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Thème

Contribution à la commande adaptative tolérante aux défauts actionneurs des systèmes non linéaires incertains

Contribution to adaptive actuator fault tolerant control of uncertain nonlinear systems

Soutenue le 30 / 09 / 2017 devant le jury composé de :

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List of acronyms

AFTC : Active Fault Tolerant Control.
ANN : Artificial Neural Networks.
DOF : Degree Of Freedom.
DSC : Dynamic Surface Control.
FDD : Fault Detection and Diagnosis.
FDI : Fault Detection and Isolation.
FLS : Fuzzy Logic System.
FMRAC : Fuzzy Model Reference Adaptive Control.
FTC : Fault Tolerant Control.
ISS : Input to State Stability.
LMI : Linear Matrix Inequalities.
LQR : Linear Quadratic Regulator.
MEMS : Micro-Electro-Mechanical Systems.
MIMO : Multi-Input Multi-Output.
MISO : Multi-Input Single-Output.
MMST : Multiple Models Switching and Tuning.
MRAC : Model Reference Adaptive Control.
PFTC : Passive Fault Tolerant Control.
PID : Proportional Integral Derivative.
RW : Reaction Wheel.
SISO : Single-Input Single-Output.
SPD : Symmetric Positive Definite.
UUB : Uniformly Ultimately Bounded.
WN : Wavelet Networks.

List of symbols

Symbols for the aircarft:

α : Angle of attack. q : Pitch rate. δ_e : Elevator deflextion angle.

ϕ : Roll angle. δ_a : Aileron deflection angle.

I_{xx} : Inertia of the wing rock model. S :wing surface. Λ : Sweep.

b : Wingspan. c_r : Root chord.

ρ : Air density. V : Air speed.

Symbols for the robot manipulator:

$q,\ \dot{q},\ \ddot{q}$: Manipulator angular position, velocity and acceleration respectively.

$D(q)$: Inertia matrix. $C(q,\dot{q})$: Coriolis term. $g(q)$: Gravity term.

τ : Torque vector. R : Joint redundancy matrix.

Symbols for the flexible spacecraft:

$\omega = [\omega_1, \omega_2, \omega_3]$: Angular velocity of the spacecraft with respect to inertial frame.

I : Inertial frame, B : Body fixed frame, O : Orbital frame.

ω_0 : Orbital rate of the spacecraft

$\phi,\ \theta,\ \psi$: Roll, pitch and yaw angles of the spacecraft respectively.

J : Inertia matrix of the spacecraft, J_s : Moment of inertia of the reaction wheel.

Ω_s : Angular velocity of the reaction wheel.

δ : Coupling matrix between elastic structures and rigid body of the spacecraft.

η : Modal displacements vector.

Λ : Modal frequency. ξ : Modal damping

R : Reaction wheels distribution matrix.

$u = -J_s\dot{\Omega}_s$: Control torque provided by reaction wheels.

T_d : External disturbances torque.

Abstract

Abstract:

The research works presented in this thesis aim to design control laws for redundant uncertain nonlinear systems to ensure an efficient management of the existing redundancy when uncertain actuator failures occur during the course of operation. The approximation based adaptive control methodology is mainly considered. Different adaptive actuator failure compensation control designs were developed for different problems. The first problem is the actuator failure compensation for uncertain redundant single variable systems in the presence of partial or total actuator loss of effectiveness. The second problem is the actuator failure compensation for uncertain redundancy multivariable systems in the presence of uncertain affine actuator failures. The third problem is the actuator failure compensation for redundancy single variable systems in the presence of generalized (non-affine) actuator failures and unknown control directions. The effectiveness of the proposed controller is proved theoretically using Lyapunov theory and through numerical simulation on several systems such as aircraft systems, redundant manipulators, and spacecraft systems.

Keywords: fault tolerant control, actuator failures, redundant actuators, adaptive control, nonlinear systems, Lyapunov stability.

Résumé :

Les travaux de recherche présentés dans cette thèse ont pour objectif la conception des lois de commande pour les systèmes non linéaires incertains afin d’assurer une gestion efficace de la redondance lorsque des défauts actionneurs apparaissent durant le fonctionnement. Dans cette thèse, la méthodologie de la commande adaptative utilisant les approximateurs de fonctions est adoptée. Plusieurs contrôleurs adaptatifs sont proposés pour différents problèmes. Le premier problème concerne les systèmes mono-variables redondants avec des défauts actionneurs de type perte d’efficacité (totale ou partielle). Le second problème est pour les systèmes multi-variables redondants avec des défauts actionneurs qui sont modélisée par un modèle affine. Le troisième problème concerne les systèmes mono-variables redondants avec des défauts actionneurs généralisés (non affines) et un gain de commande de signe inconnue. L’efficacité des contrôleurs développés a été prouvée théoriquement par la méthode de Lyapunov et validé par simulation numérique sur plusieurs systèmes tels que la dynamique de l’avion, les robots manipulateurs, et les vaisseaux spatiaux.

Mots clés : commande tolérante aux défauts, défauts actionneurs, redondance d’actionneurs, commande adaptative, systèmes non linéaires, stabilité de Lyapunov.

ملخص:

تهدف أعمال البحث المقدمة في هذه الأطروحة الى تصميم قوانين التحكم في الأنظمة اللاخطية الغير معرفة بدقة من أجل ضمان تسيير واستغلال فعالين لتكرار المشغلات الموجودة في هذه الأنظمة في حالة ظهور أعطال في هذه المشغلات أثناء عملها. اعتمدنا في هذه الأطروحة بشكل أساسي على تقنية التحكم التكيفي التقريبي، عدة قوانين تحكم تكيفي تم اقتراحها بهذا الصدد من أجل حل عدة اشكاليات : الإِشكالية الأولي تخص الأنظمة متعددة المداخل أحادية المخارج مع وجود ضياع جزئي أو كلي في فعالية المشعلات. الحالة الثانية تخص الأنظمة متعددة المداخل والمخارج مع وجود أعطال في المشغلات ذات نموذج تألفي لكنها غير معرفة. أما الحالة الثالثة فهي تخص الأنظمة متعددة المداحل أحادية المخارج مع وجود أعطال في المشعلات ذات نمودج عام (غير تألفي). كل قوانين التحكم التي تم اقتراحها تم إثباث فعاليتها نظريا بالإعتماد على نظرية ليابونوف كما تمت محاكاتها على عدة أنظمة ديناميكية متل ديناميكية الطائرة، الروبوت والمركبة الفضائية.

كلمات مفتاحية: التحكم المسامح للأعطال، أعطال المشغلات، تكرار المشغلات، التحكم التلاؤمي، الأنظمة اللاخطية، إستقرار ليابونوف.

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General Introduction

Because of the increasing needs to maintain control systems' performance at acceptable levels in a wide range of operating conditions, it has become necessary to design controllers that allow these systems to maintain their nominal performance (possibly with graceful degradation) when malfunctions or faults occur. These controllers are referred to as *fault tolerant controllers*; they have the ability to deal with systems that are subjected to faults. While classical control designs only consider systems during nominal operation, fault tolerant control (FTC) designs explicitly include the possibility of fault occurrence during the course of operation [1], [2].

Historically, safety critical systems have significantly boosted the research in the field of FTC. In particular, flight accidents that happened at the end of the 1970's have raised the need for FTC designs [3], [4]. An extensive survey of aircraft accidents and history of flight reconfigurable control has been presented in [5]–[7]. However, the research in FTC has begun to spread to other industrial fields, particularly, to systems that demand high degree of reliability and availability (sustainability) and at the same time are characterized by expensive and safety critical maintenance work such as robotic and cooperating systems, data and communication networks, oil and nuclear facilities, etc. In fact, there is a clear conflict between ensuring a high degree of availability and reducing costly maintenance times.

In a control system, actuators are important elements that feed the control actions to the plant. In safety critical systems such as aircraft, actuators (engines, rudders, stabilizers, flaps, ailerons, etc.) are more vulnerable to failures and malfunctions, and if these failures are not handled properly they can lead to catastrophic accidents. In fact, many flight accidents were caused by malfunctions or loss of actuators. In the following, some examples of flight accidents caused by actuator failures are presented:

L-1011, April 12th, 1977, USA: Lockheed L-1011 trijet, Delta Airlines Flight 1080, an elevator jammed at the full trailing edge up position [8], [9]. This failure was not indicated to the pilots, it resulted in a large nose-up pitching, and rolling moment, the airplane was just about to stall in the clouds, when the captain amazingly increased thrust on the center engine and reduced the thrust on the outboard ones. This manoeuvre allowed him to regain enough control to maintain the flight. The crew learned rapidly how to use the throttles to supplement the remaining flight controls, moved passengers forward to reduce the pitch-up tendency, and completed a safe landing. It was reported that a crew with less knowledge about the actuation redundancy in the L-1011 aircraft would likely have not been able to save this airplane from a fatal accident.

Boeing 737-200, March 3, 1991, USA: A Boeing 737-200 operated by United Airlines, Flight 585, from Denver to Colorado Springs. The aircraft started rolling to the right and pitched nose down until it reached a nearly vertical attitude. The altitude started to decrease rapidly before the plane crashed into nearby Widefield Park, a few miles from the runway threshold. The aircraft was destroyed completely, and the 2 flight crew members, 3 flight attendants and 20 passengers aboard were fatally injured. It was reported that the crash was the result of a sudden malfunction of the rudder power control unit. The pilots lost control of the airplane because the rudder surface deflected in a direction opposite to that commanded by the pilots as a result of a jam of the main rudder power control unit servo valve secondary slide to the servo valve housing offset from its neutral position and overtravel of the primary slide [10].

Boeing 747-200F, October 4th, 1992, Netherlands: A Boeing 747-200F freighter aircraft EL AL Flight 1862, from Amsterdam to Tel Aviv. Unfortunately, both right-wing engines were lost [6]. In an attempt to return to the airport for an emergency landing, the aircraft flew several right-hand circuits in order to lose altitude and to line up with the runway as intended by the crew. During the second line-up, the crew lost control of the aircraft. As a result, the aircraft crashed and hit a building. The analysis of the investigation results concluded that given the performance and controllability of the aircraft after the separation of the engines, a successful landing was highly improbable. However, later study shows that the fatal crash of EL AL Flight 1862 could have been avoided by the help of fault tolerant control [11].

McDonnell Douglas MD-83, January 31th, 2000, USA: A McDonnell Douglas MD-83 operated by Alaska Airlines, Flight 261, crashed into the Pacific Ocean about 60 miles west of Los Angeles because of a jammed horizontal stabilizer. All the passengers on board were killed, and the airplane was destroyed. The jam was later determined to be a direct result of the in-flight failure of the acme nut threads in the horizontal stabilizer trim system jackscrew assembly. The first fault the Flight 261 crew members encountered was a horizontal stabilizer jam at 0.4°, which was near the trim condition. This fault was not severe and the pilots were able to keep the aircraft aloft at 31,050 feet preparing for an emergency landing. But about twenty minutes later, the horizontal stabilizer was moved by an excessive force with huge noise from 0.4° to a new jam position, 2.5° airplane nose down, and the airplane began to pitch nose down, starting a dive. Things got worse after that – pilots lost control of the pitch axis, and the aircraft crashed into the ocean 11 minutes and 37 seconds later [12].

Beechcraft 1900D, January 8, 2003, USA: A Beechcraft 1900D, Air Midwest Flight 5481, was a flight from Charlotte/Douglas Airport in Charlotte, North Carolina, USA to Greenville-Spartanburg Airport near the cities of Greenville, South Carolina and Spartanburg, South Carolina. The aircraft stalled after take-off crashed into a US Airways hangar and burst into flames. All 19 passengers and 2 pilots aboard died in the accident, and 1 person on the ground received minor injuries. Investigation report revealed that the accident cause is an improper maintenance action of turnbuckles controlling tension on the cables to the elevators resulting in insufficient elevator travel, leading to the pilots not having sufficient pitch control [13].

***Airbus A300, November 22th, 2003, Iraq**:* On November 22th, 2003, an Airbus A300 cargo plane owned by European Air Transport and operated on behalf of DHL, was hit by a SAM-7 surface-to-air missile while climbing through 8000 feet shortly after departure from Baghdad. The missile struck the left wing and penetrated the no. 1A fuel tank. Fuel ignited, burning away a large portion of the wing. To make things worse, the plane lost all hydraulics and the pilots had to attempt a landing back at Baghdad Airport. After a missed approach, they were forced to circle the field until they finally landed heavily on runway 33L, 16 minutes later. The aircraft ran off the left side of the runway and traveled about 600 meters through soft sand, struck a razor wire fence [14].

Taking a close look at these accidents, their circumstances, and the underlying investigations the following facts and key issues on actuator failure fault tolerant control can be withdrawn:

(i) Actuator failures when they occur, they are usually unknown, i.e. it is not known which actuator has failed when and how it failed, which is a real challenge in designing an FTC.
(ii) The existing actuation redundancy within the system (in particular aircraft), if effectively used, can ensure safe operation with graceful degradation in performance and thus avoid catastrophic accidents as can be witnessed by the ***L-1011, April 12th, 1977, USA*** *case.*

Within these contexts, the research works conducted in this doctoral thesis aim to provide a theoretical framework for the design of adaptive actuator failure compensation controllers for uncertain nonlinear systems with uncertain actuator failures. Particularly, we try to provide a unified theoretical framework to the solution of this problem by considering different situations.

The thesis is organized according to four chapters: The first chapter lays the background and provides a general insight into the problem of actuator failure compensation control in particular, after a brief introduction to the topics and aims of fault tolerant control, different classifications are provided with emphasis on actuator failures. Then, the chapter provides a state of the art on existing passive and active techniques for actuator failure compensation, the superiority of adaptive control is put to the point. Different actuator failure models are given and related works from the linear and nonlinear adaptive actuator failure compensation control are provided.

The second chapter addresses the problem of actuator failure compensation control for a class of redundant nonlinear single output systems with stable internal dynamics; partial and total loss of effectiveness actuator failures are addressed. Two control designs are proposed, the first assumes that the actuator failures are unparameterized, and the second assumes that the failures are in a completely parameterized form. Numerical simulation is carried out on the dynamic model describing the angle of attack for a hypersonic aircraft with elevator failures and the wing rock motion control with aileron failures.

The third chapter addresses the control of redundant multi-input multi-output (MIMO) nonlinear systems with actuator failures. The class of compensable actuator failures is extended to affine models. The simulation is carried out on a robot manipulator with a redundant actuation system and a flexible spacecraft with redundant reaction wheels.

The fourth chapter addresses the actuator failure compensation for a class of multi-input single output nonlinear systems with unknown control direction. The class of actuator failures is further extended to general time-varying state dependent and non-affine models (unmolded actuator failures). Two designs are proposed, the first design is based on the Nussbaum-type functions while the second is based on the online estimation of the control gain sign. The simulation is carried out on the angle of attack model for a hypersonic aircraft in the present of failures of elevator segments.

The main contributions of this thesis can be summarized in the following points:

(i) The extension of many adaptive control schemes designed for the failure-free case to the case of actuator failures.
(ii) The development of a new actuator failure estimation and compensation control.
(iii) The introduction of new structural conditions on the multivariable systems that allow the design of actuator failure compensation controllers.
(iv) The extension of the class of compensable failures to general affine and non-affine failure models which are rarely considered in the research literature.

