}\}$$

9.Soit un opérateur densité arbitraire sur un espace de Hilbert de dimension finie $H_A \otimes H_B$

.

On définit M :=$\{A \mid A$ opérateur linéaire sur $H_A \otimes H_B$ avec $A^+ = A\}$ .Alors

ρ intriqués $\Leftrightarrow$ $\exists\, W \in M : Tr(\rho W) \prec 0$ et $Tr(\sigma W) \geq 0$ pour toute les opérateurs densité σ séparable

4.2 Concurrence à deux qubits utilisant l'identité de Lagrange et produit éxterieure

Produit extérieur :

Considérons deux vecteurs quelconques $\overrightarrow{a}$ et $\overrightarrow{b}$ dans $\mathbb{C}^m$ muni de la base orthonormale $\{\hat{e}_i\}_{i=1}^m$. Leur produit éxterieur est un bivecteur dans l'espace exterieur $^2(\mathbb{C}^m)$ muni de la base $\{\hat{e}_i\}_{i=1}^m \wedge \{\hat{e}_j\}_{j=1}^m$ défini par

$$\overrightarrow{a} \wedge \overrightarrow{b} = \sum_{i=1}^{m-1}\sum_{j=i+1}^{m}(a_i b_j - a_j b_i)\,\hat{e}_i \wedge \hat{e}_j \tag{4.32}$$

avec les propriétés :

$$\begin{cases} \overrightarrow{a} \wedge \overrightarrow{a} = \overrightarrow{0} \\ \overrightarrow{a} \wedge \overrightarrow{b} = (-1)\,\overrightarrow{b} \wedge \overrightarrow{a} \end{cases}$$

et ses coordonnées sont par définition :

$(a_1b_2 - a_2b_1, a_1b_3 - a_3b_1, ..., a_1b_m - a_mb_1, a_2b_3 - a_3b_2, ..., a_2b_m - a_mb_2, ..., a_{m-1}b_m - a_mb_{m-1})$

L'identité de Lagrange :

Considérons deux vecteur $\overrightarrow{a}$ et $\overrightarrow{b}$, l'identité de Lagrange stipule que :

$$\left\|\overrightarrow{a}\wedge\overrightarrow{b}\right\|^{2}=\|\overrightarrow{a}\|^{2}\left\|\overrightarrow{b}\right\|^{2}-\left|\overrightarrow{a}.\overrightarrow{b}\right|. \tag{4.33}$$

En effet,

$$\begin{aligned} RHS &= \left\|\overrightarrow{a}\wedge\overrightarrow{b}\right\|^{2}=\sum_{i=1}^{m-1}\sum_{j=i+1}^{m}\left|a_ib_j-a_jb_i\right|^2=\frac{1}{2}\sum_{i=1}^{m}\sum_{j=i+1}^{m}\left|a_ib_j-a_jb_i\right|^2 \\ &= \frac{1}{2}\sum_{i=1}^{m}\sum_{j=i+1}^{m}(a_ib_j-a_jbi)(\overline{a_i}\overline{b_j}-\overline{a_j}\overline{b}_i)=\frac{1}{2}\sum_{i=1}^{m}\sum_{j=i+1}^{m}(\left|a_i\right|^2\left|b_j\right|^2-2\,\mathrm{Re}(a_ib_j\overline{a}_i\overline{b_j})+\left|aj\right|^2\left|b_i\right|^2 \end{aligned}$$

Systéme à deux qubits :

Considérons un système à deux qubits A et B, soit $\mid\Psi\rangle$ un état pur normalisé du système

$$\begin{aligned} &\mid\ \Psi\rangle=p\mid 0_A0_B\rangle+q\mid 0_A1_B\rangle+r\mid 1_A0_B\rangle+s\mid 1_A1_B\rangle \\ &\mid\ \Psi\rangle=\mid 0_A\rangle\left(p\mid 0_B\rangle+q\mid 1_B\rangle\right)+\mid 1_A\rangle\left(r\mid 0_B\rangle+s\mid\ 1_B\rangle\right) \\ &\mid\ \Psi\rangle=\mid 0_A\rangle\langle 0_A\mid\Psi\rangle+\mid 1_A\rangle\langle 1_A\mid\Psi\rangle \end{aligned} \tag{4.34}$$

avec

$$\begin{cases} \langle 0_A\mid\Psi\rangle=p\mid 0_B\rangle+q\mid 0_B\rangle \\ \langle 1_A\mid\Psi\rangle=r\mid 0_B\rangle+s\mid\ 1_B\rangle \end{cases}$$