Chapter 1

1 Background material

Chapter Contents

1.1 Introduction

Fault tolerant control is an emerging topic in the area of automatic control that aims to design controllers taking into account the possibility of fault or failure occurrence during the course of operation. It merges between several disciplines into a common framework to achieve these goals and maintain system's operation within acceptable performance levels despite the presence of faults [15]. During the last few decades, the topic of fault tolerant control (FTC) in general and that of actuator fault/failure tolerant control, in particular, have received a great deal of attention as can be witnessed by the abundant research literature from the academia and industry. In addition, recent advances in the field of adaptive control have further paved the way through innovative solutions to many problems related to actuator failure compensation in the presence of both plant and failure uncertainties. Motivated by these observations, the research works carried out in this doctoral thesis aim to bring some contributions and solutions to open problems in the specific area of adaptive actuator failure compensation control. Therefore, it is natural to start by giving a general framework for this problem that allows further refinements of our goals and contributions. This first chapter provides an introduction to the actuator failure compensation control problem and presents a non-exhaustive state of the art on the established actuator failure compensation designs. This critical bibliographical analysis of existing results allows retrieving the contributions and novelties presented in this thesis and better place them among existing methods in the field. Backgrounds on actuator failures definitions, classifications and modeling are presented; redundancy requirements, constraints, implications, and types of redundancy are also provided. Insights into adaptive and nonlinear control of systems are also presented. Therefore, the remaining of this chapter is organized as follows: In section 1.2, the general concept of fault tolerant control and its necessity are presented. In section 1.3 fault and failure terminologies, classifications and modeling are presented according to different criteria with special emphasis on actuator failures. In section 1.5, general assumptions on system structure, redundancy, and actuator failures are introduced and discussed. In section 1.4 a bibliographical analysis of existing actuator failures compensation control designs is presented, this allows clearing up the novelties and contributions of the designs presented in this thesis. In section 1.6, research trends on adaptive control and related works on adaptive actuator failure compensation control are presented. Finally, section 1.7 gives a conclusion and an outline of the remaining of the thesis.

1.2 Background on fault tolerant control

As a result of the increasing demands for performance, precision, and reliability, modern technological systems had become more complex and intricate. They rely on more advanced control systems involving an increasing number of sensors and actuators allowing them to achieve high ends and to work properly even under harsh environments. Due to this complexity, faults or failures have become more common in these systems [15]–[17]. A fault can be conceived as an uncontrollable and often unpredictable defect in the system structure or parameters that eventually leads to degradation in the closed-loop system performance or even the loss of the system function (failure). In the literature, various definitions for faults and failures are given. For example, in [2] a fault is defined as '…*a deviation in the system structure or the system parameters from the nominal situation* …'. In [16], a fault is defined as '… *a non-permitted deviation of a characteristic property (feature), of the system from the acceptable, usual, standard condition*…'. A failure, on the other hand, is defined as '… *A failure is a permanent interruption of a system's ability to perform a required function under specified operating conditions* ...'[16].

From the above definitions, it comes to mind that a failure is a much more severe concept than a fault. Although the fault causes the deviation of nominal system performance and characteristics, we can in most situations endow the nominal controller with some remedial activities to overcome the fault effects and maintain acceptable performance. On the other hand, when a failure occurs, a totally different component is needed to be able to retain the nominal performance, so that a form of redundancy becomes necessary [17]. Redundancy (of components, actuators, and sensors) is a key element in the area of fault-tolerant control, its requirements and implications will be discussed further in this chapter. Notice that throughout this thesis, we will use the terms fault and failure interchangeably to refer to a malfunction in a component within the control system.

Systems without fault remedial mechanisms are weak and vulnerable. The occurrence of faults and failures in such systems may lead to severe performance deterioration or even system instability. They can cause catastrophic accidents involving human lives and millions worth equipment if they are not handled properly. This is particularly vital in safety-critical systems such as aircraft, spacecraft systems, nuclear power plants, and chemical plants containing dangerous materials [4], [18]. In such systems, minor faults can result in catastrophic consequences.

To deal with such weaknesses and vulnerabilities in these engineering systems, new control strategies have been developed; these strategies can tolerate faults in the system and maintain the desirable performance and stability characteristics. The control system must be designed in such a way that it can tolerate potential faults in system components and compensate for their effects while increasing the overall reliability of the system and maintaining the desirable performance and stability.

These new techniques are referred to as fault tolerant control (FTC) techniques. The main task of fault tolerant control is the synthesis of controllers that guarantee stability and closed performances not only when all system's components are operational, but also when, actuators, sensors or plant component have failed [2], [16], [17].

From a historical perspective, the topic of fault tolerant control was first motivated by the aircraft accidents that happened during the last decades [19], [20]. These accidents have raised the need for fault tolerant controllers for aircraft and spacecraft systems, this can be also witnessed by the fact that most literature in the field provides such aeronautical and spacecraft applications when designing fault tolerant controllers. But recently, the application of fault-tolerant control has spread out to other industrial domains such as power networks and power systems, wind turbines [21], robotic and cooperating systems [22], [23], vehicles and vehicle networks [24]–[26], data and communication networks, industrial processes and plants [24], MEMS devices [27], etc.

1.3 Faults classification and modeling

1.3.1 Classification of faults

In the research literature, Faults and failures can be classified according to many criteria, they can be classified according to their location within the system; they can be classified according to their effects or according to their temporal behavior. In this paragraph, these classifications are discussed and a special emphasis will be given to actuator failures.

1.3.1.1 Classification according to their location

In the literature, faults (or failures) can be classified according to their location in the control system as actuator, sensor and plant faults as shown in Figure 1.1.

Actuator faults/failures: This type of faults corresponds to uncontrollable changes undergone by the control input applied to the controlled plant. This fault alters the effect of the actuator on the plant either partially or totally [17]. The total actuator faults mean that the actuator is not responding to the inputs applied to it. This can occur as a result of breakage, or burnt out wirings. Partial actuator faults are cases in which the actuator is partially influenced by the input, so it is less effective and hence provides the plant with part of the normal actuation signal.

Sensor faults/failures: In a control system, sensors are elements that take measurements or observations from the plant for display or feedback, examples are potentiometers, accelerometers, tachometers, pressure gauges, strain gauges, etc. Sensor faults imply that incorrect readings or measurements are taken from the real dynamical system. These faults can also be subdivided into either total or partial sensor faults. The total sensor fault is the case in which the sensor provides readings that no longer correspond to the real physical parameters. The partial sensor fault is the case where the sensor provides inaccurate readings that contain the real physical parameters such that the required reading could be retrieved.

Plant faults/failures: This type of faults directly affects the physical characteristics of the system and consequently alters the dynamics and input/output properties of the system. Process faults are often termed component faults, arising as variations from the structure or parameters used during system modeling, and as such cover a wide class of possible faults e.g. dirty water having a different heat transfer coefficient compared to when it is clean, or changes in the viscosity of a liquid or components slowly degrading over time through wear and tear, aging or due to other environmental factors, etc.

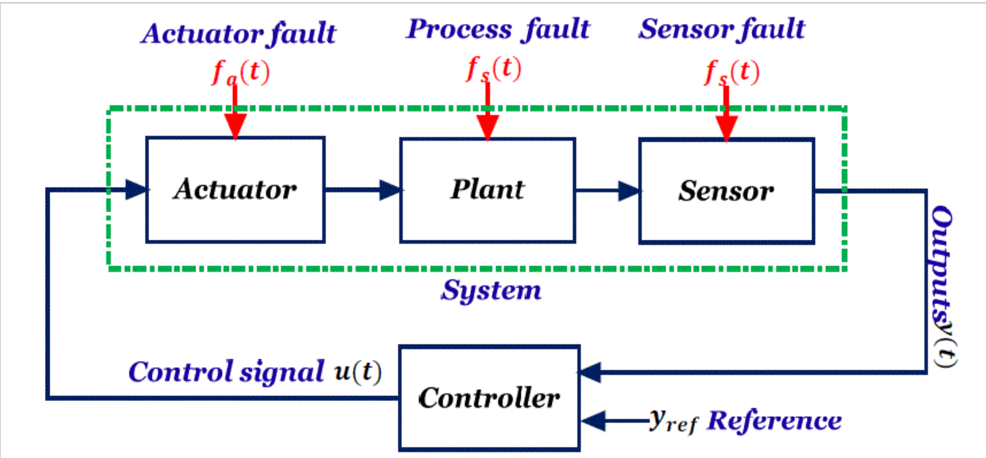

Figure 1.1: Classification of faults according to their location [21]

1.3.1.2 Classification according to their effect

Faults are usually classified according to their effect on the system dynamics as additive and multiplicative faults. Additive faults affect processes as unknown and uncontrolled inputs (actuator or sensor bias), while multiplicative faults usually have severe effects on the process dynamics and can cause instability behavior. The mathematical models of additive and multiplicative faults will be presented later on in this chapter for the case of actuator failures.

1.3.1.3 Classification according to their temporal behavior

Faults or failures can also be distinguished according to their duration and their temporal behavior, Figure 1.2. Abrupt faults are sudden changes in the behavior of the system (non-smooth step-like time behavior), for example, in a breakdown of the power supply. While incipient faults are gradual and slow drifting faults (smooth time behavior), these can be brought about by wear of mechanisms, or leakages. Permanent faults lead to the total failure of the equipment (once they occur they do not disappear), transient fault are temporary malfunctioning (appear for a short time and then disappear), and intermittent fault are the repeated occurrences of transient fault (they appear, disappear, and then reappear, pulsating time behavior), for instant this can be caused by an intermitted electrical contact.

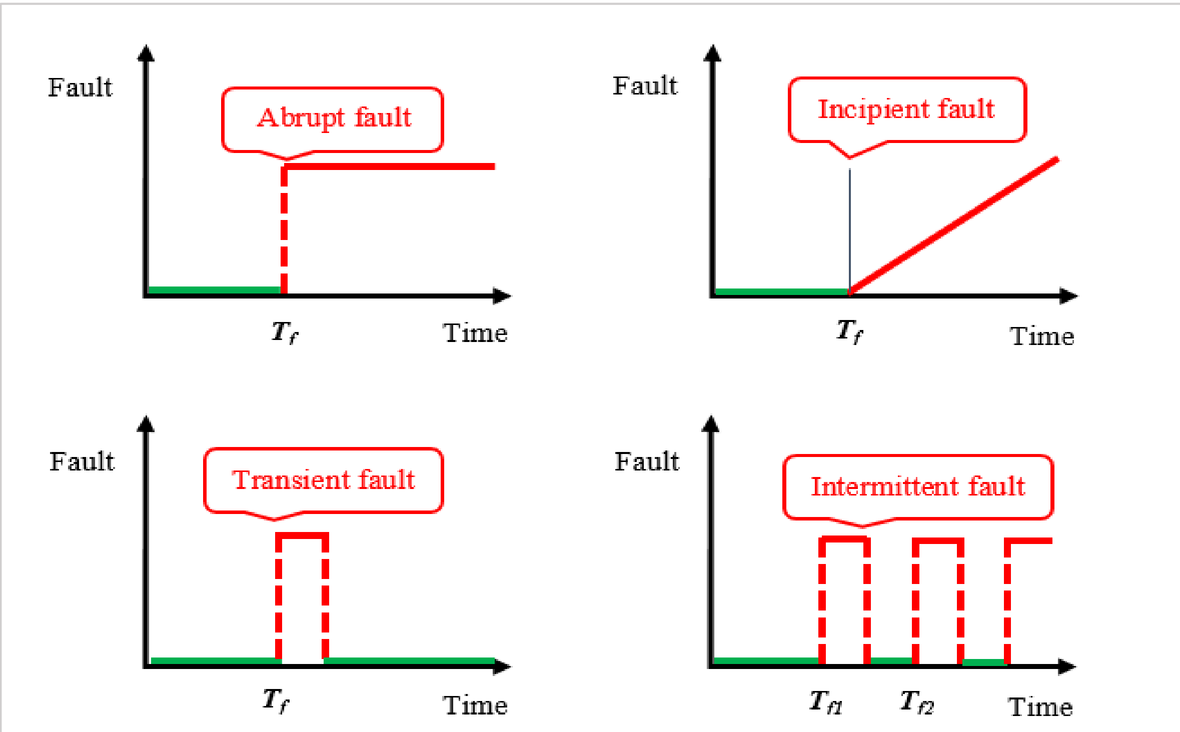

Figure 1.2: Classification of faults according to their temporal behavior [21]

1.3.2 Difference between faults, disturbances and model uncertainties

A well-known fact in the control field is that disturbances and model uncertainties can adversely affect the dynamic behavior of the system in a way somehow similar to that of failures and faults [2]. In order to distinguish between these three phenomena, consider the case of a dynamic system that is described by a mathematical model (e.g. differential equations) and represented by the block diagram in Figure 1.3. For this system, faults are usually represented as additional external signals or as parameter deviations. In the first case, the faults are called additive faults, because in the model the faults are represented by an unknown input that adds to the model equation as an additional term. In the second case, the faults are called multiplicative faults because the system parameters depending on the fault size are multiplied with the input or system state.

In principle, disturbances and model uncertainties have similar effects on the system. Disturbances are usually represented by unknown input signals which add to the system's output. Model uncertainties change the model parameters in a similar way as multiplicative faults. However, an important distinction between disturbances, model uncertainties, and faults can be seen in the fact that disturbances and model uncertainties are always present, while faults may be present or not, or more precisely, can appear during the system operation. Disturbances represent the action of the environment on the system, whereas uncertainties are a result of the modeling activities that end up with a model as an approximate representation of the actual system behavior. Hence, both phenomena are nuisances whose effects on the system performance are handled by appropriate measures like filtering or robust and adaptive control designs [28]. They do not call for FTC, but for controllers designed to attenuate their effects.

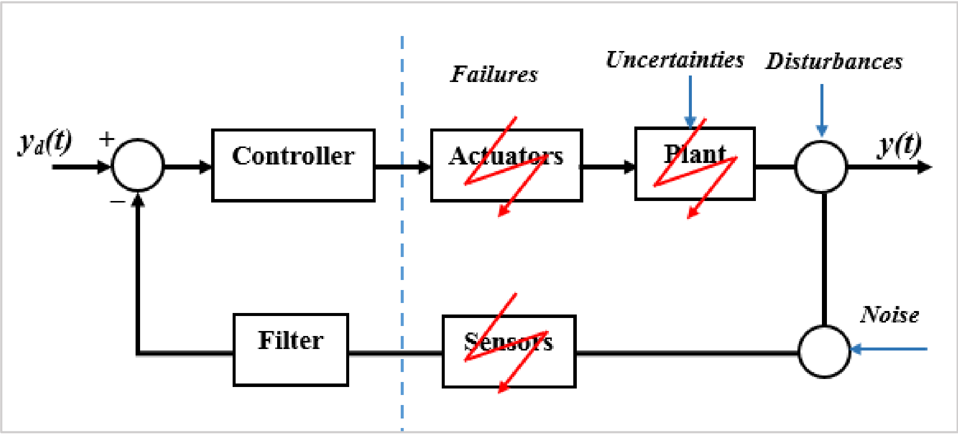

Figure 1.3: Difference between disturbances, uncertainties, and faults

After introducing the aims of fault tolerant control, and classified the faults that can occur in a control system, our attention will be focused on the actuator failures. The remaining of this thesis is focused on actuator failures.

1.4 Actuator failures descriptions and modeling

Actuators are essential elements in every control system. They map the control signals issued from the controller or the operator to actions acting directly on the plant [84]. Depending on the physical nature of the system and the effort required, actuators can be electrical, electromagnetic, mechanical, hydraulic, pneumatic, piezoelectric, MEMS actuators, etc. In a robot manipulator, for instance, actuators are motors acting on the robot links [22]. As for an aircraft, actuators can be control surfaces such as rudder, aileron, and elevators, which are actuated by pneumatic or hydraulic actuators [19], [65], see Figure 1.4. While in a spacecraft actuators are reaction wheels and thrusters [79], see Figure 1.4. Actuators are more likely to fail during the course of operation, which makes the system completely uncontrollable, thus, some redundancy is necessary. In this section, different mathematical models of actuator failures from the literature are provided, then redundancy requirements are briefly discussed.

1.4.1 Actuator failures modeling

In the literature, actuator failures are represented according to the way they affect the systems by two models, the multiplicative model, and the additive model, or combinations of both models. The multiplicative loss of effectiveness failure models are described as follows [17]

$$u^f(t) = \rho(x,t)u(t),\ t \geq t_f \tag{1.1}$$

where $u(t)$ and $u^f(t)$ are respectively the input and output of the actuator, $0 \leq \rho(x,t) \leq 1$ is the actuator effectiveness (the remaining control effort) and t_f is the time of failure occurrence. If $\rho(x,t)=1$, the actuator is completely effective (no failure) when $\rho(x,t)=0$ the actuator has completely lost its effectiveness.

On the other hand, the additive bias failure models are mathematically expressed as follows

$$u^f(t) = u(t) + \overline{u}(x,t),\ t \geq t_f \tag{1.2}$$

where $\bar{u}(x,t)$ is the fault term that describes the actuator offset.