• La bipartition A|B est séparable que si le produit extérieur de deux vecteurs est nul $\langle 0_A\mid\Psi\rangle\wedge\langle 1_A\mid\Psi\rangle=0$ ($\langle 0_A\mid\Psi\rangle$ et $\langle 1_A\mid\Psi\rangle$ parallèles)

et on écrit :

$$\frac{p}{r}=\frac{q}{s}\Longrightarrow ps-qr=0 \tag{4.35}$$

• La bipartition A|B est maximalement intriqué si les conditions suivantes sont vérifiées

$$\begin{aligned} \|\langle 0_A\mid\Psi\rangle\| &= \|\langle 1_A\mid\Psi\rangle\| \quad ,\|\langle 0_B\mid\Psi\rangle\|=\|\langle 1_B\mid\Psi\rangle\| \\ (\langle 0_A\mid\Psi\rangle)^{\dagger}\langle 1_A &\mid \Psi\rangle=0 \qquad ,\ (\langle 0_B\mid\Psi\rangle)^{\dagger}\langle 1_B\mid\Psi\rangle=0 \end{aligned}$$

• On définit la concurrence comme mesure de l'intrication de cet état pur de deux qubits par

$$E = 2 \ \|\langle 0_A \mid \Psi\rangle \wedge \langle 1_A \mid \Psi\rangle\| \ = 2\,\|\langle 0_B \mid \Psi\rangle \wedge \langle 1_B \mid \Psi\rangle\| = 2\,|ps - qr| \tag{4.36}$$

$$E^2 = 4 \ \|\langle 0_A \mid \Psi\rangle \wedge \langle 1_A \mid \Psi\rangle\|^2 = 4 \ \|\langle 0_B \mid \Psi\rangle \wedge \langle 1_B \mid \Psi\rangle\|^2 = 4\,|ps - qr|^2 \tag{4.37}$$

$\|\langle 0_A \mid \Psi\rangle \wedge \langle 1_A \mid \Psi\rangle\|$ représente l'aire d'un parallélopipède complexe. Hill et Wooters ont déterminé la concurrence des états purs de deux qubits comme :

$$C(\Psi) = \left|\langle \Psi \mid \widetilde{\Psi}\rangle\right| \ = 2\,|ps - qr| \tag{4.38}$$

où

$$\mid \widetilde{\Psi}\rangle = \sigma_y \mid \Psi^*\rangle, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

<u>Calcul matriciel :</u>

La matrice densité de l'état de deux qubits est :

$$\rho = \begin{pmatrix} |p|^2 & p\overline{q} & p\overline{r} & p\overline{s} \\ q\overline{p} & |q|^2 & q\overline{r} & q\overline{s} \\ r\overline{p} & r\overline{q} & |r|^2 & r\overline{s} \\ s\overline{p} & s\overline{q} & s\overline{r} & |s|^2 \end{pmatrix}$$

La matrice densité réduite sur A (ρ^A) prend la forme

$$\begin{aligned} \rho^A &= \langle 0_B \mid \rho \mid 0_B\rangle + \langle 1_B \mid \rho \mid 1_B\rangle \\ &= \begin{pmatrix} |p|^2 + |q|^2 & p\overline{r} + q\overline{s} \\ r\overline{p} + s\overline{q} & |r|^2 + |s|^2 \end{pmatrix} \end{aligned}$$

La mesure de l'intrication dans ce cas s'ecrit

$$E = 2\sqrt{\det\left(\rho^A\right)} = 2\sqrt{\det\left(\rho^B\right)}$$

Et on écrit aussi

$$\begin{aligned} E^2 &= 4\det\left(\rho^A\right) = 4\sum_{i<j}\left(\rho^A_{ii}\rho^A_{jj} - \rho^A_{ij}\rho^A_{ji}\right) = 4\left[\frac{1}{2}\sum_{i,j}\left(\rho^A_{ii}\rho^A_{jj} - \rho^A_{ij}\rho^A_{ji}\right)\right] \\ &= 4\left[\frac{1}{2}\left(\left[tr\left(\rho^A\right)\right]^2 - tr\left[\left(\rho^A\right)^2\right]\right)\right] = 2\left[1 - tr\left[\left(\rho^A\right)^2\right]\right] \end{aligned} \tag{4.39}$$

$$E_A^2 = 4\left[\frac{1}{2}\left(1 - \sum_i \lambda_i^2\right)\right] = 4\sum_{i<j}\lambda_i\lambda_j \tag{4.40}$$

(λ les valeurs propres de ρ^A)

4.3 Extension aux états multiparticules en dimensions arbitraires

Considérons un état pur$\mid \Psi\rangle$ à (n) particules, donné par la relation suivante

$$\mid \Psi\rangle = \sum_{j_1,j_2,..j_n=0}^{d_1-1,d_2-1,....,d_n-1} a_{j_1j_2......j_n} \mid j_1\rangle\otimes \mid j_2\rangle \otimes .....\otimes \mid j_n\rangle$$

La particule (i) appartient à un espace de Hilbert de d$_i$ -dimension.

$$\mid \Psi\rangle = \sum_{j_1,j_2,.....,j_n} a_{j_1j_2.....j_n} \left(\mid j_1\rangle\otimes \mid j_2\rangle \otimes ....\otimes \mid j_m\rangle\right) \otimes \left(\mid j_{m+1}\rangle\otimes \mid j_{m+2}\rangle \otimes .....\otimes \mid j_n\rangle\right)$$

$$= \sum_{j_1,j_2,....,j_n}\ \sum_{j_{m+1},.....,j_n} a_{j_1j_2.....j_mj_{m+1}...j_n} \mid j_1j_2....j_m\rangle\otimes \mid j_{m+1}.....j_n\rangle$$

$$= \sum_{j_1,j_2,...,j_m}\left[\mid j_1j_2...j_m\rangle \otimes \left(\sum_{j_{m+1},....,j_n} a_{j_1j_2...j_mj_{m+1}...j_n} \mid j_{m+1}...j_n\rangle\right)\right] \tag{4.41}$$

En notant que

$$\langle k_1k_2...k_m \mid \Psi\rangle = \sum_{j_m+1,.....,j_n} a_{k_1k_2..........k_mj_{m+1}........j_n} \mid j_{m+1}....j_n\rangle \tag{4.42}$$

l'équation (4.41) peut être exprimée comme :

$$\mid \Psi\rangle = \sum_{j_1,j_2,\ldots j_m} \mid j_1 j_2 \ldots j_m\rangle \otimes [\langle j_1 j_2 \ldots j_m \mid \Psi\rangle] \tag{4.43}$$

La mesure de l'intrication est donné par :

$$E_M^2 = 4 \sum_{i_1 \ldots i_m} \sum_{\substack{j_1 \geq i_1,\ldots,j_m \geq i_m \\ |i_1 - j_1| + \ldots + |i_m - j_m| \neq 0}} \left\| \langle i_1 \ i_2 \ldots i_m \mid \Psi\rangle \wedge \langle j_1 j_2 \ldots j_m \mid \Psi\rangle \right\|^2 \tag{4.44}$$

<u>**Systeme à trois qubits**</u> :

Soit un état pur normalisé d'un système à trois qubits

$$\begin{aligned} \mid \quad & \Psi\rangle = p \mid 0_A 0_B 0_C\rangle + q \mid 0_A 0_B 1_C\rangle + r \mid 0_A 1_B 0_C\rangle + s \mid 0_A 1_B 1_C\rangle \\ +t \quad \mid \quad & 1_A 0_B 0_C\rangle + u \mid 1_A 0_B 1_C\rangle + v \mid 1_A 1_B 0_C\rangle + w \mid 1_A 1_B 1_C\rangle \\ = \quad & \mid 0_A\rangle \left[p \mid 0_B 0_C\rangle + q \mid 0_B 1_C\rangle + r \mid 1_B 0_C\rangle + s \mid 1_B 1_C\rangle\right] \\ + \quad \mid \quad & 1_A\rangle \left[t \mid 0_B 0_C\rangle + u \mid 0_B 1_C\rangle + v \mid 1_B 0_C\rangle + w \mid 1_B 1_C\rangle\right] \\ = \quad & \mid 0_A\rangle\langle 0_A \mid \Psi\rangle + \mid 1_A\rangle\langle 1_A \mid \Psi\rangle \end{aligned}$$

avec $p, q, r, s, t, u, v, w \in \mathbb{C}$.

La bipartition A|BC est séparable, si les vecteurs $\langle 0_A \mid \Psi\rangle$, $\langle 1_A \mid \Psi\rangle$ sont parellèles, ce que veut dire

$$\frac{p}{t} = \frac{q}{u} = \frac{r}{v} = \frac{s}{w}$$

La condition de separabilité ($\langle 0_A \mid \Psi\rangle \wedge \langle 1_A \mid \Psi\rangle = 0$)

$$(p, q, r, s) \wedge (t, u, v, w) = (pu - qt, pv - rt, pw - st, qv - ru, qw - su, rw - sv)$$

La mesure de l'intrication

$$\begin{aligned} E &= E_A + E_B + E_C + 2 \left\| \langle 0_A \mid \Psi\rangle \ \wedge \langle 1_A \mid \Psi\rangle \right\| \\ &= 2 \left\| \langle 0_B \mid \Psi\rangle \ \wedge \langle 1_B \mid \Psi\rangle \right\| + 2 \left\| \langle 0_C \mid \Psi\rangle \ \wedge \langle 1_C \mid \Psi\rangle \right\| \\ &= 2 \left[\sqrt{\det(\rho^A)} + \sqrt{\det(\rho^B)} + \sqrt{\det(\rho^C)} \right] \end{aligned}$$