In practical situations, however, the actuator may undergo loss of effectiveness and offset simultaneously. In this case, the corresponding failures can be modeled as follows

$$u^{f}(t)=\rho(x,t)u(t)+\bar{u}(x,t),\ t\geq t_{f} \tag{1.3}$$

Notice that actuator failures described in, (1.1), (1.2) and (1.3) implicitly assume some knowledge of the type of failures undergone by the actuator, that is to say, it is known a priori that the actuator undergoes offset or/and loss of effectiveness failures. In some situations, however, the failure pattern is more complicated and cannot be modeled as these two types, this is referred to as the non-modeled actuator failures [17], [85]. The input/output characteristic is not affine in this case; this can be mathematically described as

$$u^{f}(t)=\bar{u}(u,x,t),\ t\geq t_{f} \tag{1.4}$$

where $\bar{u}(u,x,t)$ is the output of the actuator, which is assumed unknown.

Remark 1.1: In practical situations, actuator failures are usually uncertain, it is not known which actuator have failed, when the failure has occurred, what type ant what is the value of the failure. Established techniques in FDI are usually used to detect the failure, locate the failed actuator and estimate the value of the failure.

Throughout this thesis, our main objective is to provide a theoretical framework for the solution of the problem of adaptive actuator failure compensation, we will try to cover, in a progressive approach, the different types of actuator failures described in this section.

1.4.2 Redundancy requirements

The use of multiple actuators for control designs to safety critical systems such aircraft and spacecraft systems provides enough capacity for actuator fault tolerance. However, even with enough actuator redundancy, the design of controllers for dynamic systems to automatically accommodate uncertainties and compensate for the damaging effect of actuator failures is still a challenging problem, especially for nonlinear systems. A desirable actuator failure compensation control scheme which generates multiple actuating signals from multiple actuators should be able to ensure system stability and the desired control performance for both cases when all actuators are in normal operation (failure-free case) and when some of the actuators have failed.

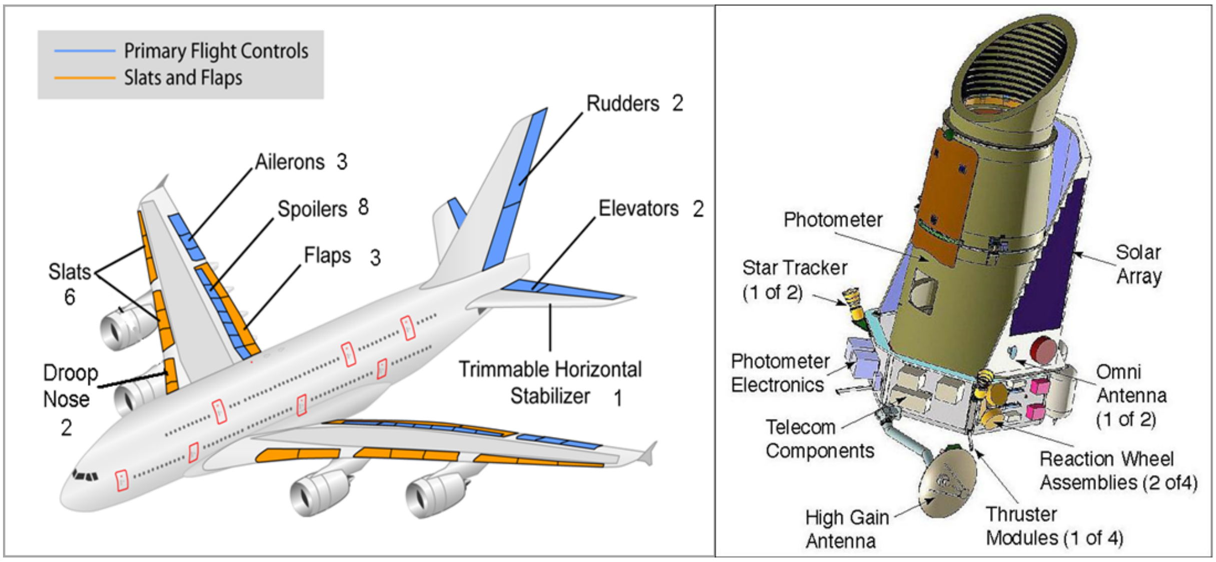

Figure 1.4: Examples of redundant actuator systems left plane A380, right spacecraft.

1.4.2.1 Actuator redundancy

From the systems architecture point of view, one way to increase systems' reliability is redundancy. That is the use of multiple actuators that can fulfill the same task, see Figure 1.4 . These actuators can be similar in physical characteristics but have the similar effect on the system or they can be similar in physical characteristic and in effect. For example, segments of a multiple-segment rudder or elevator for an aircraft. For this case, a reasonable (natural) design for the applied control inputs is one with equal or proportional actuation for each actuator, that is, all control inputs are designed to be equal or proportional to each other [19]. Thus, in the sequel, the following general assumption is stated:

Assumption 1.1: To design actuator failure compensation strategies, it is assumed that all the systems under study are provided with some type of actuation redundancy, allowing them to remain controllable in the presence of up to a certain number q of actuator failures among the existing m actuators such that $m > q$, i.e., at least one actuator should remain effective.

1.4.2.2 Compensable actuator failures

To design actuator failure compensation controllers for a given system with actuator failures. The actuator failure pattern should be compensable; otherwise, there is no need to seek controllers for non-compensable failures patterns. A failure pattern is called a compensable failure pattern of a certain control design if the associated actuator failures can be compensated (that is, closed-loop stability and asymptotic tracking are ensured despite actuator failure uncertainties) by the control design. The compensable failure pattern set of a control design for a system is the set of all failure patterns compensable by the control design. We note that the compensable failure pattern set which is the largest set of failures that can be compensated by a control design is determined by both the controlled system and the control design as indicated by the results of [77].

1.4.2.3 Hardware redundancy and actuation schemes

Redundancy can be analytical or hardware, analytical redundancy is mainly considered for sensor failures, while hardware redundancy is suitable for actuator failures. Thus the works to be undertaken in this research concern the design of actuator failure compensation techniques for multiple actuator systems. For the case of multi-input multi-output (MIMO) systems, the presence of redundancy implies that a group of actuators can have a similar effect on a given output. There, actuation grouping schemes are considered in this case, that is the actuators are divided into several groups and each group has actuators of the same physical characteristics (for example, an aircraft has a group of four engines and a group of three rudder segments); in that case, a common control signal is designed then, an equal or proportional actuation scheme is considered for each group [77], [86]. The control scheme can also be developed around an actuation matrix if the actuator distribution matrix is known [87]. In the following, a state of the art on existing actuator failure compensation strategies with critics is provided.

1.5 State of the art on actuator failure compensation control

During the last three decades, the problem of fault-tolerant control (FTC) in general and that of actuator failure compensation control in particular, have received a considerable attention from researchers in the industry and academia. Extensive research works have been carried out and various actuator failure compensation techniques have been proposed. Generally, these techniques can be classified according to two types, passive fault tolerant control (PFTC) approaches and active fault tolerant control (AFTC) approaches [4], [29]. In this section, a non-exhaustive bibliographical analysis with critics of passive and active actuator failure compensation designs is presented.

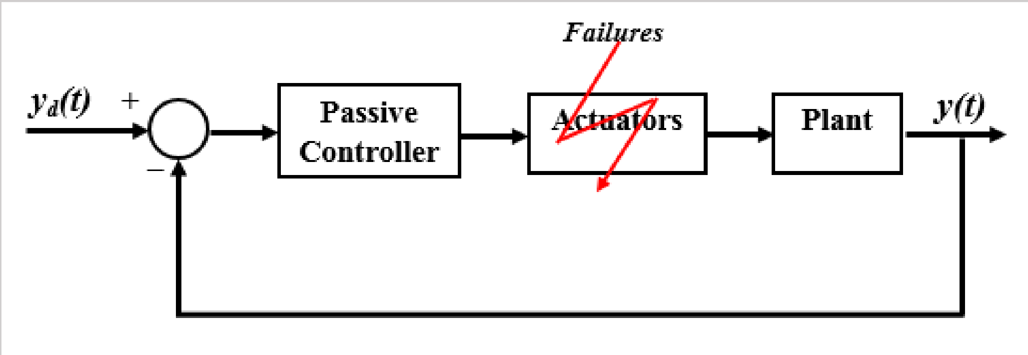

Figure 1.5: General block diagram of passive FTC systems

1.5.1 Passive approaches

Passive actuator failure compensation designs rely on a fixed structure controller, this controller should be able to ensure system operation within acceptable performance levels in the normal operation and in the presence of actuator failures. The techniques of robust and reliable control are generally used in this context. The methodology of passive FTC or reliable control is to consider a set of failure cases along with normal operating conditions at the controller's design stage [4], [29], [30]. An illustrative block diagram of passive fault tolerant control is depicted in Figure 1.5. It has much the same structure of usual control designs.

In the research literature, there are many established passive actuator failure compensation designs. In [31], the authors proposed a design approach for centralized and decentralized control systems which are robust and reliable against a prescribed set of actuator failures, the proposed design was capable of ensuring closed-loop stability and H_∞ performance in the presence of actuator failures. The concept is regarded as the baseline of the current studies on passive FTC. In [32], a linear quadratic regulator (LQR) is exploited to design a reliable controller against a class of actuator failures. In [33], a passive fault tolerant design is proposed for a class of redundant systems using the pole placement method. Based on which the passive FTC design is proposed using pole region placement method. In [34], linear Gaussian (LG) regulators and H_∞ designs are used to design passive fault tolerant controller for flight tracking control. An enhanced linear matrix inequalities (LMI) based design was developed to synthesize a fault tolerant controller in [35], umerical simulation results illustrate that the passive FTC can not only stabilize the system under actuator failures but also maintain a favorable level of tracking performance. In [35] a mixed H_2 / H_∞ passive FTC is designed to counteract actuator failures based on an enhanced LMI based design. From the aspect of performance, the approaches in [34], [35] are less conservative when compared with the conventional LMI methods. Moreover, with respect to nonlinear systems, Hamilton-Jacob inequality [36], [37], variable structure [38], passivity theory [37] and sliding mode control [39], [40], are used to design passive FTCS, which are capable of accommodating actuators malfunctions. Some recent works were also proposed in this regard, in [30], based on the parameter dependent Lyapunov and slack method, a passive controller is proposed to deal with the system and actuator failure uncertainties with application to the airplane. In [41], a reliable passive fault tolerant controller was proposed for a class of Takagi-Sugeno systems with time delay. In [42], a robust controller is developed for nonlinear multivariable systems in the presence of actuator faults and saturation with application to spacecraft system. Since neither real-time fault detection and diagnosis (FDD) nor control reconfiguration is needed, a single passive FTC is engaged in the absence or in the presence of actuator faults. Thus the term passive underlies that no further action needs to be taken by the designed FTC when the prescribed fault occurs during the course of operation. Due to that, a passive FTC has a relatively simple structure to be implemented, and no time delays exist between the fault occurrences and corresponding actions. The design of passive FTC has attracted significant attention since the 1990s. Despite their advantages, the main drawback of passive FTC techniques is that they cannot deal with a larger set of actuator failures.

1.5.2 Active approaches

In contrast to passive FTC techniques, active FTC techniques are designed to react to system, actuator or sensor malfunctions by reconfiguring the controller based on the real-time information from a fault detection and diagnosis (FDD) mechanism or based on systems errors. The term ‘active’ represents corrective actions taken actively by the reconfiguration mechanism to adapt the control system in response to the detected system faults. As shown in Figure 1.6, a general active FTCS typically consists of a FDD scheme, a reconfigurable controller, and a controller reconfiguration mechanism. The three units have to work in harmony to complete successful control tasks. Based on this architecture, the objectives of an active FTCS are: (1) Develop an effective FDD scheme to provide information about the fault with minimal uncertainties in a timely manner, (2) Reconfigure the existing control scheme to achieve stability and acceptable closed-loop system performance and (3) Commission the reconfigured controller smoothly into the system by minimizing potential switching transients. Many active FTC techniques exist in the research literature, among them, we can cite:

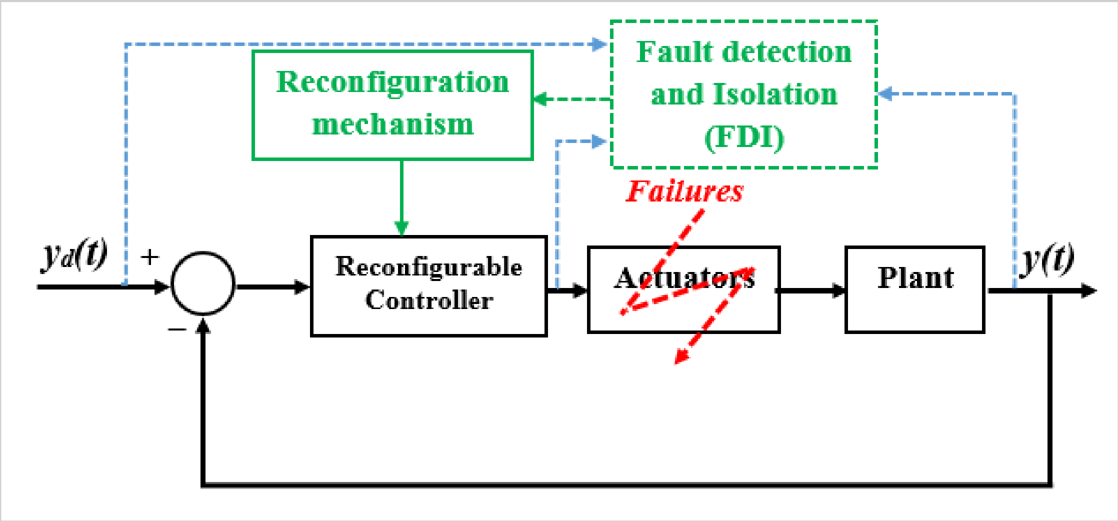

Figure 1.6: General block diagram of active FTC system

<u>Multiple model based designs:</u> These techniques are widely used in fault tolerant control, they belong to projection methods. Amongst them, we have multiple models, switching and tuning (MMST) approach [43], [44]. Basically, control schemes based on the multiple model switching and tuning (MMST) approach consists of a cluster of *N* identification models, which represent diverse operating environments, and a set of *N* controllers which are to be chosen according to the switching law regarding the identification models errors. The controller parameters are then tuned over a slower time scale to improve accuracy. The baseline multiple model fault tolerant controller is illustrated in Figure 1.7, [45], [46]. Many works exist in this regard, for example, in [47], a new multi-layer multiple model switching approach was proposed for actuator failure compensation with improved transient performance. In [48] a MMST control scheme is proposed for a class of nonlinear multivariable systems based on actuator grouping with application to the longitudinal model of twin otter aircraft with elevators' failures. In [49], a multiple model actuator failure compensation design is proposed for Two Linked 2WD Robots.

The main drawback of multiple-model based designs is that they use a large number of plant models, running online to cover possible failure cases, which increases the operations costs and computational burdens considerably. But still, it is a more general approach that is used for plant, sensor and actuator failure compensation.