Système à quatre qubits :

Considérons un système à quatre qubits

$$\begin{aligned}
& \mid \Psi\rangle = a \mid 0_A 0_B 0_C 0_D\rangle + b \mid 0_A 0_B 0_C 1_D\rangle + c \mid 0_A 0_B 1_C 0_D\rangle + d \mid 0_A 0_B 1_C 1_D\rangle \\
+e & \mid 0_A 1_B 0_C 0_D\rangle + f \mid 0_A 1_B 0_C 1_D\rangle + g \mid 0_A 1_B 1_C 0_D\rangle + h \mid 0_A 1_B 1_C 1_D\rangle \\
+i & \mid 1_A 0_B 0_C 0_D\rangle + j \mid 1_A 0_B 0_C 1_D\rangle + k \mid 1_A 0_B 1_C 0_D\rangle + l \mid 1_A 0_B 1_C 1_D\rangle \\
+m & \mid 1_A 1_B 0_C 0_D\rangle + n \mid 1_A 1_B 0_C 1_D\rangle + e \mid 1_A 1_B 1_C 0_D\rangle + p \mid 1_A 1_B 1_C 1_D\rangle \\
& = \mid 0_A\rangle \begin{bmatrix} a \mid 0_B 0_C 0_D\rangle + b \mid 0_B 0_C 1_D\rangle + c \mid 0_B 1_C 0_D\rangle + d \mid 0_B 1_C 1_D\rangle \\ +e \mid 1_B 0_C 0_D\rangle + f \mid 1_B 0_C 1_D\rangle + g \mid 1_B 1_C 0_D\rangle + h \mid 1_B 1_C 1_D\rangle \end{bmatrix} \\
+ & \mid 1_A\rangle \begin{bmatrix} i \mid 0_B 0_C 0_D\rangle + j \mid 0_B 0_C 1_D\rangle + k \mid 0_B 1_C 0_D\rangle + l \mid 0_B 1_C 1_D\rangle \\ +m \mid 1_B 0_C 0_D\rangle + n \mid 1_B 0_C 1_D\rangle + e \mid 1_B 1_C 0_D\rangle + p \mid 1_B 1_C 1_D\rangle \end{bmatrix} \\
& = \mid 0_A\rangle\langle 0_A \mid \Psi\rangle + \mid 1_A\rangle\langle 1_A \mid \Psi\rangle
\end{aligned}$$

La mesure de l'intrication

$$E_A = 2 \left\| \langle 0_A \mid \Psi\rangle \;\wedge \langle 1_A \mid \Psi\rangle \right\| = 2\sqrt{\det\left(\rho^A\right)}$$

Pour la séparabilité des qubits (AB) ou (CD) du système par conséquent,les vecteurs

$$\langle 0_A 0_B \mid \Psi\rangle, \langle 0_A 1_B \mid \Psi\rangle, \langle 1_A 0_B \mid \Psi\rangle, \langle 1_A 1_B \mid \Psi\rangle$$

dans l'espace de Hilbert du système de qubits (CD) doivent être parallèles.

La mesure de l'intrication au carré (concurrence au carré) est donnée par

$$E_A^2 = 4 \begin{bmatrix} \left\|\langle 0_A 0_B \mid \Psi\rangle \wedge \langle 0_A 1_B \mid \Psi\rangle\right\|^2 + \left\|\langle 0_A 0_B \mid \Psi\rangle \wedge \langle 1_A 0_B \mid \Psi\rangle\right\|^2 \\ + \left\|\langle 0_A 0_B \mid \Psi\rangle \wedge \langle 1_A 1_B \mid \Psi\rangle\right\|^2 + \left\|\langle 0_A 1_B \mid \Psi\rangle \wedge \langle 1_A 0_B \mid \Psi\rangle\right\|^2 \\ + \left\|\langle 0_A 1_B \mid \Psi\rangle \wedge \langle 1_A 1_B \mid \Psi\rangle\right\|^2 + \left\|\langle 1_A 0_B \mid \Psi\rangle \wedge \langle 1_A 1_B \mid \Psi\rangle\right\|^2 \end{bmatrix} \quad (4.45)$$

4.4 Géométrie de l'intrication à partir du parallélisme des vecteurs :

La concurrence d'un état multipartite pur à été discutée en utilisant le parallélisme des vecteurs. Létat pur d'un systeme à deux qubits représenté par :

$$\mid \Psi\rangle = a \mid 0_A 0_B\rangle + b \mid 0_A 1_B\rangle + c \mid 1_A 0_B\rangle + d \mid 1_A 1_B\rangle$$