<u>Fault diagnosis and isolation (FDI) based designs:</u> Fault diagnosis including fault detection and isolation (FDI) [2], [16], [17], [50] is used to detect faults and diagnose their location and significance in a system. It has the following tasks: fault detection, fault isolation, and fault estimation Figure 1.8. Fault detection is to make a decision, e.g., faults occur in the controlled

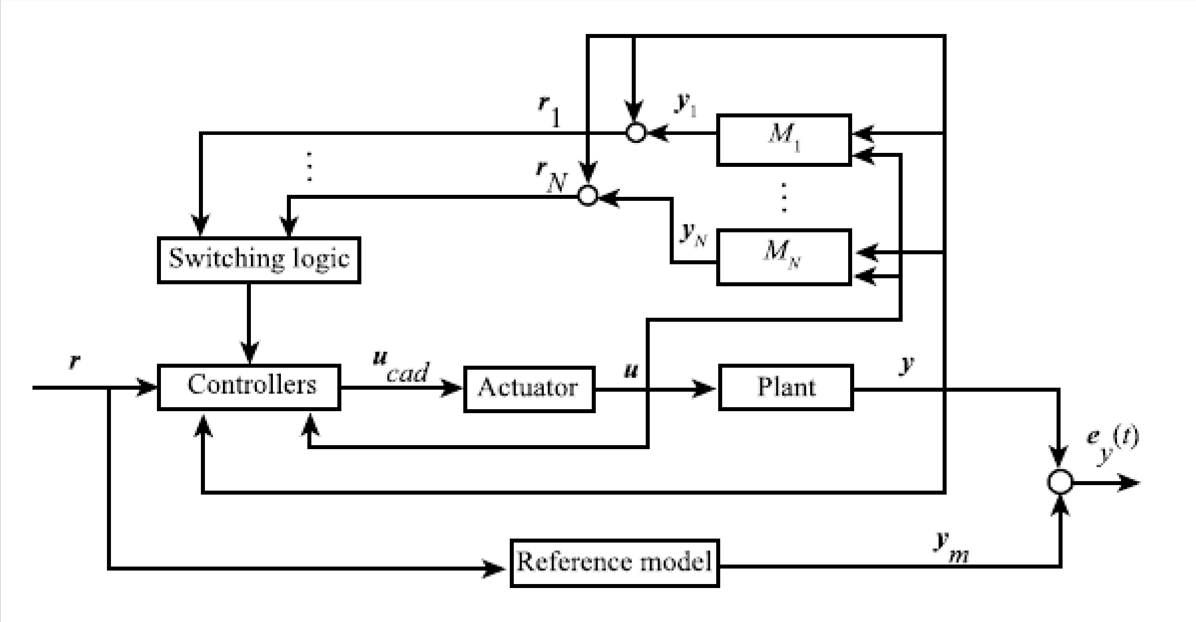

Figure 1.7: Block diagram of a baseline Multiple model based FTC [45]

systems or not. Fault isolation is used to determine the location of the faults, namely, which physical component has become faulty. The last task in fault diagnosis is to estimate the size of the fault. This information on the fault is used by the control reconfiguration/accommodation mechanism to issue the corresponding control signals to the system. Many works exist in this context, for instance, in [51], a reconfigurable PID controller was combined with an FDI module while in [24] a reconfigurable LQR was combined with an FDI module for actuator and sensor faults accommodation with application to the conveyor belt and active suspension systems. For the nonlinear case, feedback linearization and sliding mode controllers were combined with an FDI module [24]. In [52], a feedback linearization reconfigurable controller was used with a perturbation observer fault estimation and accommodation for nonlinear systems. These techniques are the baseline of active fault tolerant control, however, they have some drawback namely, these methods need some latent time fault diagnosis and isolation which limits their real-time implementation, uncertainties are not taken into consideration, besides, sometimes faults and disturbances cannot be distinguished.

The pseudo-inverse technique: Considered among the first techniques of reconfigurable control [53]–[56]. It is relatively easy to implement and can be applied to a larger set of predefined faults. The baseline approach starts from the nominal system (A, B, C), with a control law $u = Kx$ satisfying the prescribed performance in the fault-free case. The fault

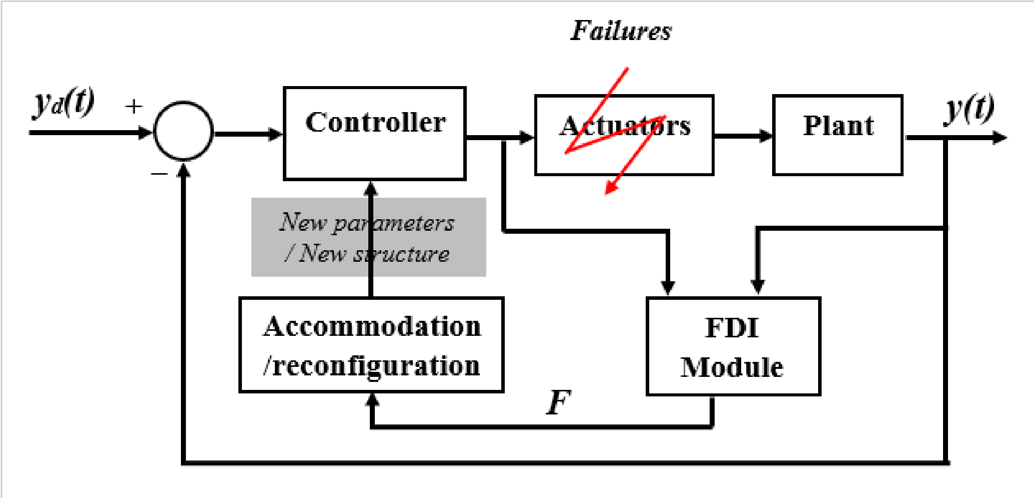

Figure 1.8: Block diagram of an FDI based FTC system

tolerant feedback $u = K^f x$ is derived by minimizing the distance between the nominal and the faulty system $\left(A_f, B_f, C_f\right)$ such as [53]

$$K^f = \arg\left[\min\left\|\left(A + BK\right) - \left(A_f + B_f K_f\right)\right\|_F\right] \tag{1.5}$$

where $\|\cdot\|_F$ denotes the Frobenius norm of a matrix.

The advantage of this method is its simplicity and its ability to compensate for different types of failures simultaneously. Its main drawbacks are that it cannot ensure closed-loop stability, this problem was investigated in [54]. The second main drawback of this technique is that it assumes a complete knowledge of the faulty system model, which is not always in our hands.

The eigenstructure assignment technique: In this approach, the aim is to bring the eigenstructure (both the eigenvalues and the eigenvectors) of the faulty system close to that of the nominal system (without failures). The idea is to assign the significant eigenvalues to desired positions and at the same time minimize the distance between the corresponding eigenvectors. This can be achieved via state feedback [3].

Input reconfiguration technique (reference redesign): It is based on updating the reference signal so that in the case of a fault, the system output is close to the desired output [58]. This is achieved by a reference redesign block. For a given system $\left(A, B, C\right)$, with a reference input $r\left(t\right)$, control input $u\left(t\right)$, state $x\left(t\right)$, and output $y\left(t\right)$. First, the state feedback controller with feedforward gain in the fault-free case is given by:

$$u = -Kx - Fr \tag{1.6}$$

where K is the state feedback gain and F is the feedforward gain. In this technique, the controller is kept as it is, while the reference is updated based on the error as follows

$$r_{new} = r_{initial} - g\left(y - r\right) \tag{1.7}$$

This new reference is then plugged into the control law as: $u = -Kx - Fr_{new}$.

Fuzzy logic and neural networks based designs: The proved capabilities of these techniques in dealing with uncertainties and complexities and their learning features made them good candidates for actuator failure compensation control designs. Many fuzzy and neural networks based designs were proposed in the literature. In [59], the authors used Takagi-Sugeno models to represent the faulty system under different fault scenarios, then a mechanism is used to select between these models or combines them. In [60], a supervisory fuzzy model reference learning controller (FMRLC) is developed for aircraft subject to actuator failures. Artificial neural networks (ANN) have been also widely used to design fault tolerant controllers, for instance, in [61], a neural networks controller is developed based on a sliding mode control to compensate for a class of incipient actuator failures, in [62], an adaptive neural network controller is developed for tailless aircraft with redundant actuation devices. It is worth to notice that most fuzzy and neural network controllers are combined with other methods such as multiple model designs [59], [63], and adaptive control technique (fuzzy/neural adaptive control) as will be detailed in the next paragraph.

Adaptive control based designs: Adaptive control updates the controller online to cope with structural, parametric and environmental changes within the system[28], [64]. It has been successfully applied in the area of actuator failure compensation [17], [19], [65]. A bloc diagram for the adaptive actuator failure compensation is given in Figure 1.9. In the research literature, numerous adaptive designs have been proposed for actuator failure compensation. A tutorial that summarizes the recent achievements in this field is given in [65]. The authors in [66]–[68] proposed adaptive control schemes of linear systems with lock in place actuator failures for state and output tracking. In [69], [70], the authors propose an adaptive control scheme for nonlinear systems with unknown actuator failures using feedback linearization techniques. An adaptive backstepping control scheme for parametric strict feedback systems with actuator failures was proposed in [71], [72]. In these works, system parameters are assumed known or partially known, i.e. defined as the product of known nonlinear functions.

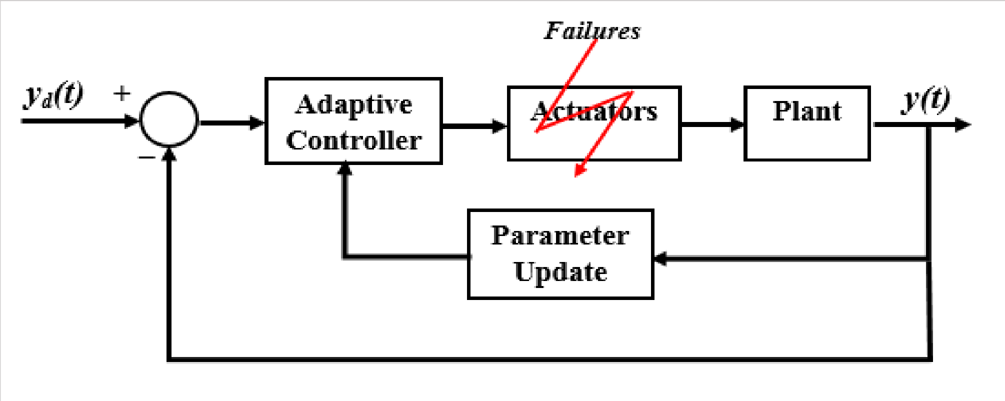

Figure 1.9: Adaptive based actuator failure compensation control

with unknown parameters. The case of unknown nonlinear systems with unknown actuator failures has been rarely dealt with. In [73], [74], an adaptive fuzzy control scheme for uncertain nonlinear systems with actuator failures has been proposed for systems in the normal form. Backstepping and dynamic surface control (DSC) designs are proposed for parametric strict systems with actuator failures in [75], [76]. In these works, the idea was to bring the faulty unknown system to an ordinary system with unknown parameters, and then adaptive neural and fuzzy control techniques are applied to this system.

For the case of multivariable nonlinear systems, a few works exist on adaptive actuator failure compensation. In [77]–[79], the authors proposed an adaptive failure compensation control designs for MIMO systems with application to aircraft and spacecraft systems. In [80]–[83], the authors proposed adaptive control of MIMO systems using fuzzy backstepping and dynamic surface control designs, respectively, where fuzzy and neural systems are used for the online approximation of system functions in the framework of indirect adaptive control. An adaptive failure compensation scheme for redundant joint robots and cooperating manipulators has been addressed in [22], [23], however, these designs require knowledge about system parameters which is not always possible. Further elaboration on the adaptive control theory and its applicability to the problem of actuator failure compensation is detailed in the last section of this chapter. The beauty of adaptive control is that it does not require an FDI module Figure 1.9. More bibliographical analysis on adaptive actuator failure compensation is provided progressively in the next chapters within their appropriate contexts.

1.6 Research trends on adaptive actuator failure compensation control

In this section, an introduction to adaptive control notions and terminology is provided. Afterward, a brief insight into related works on the adaptive actuator failure compensation control for linear and nonlinear systems is provided. This allows us to refine the objectives of the thesis and better situate the results with respect to existing ones.

1.6.1 Adaptive control terminology

Adaptive control provides mechanisms for controller update to cope with structural, parametric and environmental changes and uncertainties [28], [88]. It has been successfully applied to industrial process control, aircraft control, vehicle control, power systems, and robotic manipulators. Nowadays, is becoming more attractive in developing novel non-traditional applications, such as real-time systems, fault tolerance and accommodation, and micro-electro-mechanical systems (MEMS) [89], [90].

In its essence, a baseline adaptive controller consists of an ideal control law motivated from the nominal case, with parameters assumed to be known, and an online parameter estimator (referred to as adaptive laws or parameter update laws) to provide the estimates of these unknown parameters [28]. In the research literature, there are two classes of adaptive control designs: indirect adaptive control and direct adaptive control. The difference lies in the fact that the estimated parameters are either those of the controller (direct) or those of the plant (indirect) [91]. Notice that these direct and indirect adaptive control techniques can be combined within one composite direct/indirect control design [91], [92].

For decades, adaptive control has been employed to solve linear and nonlinear control system problems [28], [64]. For linear systems, the controller parameters, (e.g., state feedback gains, feedforward gains, proportional, integral and derivative (PID) gains [93], [94]), are updated online via an appropriate mechanism. This mechanism should ensure closed-loop stability and tracking requirements despite the presence of disturbances and uncertainties.

For the case of nonlinear systems, adaptive feedback linearization [95], [96], adaptive sliding mode [97], [98] and backstepping control designs have been developed. The idea of these strategies is to estimate some system or/and controller parameters online to cope with changes and uncertainties in the system. These approaches are effective when it is known a priori that system functions can be expressed as the product of unknown parameters with known functions.

When this is not the case, these so-called model-based adaptive approaches become limited. To remedy these limitations, as a model-free design approach, adaptive approximation based adaptive control has emerged as an effective method to design controllers for complex and ill-defined processes whose mathematical models are difficult to obtain [91], [99], [100]. It is shown that fuzzy systems, neural networks, wavelet networks, orthogonal polynomials can approximate many classes of functions to a given accuracy. Several fuzzy and neural adaptive controllers have been developed for uncertain nonlinear systems [91], [99], [100]. Another trend in approximation based adaptive control of nonlinear systems is towards the use of simple tunable linear feedback or PID controllers to approximate some ideal nonlinear controllers [93], [94], [101]. This alleviates the computational complexity of neural and fuzzy systems as a large number of basis functions are needed to have higher accuracy.

After introducing some guidelines and terminology on adaptive control, the following subsection provides some guidelines on related work from the literature on linear and nonlinear adaptive actuator failure compensation, which inspired our research.

1.6.2 Related work on adaptive actuator failure compensation design

Motivated by the above guidelines on advancement in the adaptive control technology, the research on adaptive actuator failure compensation has flourished starting from the year 2000. The research in this area was primarily pioneered by G. Tao and his coworkers and students [19], [65], several works on the topic have been carried under his guidance ranging from linear to nonlinear systems. In this subsection, we present some highlights of the main related results.

1.6.2.1 The case of linear systems

For linear systems, the designs are generally based on model reference adaptive control (MRAC). Classical adaptive controllers (state and output feedback with feedforward gains) have been extended to deal with the problem of actuator failure compensation [19], [65]. Consider a linear system (A, B, C) with state vector $x(t)$, input $u(t)$, and output $y(t)$. The actuators are subject to actuator failures described in subsection 1.4.1; the control objectives were to design an adaptive controller with adaptation mechanisms so that the dynamics of the faulty system track a reference model (A_m, B_m, C_m) in the framework of state feedback for state tracking, or the output y tracks a reference model with a transfer function $W_m(s)$. In both cases, the actuator failure compensation controller is based on a classical feedback/feedforward and an additional gain to account for failure effects, this controller is defined as follows [65]:

$$u(t) = K_1^T(t)x(t) + k_2(t)r(t) + k_3(t) \tag{1.8}$$

where K_1, k_2 and k_3 are tunable controller parameters.

The other close problem is output feedback for output tracking, in this case, an output feedback controller is derived to ensure output tracking of a reference model. In this case, the proposed controller is parameterized as follows [19]:

$$u(t) = \Theta_1^T(t)\omega_1(t) + \Theta_2^T(t)\omega_2(t) + \theta_{20}(t)y(t) + \theta_3(t)r(t) + \theta_4(t) \tag{1.9}$$

where Θ_1, Θ_2, θ_{20}, θ_3 and θ_4 are adjustable controller parameters.

Under some prescribed assumptions, both problems have been solved for the single variable and the multivariable case. Adaptive laws for the parameters that ensure tracking and stability in the presence of actuator failures have been derived, using different actuation schemes. Stability proofs are carried out using Lyapunov theory along with a piecewise analysis to deal with possible parameter jumps caused by abrupt actuator failures. Notice, however, in all these works, only lock-in-place failure types have been considered [19].

1.6.2.2 The case of nonlinear systems

For the case of nonlinear systems, the problem of actuator failure compensation can be formulated as follows: given a nonlinear system described as:

$$\begin{aligned} \dot{x} &= f(x) + g^T(x)u \\ y &= h(x) \end{aligned} \tag{1.10}$$

where $f(x)$, $g(x)$, $h(x)$ are nonlinear sufficiently smooth functions.

Different situations were considered, in some situations, it was assumed that the system dynamics are known and the actuator failures are uncertain while in other situations it was assumed that both the system dynamics and actuator failures are unknown or partially known, i.e. described as the product of unknown parameters and known functions.

The aim is to develop an adaptive controller for the system (1.10) so that, in the presence of actuator failures, the output of the system tracks a reference signal. In [19], this problem has been solved from two perspectives: For feedback linearizable systems, the solution is to bring the system into the canonical form via coordinate transformation, then classical adaptive feedback linearization control is applied. For parametric strict form, via coordinate transformations, the adaptive backstepping is applied. Notice that these developments form the baseline of many other works. Examples are in [73], [75], [76], [102], where adaptive approximation based control is derived based on fuzzy and neural techniques, in [75], the adaptive fuzzy backstepping techniques are adopted to this problem, and in [102], the dynamic surface control using neural networks is applied.

In most of the stated works, the compensable actuator failures considered are bias faults and partial loss of effectiveness. Besides, the failure patterns are assumed state-independent. In this thesis, inspired by all these works, we try to bring some contributions to these problems, mainly: We try progressively to widen the set of compensable actuator failures cover all the actuator failure models described in (1.1)-(1.4). We try further to solve the problem related to control gain sign knowledge assumption, by considering the unknown control direction case.

1.7 Conclusion

In this chapter, our main concern was to present the concept and the needs for actuator failure compensation control. The different classifications and aspects of actuator failures are presented. Actuator failures are modeled according to their effects on the system with an increasing level generalization and complexity. A bibliographical analysis on existing actuator failure compensation designs is introduced to raise the limitations of existing methods in terms of generality (set of compensable failures), computational burden and latency (FDI block requirements), and available information required. From this analysis, the superiority of adaptive control and the motivations of its use are presented. In the subsequent chapters, our main focus is to propose adaptive actuator failure compensation designs for different classes of nonlinear systems with actuation redundancy, we try also, to deal with different types of actuator failures. Some notes and tutorials on related work on linear and nonlinear adaptive actuator failure compensation control were also provided. These works with their limitations will make the departure point for the contributions that will be presented in the next chapters. The next chapter will investigate the design of actuator failure compensation controllers for a class of multi-input single-output nonlinear systems in the presence of bias and loss of effectiveness actuator failures

Chapter 2

2 Adaptive control of single-output nonlinear systems with actuator failures

Chapter Contents

2.1 Introduction

The design of adaptive actuator failure compensation controllers for uncertain nonlinear systems is a challenging task; in addition to inherent nonlinearities and uncertainties which are present in these systems, the occurrence of actuator failures leads to unpredictable dynamics and brings more uncertainties to the system [19]. An effective adaptive controller should be able to manage the existing actuation redundancy in an intelligent and flexible way so that any lack of control effort induced by one or more failed actuators will be immediately compensated by the remaining healthy actuators. Many related research works have investigated this problem from different perspectives, for example using indirect adaptive fuzzy control [73], [75], dynamic surface control [76], [102] and indirect neural control[103]. Within this context, the aim of this chapter is to develop direct adaptive actuator failure compensation controllers for a class of multi-input single-output (MISO) nonlinear control-affine systems with stable zero dynamics. The actuator failures addressed in this chapter are partial and total loss of effectiveness failure types. Two control designs are proposed in this regard, the first design assumes that the failure pattern is an unknown time-varying signal while the second design assumes that the failure pattern is in a completely parameterized form. The adaptive controllers are constructed around an online approximation of a feedback linearization controller that compensates both system uncertainties and uncertain dynamics induced by actuator failures. Therefore, the remaining of the chapter is organized as follows: In section 2.2, the actuator failure problem is formulated with basic assumptions. In section 2.3, a first design is presented for the case of non-parameterized failures with application to an aircraft angle of attack. In section 2.4, a design for parameterized failures is developed with application to wing rock motion control. Finally, section 2.5 gives a conclusion of the chapter.

2.2 Problem statement and background

2.2.1 System description and preliminaries

In this chapter, we focus on the class of multi-input single-output (MISO) nonlinear control affine systems described by the following equations [19]

$$\begin{cases} \dot{x} = f(x) + g^T(x)u \\ y = h(x) \end{cases} \tag{2.1}$$

where $x = [x_1, x_2, \ldots, x_n]^T$ is the state vector of the system assumed available for feedback, $u = [u_1, u_2, \ldots, u_m]^T \in \mathbb{R}^m$ is the input vector acting on the system, and whose actuators are somehow similar in physical construction, these actuators may fail during the course of operation, $y \in \mathbb{R}$ is the output. Besides, $f(x) \in \mathbb{R}^n$, $g(x) = [g_1(x), g_2(x), \ldots, g_m(x)]^T$, $g_i(x)$ and $h(x) \in \mathbb{R}$ are sufficiently smooth vector fields and functions respectively. Regarding the actuator redundancy requirements, the general following assumption is stated:

Assumption 2.1: The system described in (2.1) is constructed with some kind of actuation redundancy so that, in the presence of only one effective actuator which can be partially effective, the system can be driven to its desired behavior.

Remark 2.1: It is important to notice that redundancy is inherent to the system, in other words, it is part of the problem and not part of the controller design task. Our aim is thus to design an adaptive controller that manages this redundancy in an efficient way.

Remark 2.2: From assumption 2.1, in order for the system (2.1) to have some kind of actuation redundancy, the vector fields $g_i(x)$, $i = 1, \ldots, m$ must have a similar structure. This similarity can be expressed as the following condition from the distribution theory [19], [104], [105].

Assumption 2.2 [19]: Throughout this chapter, it is assumed that the vector fields $g_i(x)$, $i = 1, \ldots, m$ are linearly dependent. In terms of the distribution theory, this translates to the condition: $g_i(x) \in span(g_0(x))$, $i = 1, \ldots, m$, where $g_0(x) \in \mathbb{R}^n$ is the input gain vector field of a nominal system defined as follows

$$\begin{cases} \dot{x} = f(x) + g_0(x) u_0 \\ y = h(x) \end{cases} \tag{2.2}$$

where $u_0 \in \mathbb{R}$ denotes a nominal control input signal.

The nominal system (2.2) is assumed feedback linearizable with known relative degree $r \le n$. Therefore, it follows that there exists a diffeomorphism (coordinate transformation) $[\xi, \eta]^T = \phi(x) = [\phi_c(x), \phi_z(x)]^T$, with $\xi \in \mathbb{R}^r$ and $\eta \in \mathbb{R}^{n-r}$ that transforms the nominal system (2.2) to the following canonical form

$$\begin{cases} \dot{\xi} = A\xi + B\left(\varphi(\xi,\eta) + \beta_0(\xi,\eta)u_0\right) \\ \dot{\eta} = \psi(\xi,\eta) \\ y = C\xi \end{cases} \tag{2.3}$$

with $C = [1 \quad 0 \quad \cdots \quad 0] \in \mathbb{R}^r$, $\varphi(\xi,\eta) = L_f^r h(x)$, $\beta_0(\xi,\eta) = L_{g_0} L_f^{r-1} h(x) \neq 0$ and

$$A = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 0 & \cdots & 0 & 0 \end{bmatrix} \in \mathbb{R}^{r\times r},\; B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in \mathbb{R}^r,\; \phi_c(x) = \begin{bmatrix} h(x) \\ L_f h(x) \\ \vdots \\ L_f^{r-1} h(x) \end{bmatrix}, \phi_z(x) = \begin{bmatrix} T_{r+1}(x) \\ T_{r+2}(x) \\ \vdots \\ T_n(x) \end{bmatrix},$$

where $L_f h(x)$ represents the *Lie derivative* of the function $h(x)$ along the trajectories of $f(x)$ which is defined as: $L_f h(x) = \nabla h(x) \cdot f(x)$, here the symbol ∇ denotes the gradient operator. The transformation $\phi_z(x)$ is selected such that $\left(\partial\phi_z(x)/\partial x\right) g_0(x) = 0$. Besides, $\dot{\eta} = \psi(\xi,\eta)$ stands for the internal dynamics of the nominal system (2.2), and the autonomous equation $\dot{\eta} = \psi(0,\eta)$ is called the zero dynamics [106].

Remark 2.3: Notice that if the nominal system (2.2) has a full relative degree $n = r$, there are no internal dynamics. Notice also that the internal dynamics subsystem $\dot{\eta} = \psi(\xi,\eta)$ in (2.3) does not depend explicitly on the nominal input u_0 [106].

Now, by virtue of assumption 2.2, it follows that the diffeomorphism $\phi(x)$ can be also used to transform the redundant system (2.1) into the following canonical normal form

$$\begin{cases} \dot{\xi} = A\xi + B\left(\varphi(\xi,\eta) + \beta^T(\xi,\eta)u\right) \\ \dot{\eta} = \psi(\xi,\eta) \\ y = C\xi \end{cases} \tag{2.4}$$

with

$$\beta^T(\xi,\eta) = \left[\beta_1(\xi,\eta),\ldots,\beta_m(\xi,\eta)\right]^T = \left[L_{g_1} L_f^{r-1} h(x),\ldots,L_{g_m} L_f^{r-1} h(x)\right]^T \tag{2.5}$$

Remark 2.4: It can be seen that the initial system (2.4) and the nominal system (2.3) have the same internal dynamics $\dot{\eta} = \psi(\xi,\eta)$. It can be also seen that actuator failures do not alter the internal dynamics of the system directly and do not add as additional inputs.

2.2.2 Internal dynamics stability condition

Regarding the theory of feedback linearization control, if the nominal system (2.2) is not with a full relative degree, i.e. $r < n$, the state vector $\eta = [\eta_1, \eta_2, \ldots, \eta_{n-r}]^T$ representing the internal dynamics of the system will be unobservable from the output. Moreover, it is independent of the input. Therefore, to ensure closed-loop stability, the following assumption on the internal dynamics subsystem $\dot{\eta} = \psi(\xi, \eta)$ must be satisfied:

Assumption 2.3: Throughout this chapter, the internal dynamics subsystem defined by $\dot{\eta} = \psi(\xi, \eta)$ is assumed input to state stable (ISS) [19].

2.2.3 Actuator failures modeling

In this chapter, two types of actuator failures are considered; the first type is partial loss of effectiveness or actuator fading, in this case, the j^{th} actuator provides only a portion of the control effort. Mathematically this can be described as [75], [107]

$$u_j^f(t) = \rho_j u_j(t),\ t \geq t_j \tag{2.6}$$

where $0 < \underline{\rho}_j < \rho_j < \overline{\rho}_j \leq 1$ is the actuator effectiveness coefficient and t_j is the failure time. The index j stands for the failed actuator. Notice that all the failure parameters are supposed unknown. Examples of such failures are leakages in valves or wear and tear due to aging.

The second type of actuator failures considered in this chapter is total loss of effectiveness, in this case, at an unknown time t_j, the j^{th} actuator is no longer influenced by the issued control actions. Mathematically, this can be described as follows [73], [75], [102]

$$u_j^f(t) = \overline{u}_j(t),\ t \geq t_j \tag{2.7}$$

where $\overline{u}_j(t)$ is the failure value and t_j is the failure time. This type of actuator failures is common in practice, it can be induced by hydraulic circuit break or it can arise from a sudden power interruption. For the purpose of our study, it is further assumed that in some ideal situations, the failure model (2.7) can be expressed in a parameterized form as follows

$$\overline{u}_j(t) = \overline{u}_{j0} + \sum_{l=1}^{n_j} \overline{u}_{jl} \delta_{jl}(t) \tag{2.8}$$

for some unknown scalar constants $\bar{u}_{jl}$ and known scalar signals $\delta_{jl}(t)$, n_j is the number of $\delta_{jl}(t)$s in the j^{th} actuator failure pattern. A special case is when $\bar{u}_{jl} = 0,\ l = 1,\ldots,n_j$ which characterizes the lock in place failure types, i.e. at a time instant t_j, the j^{th} actuator is stuck at an unknown position $\bar{u}_{j0}$, a common situation is when aircraft surface segments (elevators, rudders, ailerons, etc.) are stuck at unknown positions during flight.

Remark 2.5: It should be emphasized that the failure value $\bar{u}_j(t)$ is unknown, the parameterized failure model in (2.8) represents a possible approximation (decomposition) of $\bar{u}_j(t)$ according to basis functions such as Fourier series, neural or wavelet networks, Laguerre polynomials, etc. In this decomposition, the functions $\delta_{jl}(t)$ are assumed known whereas the coefficients $\bar{u}_{jl}$ are unknown [77].

Now, in the presence of type (2.6) and/or type (2.7) actuator failures, the failed input vector issued to the system can be expressed as follows:

$$u^f(t) = \rho(t)(I - \sigma)u(t) + \sigma\bar{u}(t) \tag{2.9}$$

where $\bar{u}(t) = [\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_m]^T \in \mathbb{R}^m$, $\rho(t) = diag\{\rho_1, \rho_2, \ldots, \rho_m\}$ is the effectiveness matrix, $\sigma = diag\{\sigma_1, \sigma_2, \ldots, \sigma_m\}$ with $\sigma_j = 1$ if the j^{th} actuator fails as (2.7) and $\sigma_j = 0$ otherwise.

The control objective is to design a control vector $u(t)$ so that the output of the faulty system (2.11) tracks as closely as possible the desired reference output $y_d(t)$ with bounded derivatives up to order *r* while keeping all closed-loop signals within certain bounds.

Provided that the actuators are similar in construction, an equal or proportional actuation scheme can be considered [19], [75], [76]. For the present work, a proportional actuation scheme is considered. Thus, the control vector entries are designed as follows

$$u_j(t) = b_j(t)u_0(t),\ j = 1,\ldots,m \tag{2.10}$$

where $b_j(t)$ are proportional allocation coefficients that describe the contribution of each actuator and $u_0(t)$ is a common control signal to be designed.

In the next two sections, two different control designs are proposed according to the assumptions on the failure patterns (non-parameterized and fully parameterized patterns).

2.3 Design for non-parameterized actuator failures

In this section, an adaptive controller is developed for the system (2.4) in the presence of type (2.6) and/or non-parameterized type (2.7) actuator failures. First, the ideal controller structure is provided, then an adaptive version of this controller with an adaptive law is developed, stability and tracking analyses are carried out and a simulation study is carried out.

2.3.1 Ideal controller structure

Recall that the internal dynamics $\dot{\eta} = \psi(\xi,\eta)$ in (2.4) are assumed ISS stable and they are not affected by the failures. Moreover, they are not observable from the output. Therefore, the system (2.4) with faulty input $u^f(t)$ as defined in (2.9) can be written into the following form

$$y^{(r)} = \varphi(\xi,\eta) + \beta^T(\xi,\eta)\big(\rho(t)(I-\sigma)u(t) + \sigma\bar{u}(t)\big) \tag{2.11}$$

Our task is to design an adaptive actuator failure compensation scheme that generates the control signals to compensate actuator failure and ensure the control objectives.

Using the proportional actuation scheme (2.10), equation (2.11) can be written as follows

$$y^{(r)} = \varphi(\xi,\eta) + \sum_{j\notin\Omega_F} \beta_j(\xi,\eta)\rho_j b_j(t) u_0 + \sum_{j\in\Omega_F} \beta_j(\xi,\eta)\bar{u}_j \tag{2.12}$$

where Ω_F represents the set of indices referring to actuators that have failed according to the pattern (2.7) at any given time.