Un critère de séparabilité apparait pour l'état pur de deux qubits à partir d'ici comme

$$|ad - bc| = 0$$

A partir de ce critère de séparabilité, Bhaskara et Panighrahi ont défini la mesure de concurrence de l'intrication (C_A) pour les système à deux qubits

$$C_A = 2 \left\| \langle 0_A \mid \Psi \rangle \wedge \langle 1_A \mid \Psi \rangle \right\| \tag{4.46}$$

Etat pur à trois qubits :

$$\begin{aligned} \mid & \Psi\rangle = a \mid 0_A 0_B 0_C\rangle + b \mid 0_A 0_B 1_C\rangle + c \mid 0_A 1_B 0_C\rangle + d \mid 0_A 1_B 1_C\rangle \\ +p \mid & 1_A 0_B 0_C\rangle + q \mid 1_A 0_B 1_C\rangle + r \mid 1_A 1_B 0_C\rangle + s \mid 1_A 1_B 1_C\rangle \end{aligned} \tag{4.47}$$

On considère alors la bi-partition A|BC et le système de prémesure (BC) dans le base de calcul. Les vecteurs non normalisées post-mesure pour le système A seront :

$$\chi_0^A = a \mid 0\rangle + p \mid 1\rangle, \chi_1^A = b \mid 0\rangle + q \mid 1\rangle, \chi_2^A = c \mid 0\rangle + r \mid 1\rangle, \chi_3^A = d \mid 0\rangle + s \mid 1\rangle$$

La concurrence au carré dans cette bi-partition du systéme est donnée par :

$$C_{A|BC}^2 = 4 \left[\left|\chi_0^A \wedge \chi_1^A\right|^2 + \left|\chi_0^A \wedge \chi_2^A\right|^2 + \left|\chi_0^A \wedge \chi_3^A\right|^2 + \left|\chi_1^A \wedge \chi_2^A\right|^2 + \left|\chi_1^A \wedge \chi_3^A\right|^2 + \left|\chi_2^A \wedge \chi_3^A\right|^2 \right] \tag{4.48}$$

Si les quatre vecteurs sont paralléles, le cas est séparable.

En 2002, Coffman, Kundu, et Wooters ont montré que si trois particules A, B, C sont intriquées, la somme de la concurrenceau carré entre A et$B(C_{A/B}^2)$ et la concurrence au carré entre A et $C(C_{A/C}^2)$ obéissent à l'inégalité de Coffman-Kundu-Wooters suivante comme :

$$C_{A/BC}^2 \geq C_{A/B}^2 + C_{A/C}^2 \tag{4.49}$$

où, C_{A/B_i} est la concurrence entre les systèmes A et B, lorsque C est fixé à l'état $|i\rangle$ $i = 0, 1$. De même, la concurrence entre les systèmes A et C quand B est fixé à l'état $|i\rangle$ $i = 0, 1$.

$$C_{A/C}^2 =\mid C_{A/C_0} + C_{A/C_1} \mid^2 \tag{4.50}$$

et

$$C_{A/B}^2 = \mid C_{A/B_0} + C_{A/B_1} \mid^2 \tag{4.51}$$

On définit le 3-tangle par

$$\tau = C_{A/BC}^2 \ - C_{A/B}^2 - C_{A/C}^2 \tag{4.52}$$

De l'équation (4.47), et en utilisant les définitions des vecteurs d'états des mesures passées du système A, on trouve,

$$C_{A/B}^2 = 4 \mid (\chi_0^A \wedge \chi_2^A) + (\chi_1^A \wedge \chi_3^A) \mid^2 \tag{4.53}$$

et,

$$C_{A/C}^2 = 4 \mid (\chi_0^A \wedge \chi_1^A) + (\chi_2^A \wedge \chi_3^A) \mid^2 \tag{4.54}$$

Le remplacement des équations (4.48),(4.53), et (4.54) dans l'équation (4.52) nous conduit à une définition de 3-tangle en utilisant les vecteurs d'état de post mesure du système A comme,

$$\begin{aligned} \tau \ = \ & 4[\mid \chi_0^A \wedge \chi_3^A \mid^2 + \mid \chi_1^A \wedge \chi_2^A \mid^2 -2(\chi_0^A \wedge \chi_1^A).(\chi_2^A \wedge \chi_3^A) \\ & -2(\chi_0^A \wedge \chi_2^A).(\chi_1^A \wedge \chi_3^A)] \end{aligned} \tag{4.55}$$

4.5 Application du code stabilisateur à cinq Qubits

Soit le qubit $\mid \Psi_0\rangle$ c'est l'etat sans erreurs.