Now, let us define the following two functions

$$\bar{\varphi}(\xi,\eta) = \varphi(\xi,\eta) + \sum_{j\in\Omega_F} \beta_j(\xi,\eta)\bar{u}_j \tag{2.13}$$

$$\bar{\beta}(\xi,\eta) = \sum_{j\notin\Omega_F} \beta_j(\xi,\eta)\rho_j b_j(t) \tag{2.14}$$

Then, the dynamic equation (2.12) can be written in the following compact form

$$y^{(r)} = \bar{\varphi}(\xi,\eta) + \bar{\beta}(\xi,\eta)u_0 \tag{2.15}$$

It can be seen from (2.15) that the actuation scheme (2.10) simplifies the controller design task, and brings the problem of designing *m* controllers into that of a single controller u_0. Regarding the adaptive control of nonlinear systems, the following assumption is imposed

Assumption 2.4: It is assumed that for all compensable actuator failure patterns $sign\left(\bar{\beta}\left(\xi,\eta\right)\right)$ does not change and is known. Without loss of generality, it is assumed that $sign\left(b_j\left(t\right)\right)$ is the same as $sign\left(\beta_j\left(\xi,\eta\right)\right)$ so that the quantity $\sum_{j\notin\Omega_F}\beta_j\left(\xi,\eta\right)\rho_j b_j\left(t\right)$ remains strictly positive for every failure pattern in the first case. More particularly, it is assumed that for all possible actuator failures, the following inequality holds:

$$\exists\bar{\beta}_{\min}>0,\ \bar{\beta}_{\max}>0\,|\,\forall x\in\mathbb{R}^n:\ \bar{\beta}_{\min}\le\beta\left(\xi,\eta\right)\le\bar{\beta}_{\max} \tag{2.16}$$

Now, define the tracking error between the desired output and the actual output as follows

$$e\left(t\right)=y_d\left(t\right)-y\left(t\right) \tag{2.17}$$

The corresponding filtered tracking error $s\left(t\right)$ is given by

$$s\left(t\right)=\left(\frac{d}{dt}+\lambda\right)^{r-1}e\left(t\right),\lambda>0 \tag{2.18}$$

From (2.18), $s\left(t\right)=0$ implies that $e^{(k)}\left(t\right),\ k=0,1,\ldots,r-1$ converge to zero [108]. Moreover, if $\left|s\left(t\right)\right|\le\Phi$ with $\Phi>0$, it can be concluded that: $\left|e^{(j)}\left(t\right)\right|\le 2^j\lambda^{j-r-1},\ k=0,1,\ldots,r-1$. These bounds can be reduced by increasing the design parameter λ [108].

The time derivative of $s\left(t\right)$ can be simplified as follows

$$\dot{s}=v-\bar{\varphi}\left(\xi,\eta\right)-\bar{\beta}\left(\xi,\eta\right)u_0 \tag{2.19}$$

with $v=y_d^{(r)}+\sum_{i=1}^{r-1}k_i e^{(i)}$ and $k_i=C_{r-1}^{i-1}\lambda^{r-i}$.

In the case where the functions $\varphi\left(\xi,\eta\right)$, $\beta\left(\xi,\eta\right)$ and the actuator failures are known, it follows that the functions $\bar{\varphi}\left(\xi,\eta\right)$ and $\bar{\beta}\left(\xi,\eta\right)$ will be also known. The control objectives can then be met through the following ideal control law [74], [109]

$$u_0^*=\frac{1}{\bar{\beta}\left(\xi,\eta\right)}\left(-\bar{\varphi}\left(\xi,\eta\right)+v+ks+k_0\tanh\left(\frac{s}{\varepsilon_0}\right)\right) \tag{2.20}$$

where $k,k_0>0$, ε_0 is a small positive constant, besides $\tanh\left(\bullet\right)$ stands for the hyperbolic tangent function which is introduced in the control law (2.20) as a smooth approximation of the sign function for controller robustness.

In fact, with the ideal control law (2.20), the time derivative of $s(t)$ can be written as follows

$$\dot{s} = -ks + k_0 \tanh\left(\frac{s}{\varepsilon_0}\right) \tag{2.21}$$

From (2.21) it follows that $s(t)$ converges asymptotically to zero and consequently $e(t)$ and its derivatives up to $r-1$ are bounded and converge to zero as time tends to infinity. However, since $\bar{\alpha}(\xi,\eta)$ and $\bar{\beta}(\xi,\eta)$ are assumed unknown, the ideal control law (2.20) cannot be implemented. In what follows, this ideal controller will be adaptively constructed online.

2.3.2 Adaptive controller design

In this subsection, we assume that the ideal controller (2.20) can be approximated as [109]

$$u_0 = \Pi^T(z)\theta(t) \tag{2.22}$$

where $\Pi(z)$ is a regressor vector, z is the input to the regressor and $\theta(t)$ is a vector of adjustable parameters. Assume also that there exists a piecewise continuous time-varying parameter vector $\theta^*(t)$ with possible jumps when one or more abrupt actuator failures occur with bounded time derivative inside the interval of continuity, such that

$$u_0^* = \Pi^T(z)\theta^*(t) + \varepsilon(z) \tag{2.23}$$

with $\varepsilon(z)$ is the approximation error which is assumed bounded as $|\varepsilon(z)| < \bar{\varepsilon}$.

Remark 2.6: The regressor vector $\Pi(z)$ can be constructed in different ways, it can be based on universal function approximators such fuzzy logic systems (FLS), neural networks (NN), wavelet networks (WN), or a simple combination of system states, errors, their integrals, and derivatives. Through this thesis, we will be examining different types of regressors to approximate some ideal controllers.

By using (2.19), (2.20), (2.22), and (2.23), $\dot{s}(t)$ can be written in the following form

$$\dot{s} = -ks - k_0 \tanh\left(\frac{s}{\varepsilon_0}\right) + \bar{\beta}(\xi,\eta)e_{u_0} \tag{2.24}$$

with $e_{u_0} = u_0^* - u_0 = \Pi^T(z)\tilde{\theta} + \varepsilon(z)$, and $\tilde{\theta} = \theta^* - \theta$.

Consider now the following quadratic cost function of the error e_{u_0} defined as follows [109]

$$J(\theta)=\frac{1}{2}\bar{\beta}(\xi,\eta)e_{u_0}^2=\frac{1}{2}\bar{\beta}(\xi,\eta)\left(u_0^*-\Pi^T(z)\theta\right)^2 \tag{2.25}$$

Based on the gradient descent method, the parameter update law for θ that minimizes the cost function $J(\theta)$ is designed as follows

$$\dot{\theta}=-\gamma\nabla_\theta J(\theta),\ \gamma>0 \tag{2.26}$$

Provided that $\nabla_\theta J(\theta)=-\bar{\beta}(\xi,\eta)\Pi(z)e_{u_0}$, (2.26) becomes as follows

$$\dot{\theta}=\gamma\bar{\beta}(\xi,\eta)\Pi(z)e_{u_0} \tag{2.27}$$

In the parameter update law (2.27), the quantity $\bar{\beta}(\xi,\eta)e_{u_0}$ is not at our disposal, however, from (2.24) this term can be pulled out as:

$$\bar{\beta}(\xi,\eta)e_{u_0}=\dot{s}+ks+k_0\tanh\left(\frac{s}{\varepsilon_0}\right) \tag{2.28}$$

Then, by substituting (2.28) into the parameter update law (2.27) and introducing a $\sigma-\text{modification}$ to ensure robustness we obtain the following parameter update law

$$\dot{\theta}=\gamma\Pi(z)\left\{\dot{s}+ks+k_0\tanh\left(\frac{s}{\varepsilon_0}\right)\right\}-\gamma\sigma\theta \tag{2.29}$$

where σ is a small positive constant. Notice that the $\sigma-\text{modification}$ term is introduced to ensure boundedness of the parameter $\tilde{\theta}$ in the presence of the approximation error $\varepsilon(z)$.

2.3.3 Stability and tracking analysis

Suppose that one or more actuators fail at time instants $t_i,\ i=1,2,\ldots,N$, and at the time $t\in[t_i,t_{i+1})$ there are $p_1(0\le p_1\le m)$ actuators that have failed as (2.6) and $p_2(0\le p_2\le m-1)$ actuators that have failed as (2.7). For stability and tracking analysis, we introduce a first Lyapunov candidate function over the interval $[t_i,t_{i+1})$ defined as follows

$$V_\theta^i=\frac{1}{2\gamma}\tilde{\theta}^T\tilde{\theta} \tag{2.30}$$

The time derivative of V_θ^i over the time interval $t \in [t_i, t_{i+1})$ is given by

$$\dot{V}_\theta^i = \frac{1}{\gamma}\tilde{\theta}^T \dot{\theta}^* - \frac{1}{\gamma}\tilde{\theta}^T \dot{\theta} \tag{2.31}$$

By virtue of (2.29), equation (2.31) can be further simplified as follows

$$\dot{V}_\theta^i = \frac{1}{\gamma}\tilde{\theta}^T \dot{\theta}^* - \overline{\beta}(\xi,\eta) e_{u_0}^2 + \overline{\beta}(\xi,\eta)\varepsilon(z) e_{u_0} + \sigma \tilde{\theta}^T \tilde{\theta} \tag{2.32}$$

Let us consider the following inequalities

$$\sigma\tilde{\theta}^T \theta \le -\frac{\sigma}{2}\left\|\tilde{\theta}\right\|^2 + \frac{\sigma}{2}\left\|\theta^*\right\|^2,\ e_{u_0}\varepsilon(z) \le e_{u_0}^2 + \frac{1}{4}\varepsilon^2(z), \frac{1}{\gamma}\tilde{\theta}^T \dot{\theta}^* \le \frac{\sigma}{4}\left\|\tilde{\theta}\right\|^2 + \frac{1}{\sigma\gamma^2}\left\|\dot{\theta}^*\right\|^2 \tag{2.33}$$

Using (2.33), equation (2.32) can be bounded as follows

$$\dot{V}_\theta^i \le -\frac{\sigma}{4}\left\|\tilde{\theta}\right\|^2 + \frac{1}{\sigma\gamma^2}\left\|\dot{\theta}^*\right\|^2 + \frac{\sigma}{2}\left\|\theta^*\right\|^2 + \frac{1}{4}\overline{\beta}(\xi,\eta)\varepsilon^2(z) \tag{2.34}$$

Since $\overline{\beta}(\xi,\eta)$ is supposed bounded from below and above, and since $\varepsilon(z)$, θ^* and $\dot{\theta}^*$ are assumed bounded over an interval between successive abrupt failures, i.e. over the time interval $[t_i, t_{i+1})$, a positive bound ψ_i can be defined over this interval as follows

$$\psi_i = \sup_{t\in[t_i, t_{i+1})}\left(\frac{1}{\sigma\gamma^2}\left\|\dot{\theta}^*\right\|^2 + \frac{\sigma}{2}\left\|\theta^*\right\|^2 + \frac{1}{4}\overline{\beta}_{\max}\overline{\varepsilon}^2\right) \tag{2.35}$$

Then, (2.34) can be rewritten as follows

$$\dot{V}_\theta^i \le -\alpha_\theta V_\theta^i + \psi_i \tag{2.36}$$

with $\alpha_\theta = \sigma\gamma/2$. We can now prove the following result on closed loop stability

Theorem 2.1: Consider the system (2.1) subject to possible actuator failures of type (2.6) and (2.7), if the prescribed assumptions hold for all actuator failures, then the actuation scheme (2.10) and the adaptive controller (2.22) with the parameter update law (2.29) guarantees the following properties: (i) The parameter error vector is bounded and converges to the residual set $\Omega_\theta = \left\{\tilde{\theta} : \left\|\tilde{\theta}\right\|^2 \le 2\eta\psi_{N+1}/\alpha_\theta\right\}$ after the time t_N where no further actuator failures occur. (ii) The control signal error e_{u_0} is bounded and the tracking error converges to a small residual set.

Proof:

First, recall that two patterns of actuator failures should be distinguished. The first is when the actuator failures are incipient; in this case, there is no parameter jump. That stability and tracking proofs are relatively easy and conducted in the same way as the failure-free case [18], [110]. The second case is when there are abrupt failures, this may result in parameter jumps, and thus the classical stability proofs are not sufficient. Careful attention should be made to those jumps. In the following we provide the proof for the case of abrupt actuator failures, the incipient case can be considered as a special case by disregarding the jumps.

Equation (2.36) implies that, over the time interval $\left[t_i, t_{i+1}\right)$, where the failure pattern in fixed, given that $V_\theta^i \geq \psi_i / \alpha_\theta$, implies that $\dot{V}_\theta^i < 0$. Which means that V_θ^i and $\tilde{\theta}$ are bounded over that interval. In addition, since $\varepsilon(z)$ and $\Pi(z)$ are bounded, it follows that e_{u_0} is bounded given that the jumps are finite. Integrating (2.36) over the time interval $t \in \left[t_i, t_{i+1}\right)$ yields

$$\left\|\tilde{\theta}(t)\right\|^2 \leq \left\|\tilde{\theta}(t_i)\right\|^2 e^{-\alpha_\theta (t-t_i)} + \frac{2\gamma\psi_i}{\alpha_\theta} \tag{2.37}$$

Let us denote V_θ the extension of $V_\theta^i\ s$ over the whole time domain. Due to abrupt actuator failures, V_θ or equivalently $\tilde{\theta}(t)$, exhibits finite jumps at each time instant $t_i,\ i = 1, \ldots, N$, with $t_{N+1} = \infty$, i.e. there are no failures after t_N [19], let us denote that jump by $\Delta\left\|\tilde{\theta}(t_i)\right\|^2$, i.e. $\left\|\tilde{\theta}(t_i^+)\right\|^2 = \left\|\tilde{\theta}(t_i^-)\right\|^2 + \Delta\left\|\tilde{\theta}(t_i)\right\|^2$, where t_i^- and t_i^+ are the time instants just before and after the occurrence of the failure at the time t_i, respectively. Starting from t_1, one can write

$$\left\|\tilde{\theta}(t_1^+)\right\|^2 = \left\|\tilde{\theta}(t_1^-)\right\|^2 + \Delta\left\|\tilde{\theta}(t_1)\right\|^2 \tag{2.38}$$

where $\Delta\left\|\tilde{\theta}(t_1)\right\|$ is the jump on $\left\|\tilde{\theta}(t_1)\right\|$ caused by abrupt failures. From (2.37), one can write

$$\left\|\tilde{\theta}(t_1^-)\right\|^2 \leq \left\|\tilde{\theta}(t_0)\right\|^2 e^{-\alpha_\theta (t_1-t_0)} + \frac{2\gamma\psi_1}{\alpha_\theta} \leq \left\|\tilde{\theta}(t_0)\right\|^2 + \frac{2\gamma\psi_1}{\alpha_\theta} \tag{2.39}$$

or

$$\left\|\theta(t_1^+)\right\|^2 = \left\|\theta(t_1^-)\right\|^2 + \Delta\left\|\theta(t_1)\right\|^2 \leq \left\|\tilde{\theta}(t_0)\right\|^2 + \frac{2\gamma\psi_1}{\alpha_\theta} + \Delta\left\|\theta(t_1)\right\|^2 \tag{2.40}$$

Likewise, we proceed the same way for $t_2, \ldots, t_N$, at t_N we end up with the following inequality

$$\left\|\theta\left(t_N^+\right)\right\|^2 \le \left\|\tilde{\theta}\left(t_0\right)\right\|^2 + \frac{2\gamma}{\alpha_\theta}\sum_{i=1}^{N}\psi_i + \sum_{k=1}^{N-1}\left\|\Delta\theta\left(t_i\right)\right\|^2 \tag{2.41}$$

After t_N, there are no actuator failures occurring, integrating (2.36) over $t \in [t_N, \infty)$, we get

$$\left\|\tilde{\theta}(t)\right\|^2 \le \left\|\tilde{\theta}\left(t_N^+\right)\right\|^2 e^{-\alpha_\theta(t-t_N)} + \frac{2\gamma\psi_{N+1}}{\alpha_\theta} \tag{2.42}$$

This implies that $\left\|\tilde{\theta}(t)\right\|^2 \in L^\infty$ for $t \in [t_N, \infty)$, besides it converges to a residual set defined as

$$\Omega_\theta = \left\{\tilde{\theta} : \left\|\tilde{\theta}\right\|^2 \le \frac{2\gamma\psi_{N+1}}{\alpha_\theta}\right\} \tag{2.43}$$

Now, for the output tracking error convergence proof, we consider a second Lyapunov function candidate function defined as follows

$$V_s = \frac{1}{2}s^2 \tag{2.44}$$

The time derivative of V_s along system trajectories can be simplified as follows

$$\dot{V}_s = -ks^2 - k_0 s\tanh\left(\frac{s}{\varepsilon_0}\right) + s\bar{\beta}(\xi,\eta)\left(\Pi^T(z)\tilde{\theta} + \varepsilon(z)\right) \tag{2.45}$$

From (2.42), we can write

$$\left\|\tilde{\theta}(t)\right\| \le \left\|\tilde{\theta}(t_0)\right\| e^{-0.5\alpha_\theta(t-t_N)} + \chi \tag{2.46}$$

where $\chi = \sqrt{2\gamma\psi_{N+1}/\alpha_\theta}$.