$$\mid \Psi_0\rangle = \frac{1}{2\sqrt{2}} \begin{pmatrix} \alpha \mid 00000\rangle + \alpha \mid 01001\rangle + \alpha \mid 11010\rangle - \alpha \mid 10011\rangle \\ +\alpha \mid 01110\rangle + \alpha \mid 00111\rangle + \alpha \mid 10100\rangle - \alpha \mid 11101\rangle \\ +\beta \mid 11000\rangle + \beta \mid 10001\rangle + \beta \mid 00010\rangle - \beta \mid 01011\rangle \\ -\beta \mid 10110\rangle - \beta \mid 11111\rangle - \beta \mid 01100\rangle + \beta \mid 0101\rangle \end{pmatrix} \tag{4.56}$$

Sur chaque qubit (i) de cet état, on a les conditions :

$$({}_i\langle 0 \mid \Psi_0\rangle) \dagger . ({}_i\langle 1 \mid \Psi_0\rangle) = 0$$

$$\|({}_i\langle 0 \mid \Psi_0\rangle)\| = \|({}_i\langle 1 \mid \Psi_0\rangle)\| = \frac{1}{\sqrt{2}}$$

<u>vérification</u> :

$$\mid \ \Psi_0\rangle \ = \frac{1}{2\sqrt{2}} \left(\begin{array}{ll} \mid 0 \ \rangle & \left[\begin{array}{c} \alpha \mid 0000\rangle + \alpha \mid 1001\rangle + \alpha \mid 1110\rangle + \alpha \mid 0111\rangle \\ +\beta \mid 0010\rangle - \beta \mid 1011\rangle - \beta \mid 1100\rangle + \beta \mid 0101\rangle \end{array} \right] \\ \mid 1 \ \rangle & \left[\begin{array}{c} \alpha \mid 1010\rangle - \alpha \mid 0011\rangle + \alpha \mid 0100\rangle - \alpha \mid 1101\rangle \\ +\beta \mid 1000\rangle + \beta \mid 0001\rangle - \beta \mid 0110\rangle - \beta \mid 1111\rangle \end{array} \right] \end{array} \right)$$

$$\mid \ \Psi_0\rangle = \mid 0\rangle\langle 0 \mid \Psi_c\rangle + \mid 1\rangle\langle 1 \mid \Psi_c\rangle) \qquad (4.57)$$

avec :

$$\left\{ \begin{array}{l} \langle 0 \mid \Psi_c\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 0000\rangle + \alpha \mid 1001\rangle + \alpha \mid 1110\rangle + \alpha \mid 0111\rangle \\ \qquad +\beta \mid 0010\rangle - \beta \mid 1011\rangle - \beta \mid 1100\rangle + \beta \mid 0101\rangle) \\ \langle 1 \mid \Psi_c\rangle = \frac{1}{2\sqrt{2}}(\alpha \mid 1010\rangle - \alpha \mid 0011\rangle + \alpha \mid 0100\rangle - \alpha \mid 1101\rangle \\ \qquad +\beta \mid 1000\rangle + \beta \mid 0001\rangle - \beta \mid 0110\rangle - \beta \mid 1111\rangle) \end{array} \right.$$

$$\begin{array}{rcl} \|\langle 0 \mid \Psi_c\rangle\|^2 & = & \langle \Psi_c \mid 0\rangle\langle 0 \mid \Psi_c\rangle \\ & = & \frac{1}{8}[(\alpha^*\langle 0000 \mid +\alpha^*\langle 1001 \mid +\alpha^*\langle 1110 \mid +\alpha^*\langle 0111 \mid +\beta^*\langle 0010 \mid -\beta^*\langle 1011 \mid -\beta^*\langle 1100 \mid \\ +\beta^*\langle 0101 & \mid &) \ \times (\alpha \mid 0000\rangle + \alpha \mid 1001\rangle + \alpha \mid 1110\rangle + \alpha \mid 0111\rangle + \beta \mid 0010\rangle \\ -\beta & \mid & 1011\rangle - \beta \mid 1100\rangle + \beta \mid 0101\rangle)] \\ & = & \frac{1}{8} (\alpha^*\alpha + \alpha^*\alpha + \alpha^*\alpha + \alpha^*\alpha + \beta^*\beta + \beta^*\beta + \beta^*\beta + \beta^*\beta) \\ & = & \frac{1}{8} \left(|\alpha|^2 + |\alpha|^2 + |\alpha|^2 + |\alpha|^2 + |\beta|^2 + |\beta|^2 + |\beta|^2 + |\beta|^2\right) \\ & = & \frac{1}{2} \end{array}$$