Given that $\bar{\beta}(\xi,\eta)$, $\Pi(z)$ and $\tilde{\theta}$ are bounded, from (2.46) we can establish that after the time t_N, the following inequality holds

$$\left|\bar{\beta}(\xi,\eta)\Pi^T(z)\tilde{\theta} + \varepsilon(z)\right| \le \phi_1 e^{-0.5\alpha_\theta(t-t_N)} + \phi_2 \tag{2.47}$$

where ϕ_1 and ϕ_2 are some finite positive constants. Using (2.47), (2.45) can be bounded as

$$\dot{V}_s \le -ks^2 - k_0 s\tanh\left(\frac{s}{\varepsilon_0}\right) + |s|\left(\phi_1 e^{-0.5\alpha_\theta(t-t_N)} + \phi_2\right) \tag{2.48}$$

If k_0 is chosen such that $k_0 \geq \phi_2$, by virtue of the following inequality [111]: $0 \leq |s| - s\tanh(s/\varepsilon_0) \leq \kappa\varepsilon_0;\ \kappa = 0.2785$, and proceeding as in [109], given that $k \geq 1/4$, we end up with the following inequality

$$\dot{V}_s \leq -\alpha_s V_s + \phi_1^2 e^{-0.5\alpha_\theta(t-t_N)} + \kappa\phi_2\varepsilon_0 \tag{2.49}$$

From which we can prove the following theorem on the convergence of the tracking errors

Theorem 2.2: For the system (2.1) subject to type (2.6) and/or type (2.7) actuator failures, with the prescribed assumptions and actuation scheme (2.10), the controller (2.22) with adaptation law (2.29) satisfies the boundedness of the state *x* and the signal u_0. Besides, the tracking errors satisfy: $e^{(j)}(t) \leq 2^j \lambda^{j-r+1}\Phi,\ j = 1,\ldots,r-1$, with $\Phi = \sqrt{2\phi_2\kappa\varepsilon_0/\alpha_s}$.

Proof:

From (2.49), it follows that for $V_s \geq \left(\phi_1^2 e^{-0.5\alpha t} + \kappa\phi_2\varepsilon_0\right)/\alpha_s$, we have $\dot{V}_s < 0$, therefore $s(t)$ is bounded. Given the boundedness of $y_d(t)$ and its derivatives, it follows that $x \in L^\infty$. Moreover, since the term $\phi_1^2 e^{-\alpha_\theta t}$ decays to zero as $t \to \infty$, V_s will finish by converging to a bound defined as: $V_s < \phi_2\kappa\varepsilon_0/\alpha_s$, and therefore $s(t)$ converges asymptotically to the residual set $\Omega_s = \left\{ s \mid |s| \leq \sqrt{2\phi_2\kappa\varepsilon_0/\alpha_s} \right\}$. This implies that $e(t)$ and its time derivatives are asymptotically bounded as [108]: $e^{(j)}(t) \leq 2^j \lambda^{j-r+1}\Phi,\ j = 1,\ldots,r-1$, with $\Phi = \sqrt{2\phi_2\kappa\varepsilon_0/\alpha_s}$. This bound on the tracking error can be reduced by a proper choice of the design parameters k, λ and ε_0. On the other hand, from Theorem 2.1, we have $\tilde{\theta}$ and consequently θ are bounded, with the boundedness of $\Pi(z)$ it follows that u_0 is bounded.

2.3.4 Simulation study: Application to hypersonic aircraft angle of attack model

In this section, the proposed adaptive controller is applied to the angle of attack of a hypersonic aircraft model [112]. The equations for velocity, flight path angle, altitude, the angle of attack and pitch rate for the longitudinal dynamics of the aircraft can be split into two time scales namely the fast dynamics and the slow dynamics. Therefore, the controller design can be designed according to two time scales. For our study, we focus the fast dynamics controller design, the fast dynamics time scale includes the angle of attack α and the pitch rate q. When the fast dynamics time scale is considered, all variables corresponding to slow dynamics are assumed to be constant. Therefore the flight path angle dynamics $\dot{\gamma}$ can be set to zero. Assuming that all other slow dynamics variables are at their trim condition, assume also that the control effort is provided by two elevator segments, so that $u_1 = \delta_{e1}$ and $u_2 = \delta_{e2}$. Then the fast dynamics are described by the following dynamical model [112]:

$$\begin{aligned} &\dot{\alpha} = q \\ &\dot{q} = 113.5891\Big[5.33\times 10^{-6}\left(-6565\alpha^2 + 6875\alpha - 1\right) + 0.0026q \\ &\quad \times\left(-6.83\alpha^2 + 0.303\alpha - 0.23\right) + 0.0292\left(b_1\delta_{e1} + b_2\delta_{e2} - \alpha\right)\Big] \end{aligned} \tag{2.50}$$

It can be seen that under the prescribed flight assumptions see [112], the angle of attack model is already in the canonical form, without zero dynamics, therefore there is no need to check their stability. A proportional actuation scheme with $b_1(t) = 1$, $b_2(t) = 2$ is chosen. The controller parameters are chosen as $k = 5,\ k_0 = 2,\ \lambda = 2,\ \gamma = 5$ and $\varepsilon_0 = 0.01$. In this first design, the regressor is constructed based on a fuzzy logic system (FLS) whose input is $z = [\alpha, q]^T$, the FLS is composed of five membership functions (MFs) for both the angle of attack α and the pitch rate q defined as follows: $\mu_1(z_i) = \exp\left(-(z_i + 1.25)^2\right)$, $\mu_2(z_i) = \exp\left(-(z_i + 0.625)^2\right)$, $\mu_3(z_i) = \exp\left(-z_i^2\right)$, $\mu_4(z_i) = \exp\left(-(z_i - 0.625)^2\right)$, $\mu_5(z_i) = \exp\left(-(z_i + 1.25)^2\right)$, for i=1,2. The initial values of the parameters are chosen as $\theta_i(0) = 0.01,\ i = 1,\ldots,25$, The initial angle of attack and the pitch rate are $\alpha(0) = 0.03$ rad and $q(0) = 0$ rad/s. The desired angle of attack is $\alpha_d = \pi/36\sin(0.2t)$ rad. The simulation is carried out for 120 s and the following two failure scenarios are considered:

Scenario 1: At the time $t = 40$ s, the second elevator undergoes an abrupt failure according to the pattern defined as follows: $u_2^f = 0.1 + 0.05\cos(0.5t) - 0.08\sin(0.5t)$, at the time $t = 80$ s, the first elevator fails according to the following pattern: $u_1^f = 0.75u_1 - 0.2$. The simulation results are shown in Figure 2.1 for the state variables tracking and in Figure 2.2 for the control signals (elevator angles deflections), it can be seen that the proposed controller indeed ensures the control objectives despite the presence of actuator failures, at times $t = 40$ s and $t = 80$ s corresponding to the appearance of elevator failures, the tracking performance is reestablished after a transient and the control effort is adaptively redistributed among healthy actuators.

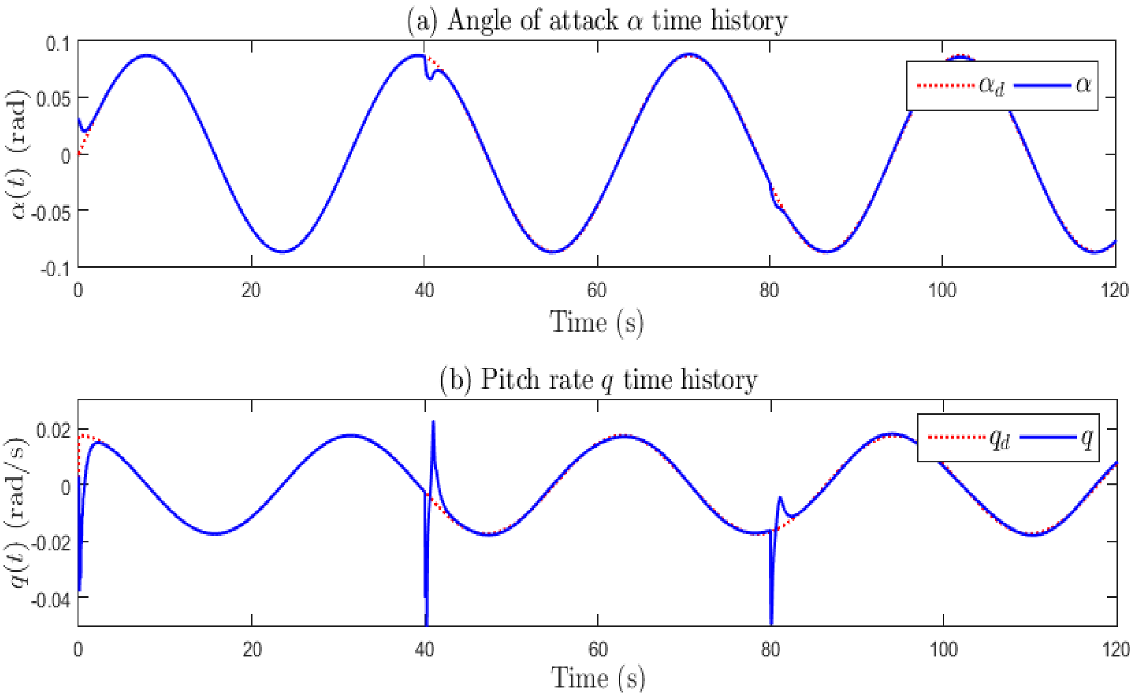

Figure 2.1: Angle of attack and pitch rate evolution (scenario 1)

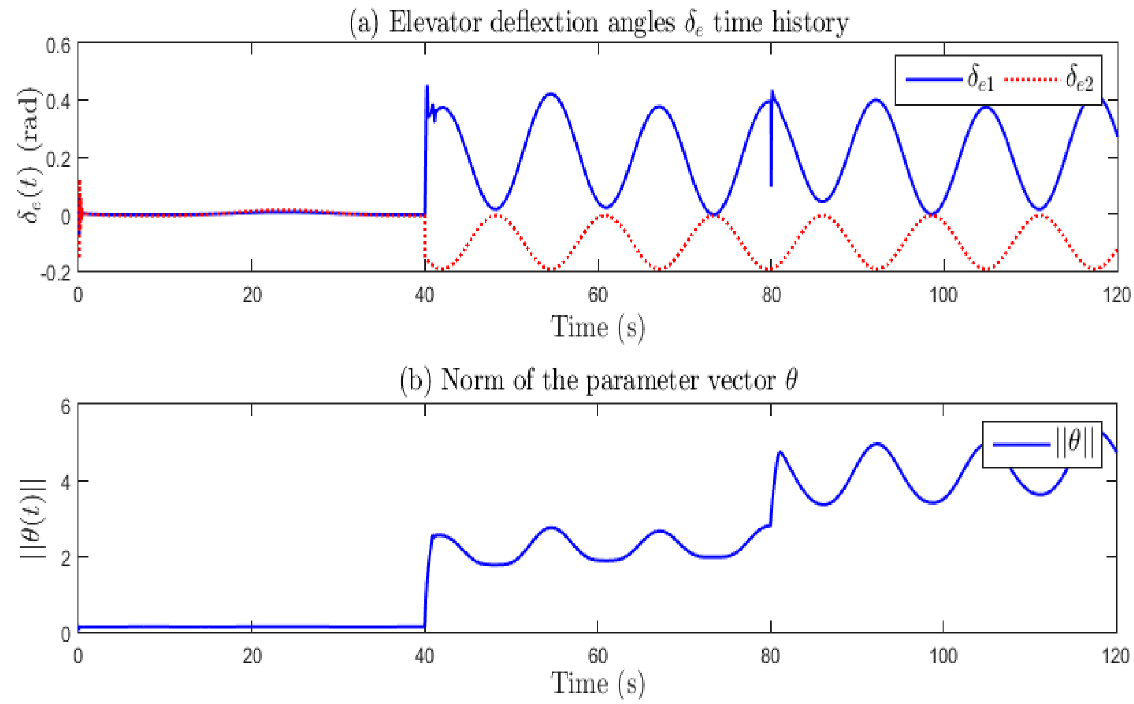

Figure 2.2: Control signals and parameters evolution (scenario 1)

Scenario 2: Consider a more intricate failure scenario, assume that at the time $t = 40\ s$, the second elevator fails according to the pattern $u_2^f = 0.2x_1 + 0.05u_2 - 0.03\sin(0.5t)$, then it recovers from this failure at the time $t = 100\ s$, for the first elevator, we assume that at the time $t = 70\ s$ it fails according to the pattern $u_1^f = 0.4u_1 + 0.5x_1x_2$. The simulation results are shown in Figure 2.3 and Figure 2.4, it can be seen that even with state dependent failures, the controller was able to ensure the control objectives (tracking and stability) in the presence of both types of actuator failures. Besides, it can be witnessed from the results that when a failed actuator recovers from a failure, the control effort is redistributed and this actuator is exploited again.

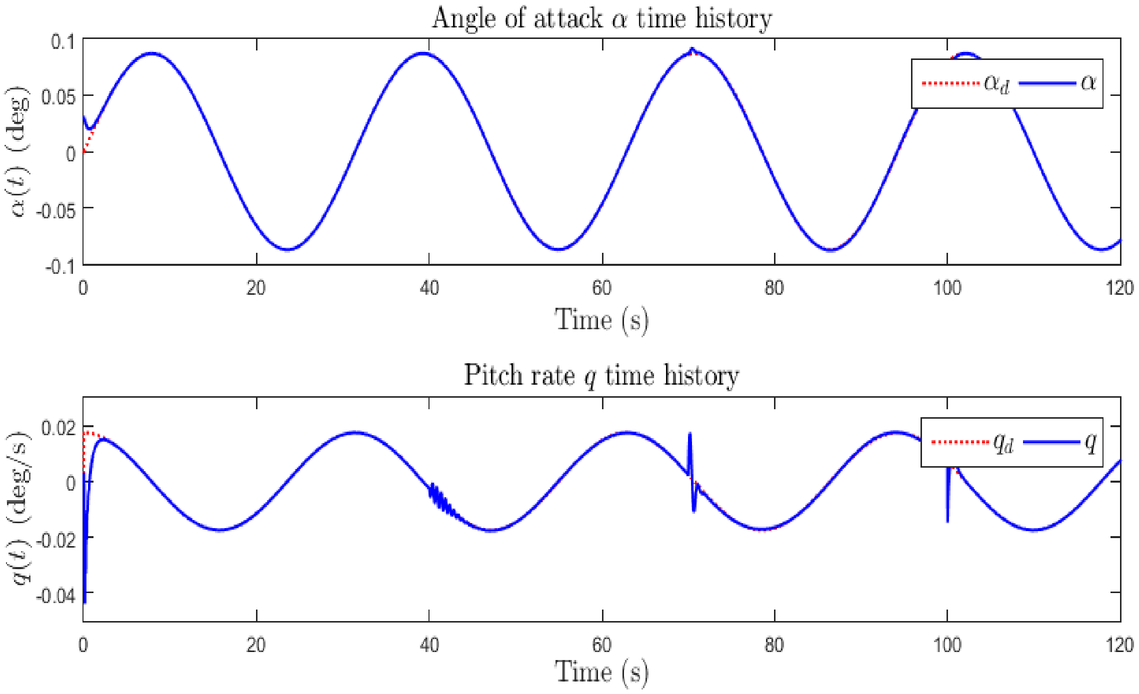

Figure 2.3: Angle of attack and pitch rate evolution (scenario 2)

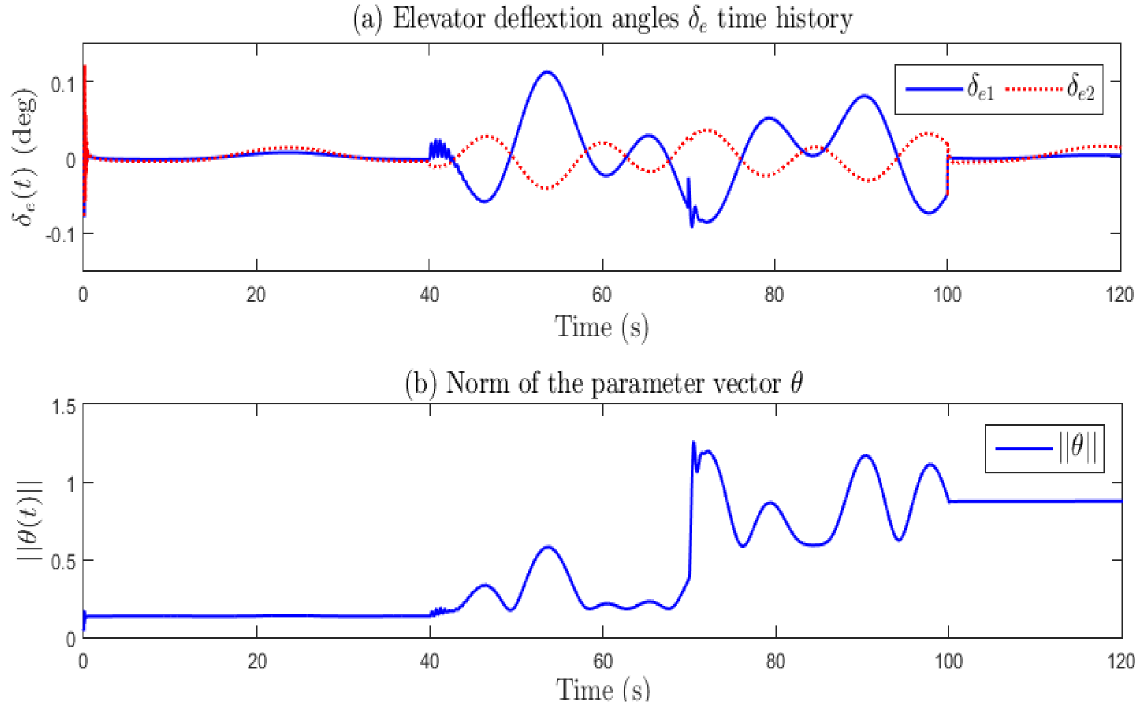

Figure 2.4: Control signals and parameters evolution (scenario 2)

2.4 Design with parameterized actuator failures

In this section, the failure pattern $\bar{u}_j(t)$ is assumed completely parameterized. The proposed controller consists of a plant controller and a failure estimation and compensation term.