$$\begin{array}{rcl} \|\langle 1 \mid \Psi_c\rangle\|^2 & = & \langle \Psi_c \mid 1\rangle\langle 1 \mid \Psi_c\rangle \\ & = & \frac{1}{8}[(\alpha^*\langle \ 1010 \mid -\alpha^*\langle 0011 \ \mid +\alpha^*\langle 0100 \ \mid -\alpha^*\langle 1101 \ \mid +\beta^*\langle 1000 \ \mid +\beta^*\langle \ 0001 \mid \\ -\beta^*\langle \ 0110 & \mid & -\beta^*\langle \ 1111 \mid) \times (\alpha \mid 1010 \ \rangle - \alpha \mid 00111 \ \rangle + \alpha \mid \ 0100\rangle - \alpha \mid 1101 \ \rangle + \beta \mid 1000 \ \rangle \\ +\beta & \mid & 0001\rangle - \beta \mid \ 0110 \ \rangle - \beta \mid 1111 \ \rangle)] \\ & = & \frac{1}{8} (\alpha^*\alpha + \alpha^*\alpha + \alpha^*\alpha + \alpha^*\alpha + \beta^*\beta + \beta^*\beta + \beta^*\beta + \beta^*\beta) \\ & = & \frac{1}{8} \left(|\alpha|^2 + |\alpha|^2 + |\alpha|^2 + |\alpha|^2 + |\beta|^2 + |\beta|^2 + |\beta|^2 + |\beta|^2\right) \\ & = & \frac{1}{2} \end{array}$$

$$\begin{aligned}(\langle 0 \mid \Psi_0\rangle)\dagger . (\langle 1 \mid \Psi_0\rangle) &= \frac{1}{8}[(\alpha^*\langle 0000 \mid +\alpha^*\langle 1001 \mid +\alpha^*\langle 1110 \mid +\alpha^*\langle 0111 \mid +\beta^*\langle 0010 \mid \\ -\beta^*\langle 1011 &\mid -\beta^*\langle 1100 \mid +\beta^*\langle 0101 \mid) \times (\alpha \mid 1010\rangle - \alpha \mid 00111\rangle + \alpha \mid 0100\rangle \\ -\alpha &\mid 1101\rangle + \beta \mid 1000\rangle + \beta \mid 0001\rangle - \beta \mid 0110\rangle - \beta \mid 1111\rangle)] \\ &= 0\end{aligned}$$

Les conditions sont alors vérifiées.Par conséquent,le processus de codage est effectué avec une intrication maximale des qubits utilisés.

Regardons maintenant l'intrication de chaque qubit, ajouté pour calculer les syndromes sur l'état encodé. Soit U, l'opétateur correspondant. En appliquant le circuit présenté en **Fig2.1,** les états obtenus sont :

$$\mid 0\rangle \mid \Psi_1\rangle \tag{4.58}$$

$$\frac{1}{\sqrt{2}}(\mid 0\rangle + \mid 1\rangle) \mid \Psi_1\rangle \tag{4.59}$$

$$\frac{1}{\sqrt{2}}(\mid 0\rangle \mid \Psi_1\rangle + \mid 1\rangle U_j \mid \Psi_1\rangle) \tag{4.60}$$

La matrice densité de cet état est :

$$\rho = \frac{1}{2}(\mid 0\rangle \mid \Psi_1\rangle + \mid 1\rangle U_j \mid \Psi_1\rangle)\left(\langle 0 \mid \langle\Psi_1 \mid + \langle 1 \mid \langle\Psi_1 \mid U_j^\dagger\right) \tag{4.61}$$

Le calcule de la trace partielle sur qubit, donne :

$$tr\,(\rho)_{Qubit_j} = \sum_{k=0}^{k=1}\left(\langle k \mid \otimes I^{\otimes 5}\right)\rho\left(\mid K\rangle \otimes I^{\otimes 5}\right) = \mid \Psi_1\rangle\langle\Psi_1 \mid \tag{4.62}$$

Ce résultat montre que malgré :

•La superposition de l'état de qubit.

•L'application d'une opération U contrôlée par ce qubit sur $\mid \Psi_1\rangle$.

Il n'y a pas d'intraction. Cela revient au fait que $\mid \Psi_1\rangle$ est un vecteur propre de l'opérateur U.

Généralisation :

Soit le qubit $\mid \Psi\rangle$, on applique le circuit quantique pour la détection des syndromes d'erreur.

$$\mid \Psi_0\rangle = \mid 000.........0\rangle \mid \Psi\rangle \tag{4.63}$$

$$\mid \Psi_1\rangle = \frac{1}{2^{\frac{n}{2}}} \sum_{\{I\} \ =\{0,1\} \ ^n} \mid I\rangle \mid \Psi\rangle \tag{4.64}$$

$$\mid \Psi_2\rangle = \frac{1}{2^{\frac{n}{2}}} \sum_{\{I\}} \mid I\rangle \, (U)^{\delta_{\{I\};1}} \mid \Psi\rangle \tag{4.65}$$

avec :

$$\begin{aligned} (U)^{\delta_{\{I\};1}} \quad & \mid \quad \Psi\rangle = \lambda_1^{\delta_{i_1;1}} \lambda_2^{\delta_{i_2;1}} ......\lambda_n^{\delta_{i_n;1}} \mid \Psi\rangle \\ & \equiv \quad \{\lambda\}^{\delta_{\{I\};1}} \mid \Psi\rangle \end{aligned} \tag{4.66}$$

La matrice densité de cet état est :

$$\rho^{AB} = \mid \Psi_2\rangle\langle\Psi_2 \mid \tag{4.67}$$

La trace partielle donne :

$$tr_A\left(\rho^{AB}\right) = \frac{1}{2^{\frac{n}{2}}} \sum_{\{I\}} \{\lambda\}^{\delta_{\{I\};1}} \times \left(\{\lambda\}^{\delta_{\{I\};1}}\right)^* \mid \Psi\rangle\langle\Psi \mid \tag{4.68}$$

Sachant que $|\lambda_i| = 1 \, (i = 1 - n)$, alors on a

$$tr_A\left(\rho^{AB}\right) = \mid \Psi\rangle\langle\Psi \mid$$

Chapitre 5

Conclusion générale

Les énormes difficultés qu'il y a à préserver les qubits des influences du milieu environnant représentent l'entrave majeure à la réalisation d'un prototype d'ordinateur quantique. Le problème est si aigu que la solution n'est nullement écartée de l'adoption d'un protocole massif de corrections d'erreurs.

Dans ce mémoire, nous avons étudié trois protocoles de correction des erreurs quantiques. La différence réside dans le nombre de qubits supplémentaire ajoutés dans l'étape d'encodage. Dans la première variante c'est un codage à 9 qubits, la seconde un codage à 7 qubits et enfin la plus optimale à 5 qubits. Nous avons présenté un nouveau correcteur quantique basé sur les codes stabilisateurs. Dans la solution proposée par la plate forme de M. Kh. Khalfaoui, il s'agit d'un encodage à 5 qubits mais n'utilise que 10 portes quantiques au lieu de 18 portes quantiques. Les propriétés spécifiques des opérateurs de Pauli nous a permis de repérer facilement l'ensemble des stabilisateurs. Pour valider notre proposition, une vérification manuelle des résultats a été réalisé. Ensuite, on a présenté une technique géométrique pour étudier l'intrication (la matrice densité, L'entropie de von Neumann , le produit extérieur, parallélisme des vecteurs) et appliqué cette technique au cas du circuit d'encodage en question.

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Résumé

L'objet de ce mémoire concerne la correction d'Erreur quantique d'un qubit basée sur les codes stabilisateurs. Il existe trois protocoles de correction dit Codage (codage 9 qubits, codage 7 qubits et codage 5 qubits). Nous présentons un nouveau codage 5 qubits dans lequel l'encodage ne nécessite que dix portes quantique seulement. Le processus de détection d'erreurs s'appuie sur la mesure d'un ensemble de syndromes utilisés pour discriminer cette Erreur dans tous les cas possibles. Les propriétés du groupe stabilisateur de Pauli sont déduites. Par ailleurs, on a présenté une technique géométrique pour étudier l'intrication qui nous permis de caractériser le circuit d'encodage utilisé.

Mots clés : Information quantique, calcul quantique, codes stabilisateurs L'intrication quantique.

Abstract

The subject of this thesis concerns quantum error correction of a qubit based on stabilizing codes. There are three correction protocols known as Coding (9-qubit coding, 7-qubit coding and 5-qubit coding). We present a new coding 5 qubits in which encoding requires only ten quantum gates. The error detection process is based on the measurement of a set of syndromes used to discriminate this error in all possible cases. group properties Pauli stabilizer are deducted. In addition, we presented a geometric technique to study the entanglement which allowed us to characterize the encoding circuit used.

Keywords : Quantum information, quantum computation, stabilizing codes Quantum entanglement

الملخص

يتعلق موضوع هذه المذكرة بتصحيح الخطأ الكمي للكيوبت بناءً على تثبيت الرموز. هناك ثلاثة بروتوكولات تصحيح تُعرف باسم التشفير (تشفير 9 كيوبت ، وتشفير 7 كيوبت ، وتشفير 5 كيوبت). نقدم ترميزًا جديدًا مكونًا من 5 كيوبت لا يتطلب فيه الترميز سوى عشرة بوابات كمومية. تعتمد عملية اكتشاف الخطأ على قياس مجموعة من المتلازمات المستخدمة لتمييز هذا الخطأ في جميع الحالات الممكنة. يتم استنتاج خصائص مجموعة تثبيت Pauli. بالإضافة إلى ذلك ، قدمنا تقنية هندسية لدراسة التشابك والتي سمحت لنا بتوصيف دائرة الترميز المستخدمة.

الكلمات المفتاحية: معلومات الكم ، الحساب الكمي ، تثبيت الرموز ، تشابك الكم.