2.4.1 Ideal controller structure

To start, let us rewrite the parameterized actuator failure signals (2.8) in the following form

$$\bar{u}_j(t) = \vartheta_j^T \varpi_j(t) \tag{2.51}$$

where $\vartheta_j = \left[\bar{u}_{j0}, \bar{u}_{j1}, \ldots, \bar{u}_{jn_j}\right]^T \in \mathbb{R}^{n_j+1}$ represents an unknown parameter vector and $\varpi_j(t) = \left[1, \delta_{j1}(t), \ldots, \delta_{jn_j}(t)\right]^T \in \mathbb{R}^{n_j+1}$ is a known basis function vector.

Using the parameterized failure (2.51) and the actuation scheme (2.10), the faulty system (2.11) can be written as follows

$$y^{(n)} = \varphi(\xi,\eta) + \beta^T(\xi,\eta)\rho(I-\sigma)b(t)u_0 + \sum_{j=1}^{m} \beta_j(\xi,\eta)\sigma_j \vartheta_j^T \varpi_j(t) \tag{2.52}$$

Let us denote $\bar{\beta}(\xi,\eta) = \beta^T(\xi,\eta)\rho(I-\sigma)b(t)$ and $\kappa_j = \beta_j(\xi,\eta)\sigma_j\vartheta_j / \bar{\beta}(\xi,\eta)$ which is a time varying parameter. Then (2.52) simplifies to the following equation

$$y^{(n)} = \varphi(\xi,\eta) + \bar{\beta}(\xi,\eta)u_0 + \bar{\beta}(\xi,\eta)\sum_{j=1}^{m} \kappa_j^T \varpi_j(t) \tag{2.53}$$

Equation (2.53) can be thought of as being a simple SISO nonlinear system with a disturbance like term for which a scalar control signal $u_0(t)$ is sought to force the output $y(t)$ to track the desired reference signal $y_d(t)$ and ensure closed-loop system stability. Now, by virtue of (2.53), the time derivative of $s(t)$ can be written as follows

$$\dot{s} = v - \varphi(\xi,\eta) - \bar{\beta}(\xi,\eta)u_0 - \bar{\beta}(\xi,\eta)\sum_{j=1}^{m} \kappa_j^T \varpi_j(t) \tag{2.54}$$

In the case where the system parameters and actuator failures are known, $\varphi(\xi,\eta)$, $\bar{\beta}(\xi,\eta)$ and κ_j will be also known, let us denote κ_j^* the actual value of κ_j. Therefore, the control objectives can be met through the following ideal control law [101], [109]:

$$u_0^* = \frac{1}{\bar{\beta}(\xi,\eta)}\left(-\varphi(\xi,\eta)+v+ks+k_0\tanh\left(\frac{s}{\varepsilon_0}\right)\right)-\sum_{j=1}^{m}\kappa_j^{*T}\varpi_j(t) \tag{2.55}$$

where $k,k_0>0$ and $k,k_0>0$ is a small positive constant. In fact, with the ideal controller (2.55), the time derivative of $s(t)$ along system trajectories becomes

$$\dot{s} = -ks - k_0\tanh\left(\frac{s}{\varepsilon_0}\right) \tag{2.56}$$

From (22), it can be easily checked that $s(t)$ and consequently $e(t)$ and its time derivatives up to order $r-1$ are all bounded and converge to zero as time tends to infinity.

The ideal control law (2.55) can be further split into two control terms as $u_0^* = u_p^* + u_f^*$, where u_p^* and u_f^* are defined as follows

$$u_p^* = \frac{1}{\bar{\beta}(\xi,\eta)}\left(-\varphi(\xi,\eta)+v+ks+k_0\tanh\left(\frac{s}{\varepsilon_0}\right)\right) \tag{2.57}$$

$$u_f^* = \sum_{j=1}^{m}u_{f,j} = -\sum_{j=1}^{m}\kappa_j^{*\mathrm{T}}\varpi_j(t) \tag{2.58}$$

The term control u_p accounts for the effective actuators (partially or completely effective) while the term u_f compensates for actuators that have totally lost their effectiveness.

Since $\varphi(\xi,\eta)$, $\bar{\beta}(\xi,\eta)$ and κ_j^*, $j=1,2,\ldots,m$ are unknown. The control law (2.55) cannot be implemented. In what follows, an adaptive version of the controller (2.55) is proposed.

2.4.2 Adaptive controller design

In this section, an adaptive version of the controller (2.55) is developed. For this, the term u_p^* is approximated by an adaptive controller while the failure terms κ_j^* are estimated online. To start, assume that u_p^* can be approximated as in (2.22), which is rewritten for convenience

$$u_p = \Pi^T(z)\theta(t) \tag{2.59}$$

again, $\Pi(z)$ is a regressor vector, and $\theta(t)$ is a vector of adjustable parameters.

If we denote $\theta^*(t)$ the optimal value of $\theta(t)$, one can write

$$u_0^* = \Pi^T(z)\theta^*(t) - \sum_{j=1}^{m} \kappa_j^{*T} \varpi_j(t) + \varepsilon(z) \tag{2.60}$$

where $\varepsilon(z)$ is the optimal approximation error for the plant controller u_p. For further technical developments, it is assumed that $\varepsilon(z)$ satisfies the following inequality

$$\varepsilon^2(z) \le \bar{\varepsilon}_0 e_{u_0}^2 + \bar{\varepsilon}_1 \tag{2.61}$$

where $\bar{\varepsilon}_0$ and $\bar{\varepsilon}_1$ are positive constants.

Remark 2.6: The approximation error $\varepsilon(z)$ can be considered as the sum of the approximation errors for both u_p and u_f if we assume that there is a parameterization error for the failure signal, i.e. $\bar{u}_j(t) = \vartheta_j^T \varpi_j(t) + \varepsilon_{fj}(t)$. In this case, we can write: $\varepsilon(z) = \sum_{j=1}^{m} \varepsilon_{fj}(t) + \varepsilon_p(z)$.

The adaptive version of the controller (2.60) can be written as follows

$$u_0 = \Pi^T(z)\theta - \sum_{j=1}^{m} \kappa_j^T \varpi_j(t) \tag{2.62}$$

Now, let us define the following control errors

$$e_{u_p} = u_p^* - \Pi^T(z)\theta = \Pi^T(z)\tilde{\theta} + \varepsilon(z) \tag{2.63}$$

$$e_{u_{f,j}} = -\tilde{\kappa}_j^T \varpi_j(t),\ j = 1,2,\ldots,m \tag{2.64}$$

with $\tilde{\theta} = \theta^* - \theta$ and $\tilde{\kappa}_j = \kappa_j^* - \kappa_j$, $j = 1,2,\ldots,m$. The control errors e_{u_p} and $e_{u_{f,j}}$, $j = 1,2,\ldots,m$ measure the discrepancy between the unknown functions u_p^*, $u_{f,j}^*$, $j = 1,2,\ldots,m$ and the control signals u_p, $u_{f,j}$, $j = 1,2,\ldots,m$ respectively.

By adding and subtracting $\bar{\beta}(\xi,\eta)u_p^*$ to (2.54) and using (2.55), (2.63) and (2.64), we obtain

$$\dot{s} = -ks - k_0 \tanh\left(\frac{s}{\varepsilon_0}\right) + \bar{\beta}(\xi,\eta)\left(e_{u_p} + \sum_{j=1}^{m} e_{u_{f,j}}\right) \tag{2.65}$$

Consider the following quadratic cost function of the control error

$$J(\theta,\kappa) = \frac{1}{2}\bar{\beta}(\xi,\eta)\left(e_{u_p} + \sum_{j=1}^{m} e_{u_{f,j}}\right)^2 = \frac{1}{2}\bar{\beta}(\xi,\eta)\left(u_p^* - \Pi^T(z)\theta - \sum_{j=1}^{m} \tilde{\kappa}_j \varpi_j(t)\right)^2 \tag{2.66}$$

The parameter update laws for θ and κ_j, $j = 1,2,\ldots,m$ which minimize $J(\theta,\kappa)$ are given as:

$$\dot{\theta} = -\eta_1 \nabla_\theta J(\theta, \kappa) \tag{2.67}$$

$$\dot{\kappa}_j = -\eta_2 \nabla_{\kappa_j} J(\theta, \kappa),\ j = 1, 2, \ldots, m \tag{2.68}$$

where η_1 and η_2 are positive adaptation gains. Now by using (2.66), one can write: $\nabla_\theta J(\theta, \kappa) = -\bar{\beta}(\xi, \eta)\Pi(z)\left(e_{u_p} + \sum_{j=1}^{m} e_{u_{f,j}}\right)$, $\nabla_{\kappa_j} J(\theta, \kappa) = \bar{\beta}(\xi, \eta)\varpi_j(t)\left(e_{u_p} + \sum_{j=1}^{m} e_{u_{f,j}}\right)$. Therefore, (2.67) and (2.68) can be written respectively as

$$\dot{\theta} = \eta_1 \bar{\beta}(\xi, \eta)\Pi(z)\left(e_{u_p} + \sum_{j=1}^{m} e_{u_{f,j}}\right) \tag{2.69}$$

$$\dot{\kappa}_j = -\eta_2 \bar{\beta}(\xi, \eta)\varpi_j(t)\left(e_{u_p} + \sum_{j=1}^{m} e_{u_{f,j}}\right),\ j = 1, 2, \ldots, m \tag{2.70}$$

In (2.69) and (2.70) the term $\bar{\beta}(\xi, \eta)\left(e_{u_p} + \sum_{j=1}^{m} e_{u_{f,j}}\right)$ is unknown. However, from (2.65), it can be pulled out as follows

$$\bar{\beta}(\xi, \eta)\left(e_{u_p} + \sum_{j=1}^{m} e_{u_{f,j}}\right) = \dot{s} + ks + k_0 \tanh\left(\frac{s}{\varepsilon_0}\right) \tag{2.71}$$

Substituting (2.71) into (2.69) and (2.70), and introducing a $\sigma-$modification to ensure the boundedness of the parameters θ and $\kappa_j,\ j = 1, \ldots, m$ to improve the robustness of the controller in the presence of approximation errors, one can write

$$\dot{\theta} = \eta_1 \Pi(z)\left(\dot{s} + ks + k_0 \tanh\left(\frac{s}{\varepsilon_0}\right)\right) - \eta_1 \sigma \theta \tag{2.72}$$

$$\dot{\kappa}_j = -\eta_2 \varpi_j(t)\left(\dot{s} + ks + k_0 \tanh\left(\frac{s}{\varepsilon_0}\right)\right) - \eta_2 \sigma \kappa_j,\ j = 1, 2, \ldots, m \tag{2.73}$$

In the following subsection, closed-loop stability and tracking analysis are carried out.

2.4.3 Stability and tracking analysis

For stability and tracking analysis, assume that abrupt actuator failures occur at time instants $t_i,\ i = 1, \ldots, N$, we introduce a piecewise Lyapunov-like function defined over the time intervals $[t_i, t_{i+1})$ where there are no abrupt actuator failures as

$$V_i = \frac{1}{2}s^2 + \frac{1}{2\eta_1}\tilde{\theta}^T \tilde{\theta} + \frac{1}{2\eta_2}\sum_{j=1}^{m} \tilde{\kappa}_j^{\ T} \tilde{\kappa}_j \tag{2.74}$$

The derivative of (2.74) over the time interval $t \in [t_i, t_{i+1})$ is given as

$$\dot{V}_i = s\dot{s} - \frac{1}{\eta_1}\tilde{\theta}^T\dot{\theta} + \frac{1}{\eta_1}\tilde{\theta}^T\dot{\theta}^* - \frac{1}{\eta_2}\sum_{j=1}^{m}\tilde{\kappa}_j^T\dot{\kappa}_j + \frac{1}{\eta_2}\sum_{j=1}^{m}\tilde{\kappa}_j^T\dot{\kappa}_j^* \tag{2.75}$$

Using (2.65), (2.72) and (2.73), equation (2.75) can be written as follows

$$\begin{aligned}\dot{V}_i = {} & s\left(-ks - k_0 \tanh\left(\frac{s}{\varepsilon_0}\right) + \overline{\beta}(\xi,\eta)e_{u_0}\right) - \tilde{\theta}^T\left(\Pi(z)\overline{\beta}(\xi,\eta)e_{u_0} - \sigma\theta\right) \\ & + \frac{1}{\eta_1}\tilde{\theta}^T\dot{\theta}^* + \sum_{j=1}^{m}\tilde{\kappa}_j^T\left(\varpi_j(t)\overline{\beta}(\xi,\eta)e_{u_0} + \sigma\kappa_j\right) + \frac{1}{\eta_2}\sum_{j=1}^{3}\tilde{\kappa}_j^T\dot{\kappa}_j^*\end{aligned} \tag{2.76}$$

with $e_{u_0} = e_{u_p} + \sum_{j=1}^{m} e_{u_{f,j}}$. Now, by virtue of (2.63) and (2.64), (2.76) can be written as

$$\begin{aligned}\dot{V}_i = {} & -ks^2 - k_0 s \tanh\left(\frac{s}{\varepsilon_0}\right) + s\overline{\beta}(\xi,\eta)e_{u_0} - \left(e_{u_p} - \varepsilon(z)\right)\overline{\beta}(\xi,\eta)e_{u_0} \\ & + \sigma\tilde{\theta}^T\theta + \frac{1}{\eta_1}\tilde{\theta}^T\dot{\theta}^* - \sum_{j=1}^{m} e_{u_{f,j}}\overline{\beta}(\xi,\eta)e_{u_0} + \sigma\sum_{j=1}^{m}\tilde{\kappa}_j^T\kappa_j + \frac{1}{\eta_2}\sum_{j=1}^{m}\tilde{\kappa}_j^T\dot{\kappa}_j^*\end{aligned} \tag{2.77}$$

Provided the fact that $e_{u_0} = e_{u_p} + \sum_{j=1}^{m} e_{u_{f,j}}$, equation (2.77) can be written as

$$\begin{aligned}\dot{V}_i = {} & -ks^2 - k_0 s \tanh\left(\frac{s}{\varepsilon_0}\right) + s\overline{\beta}(\xi,\eta)e_{u_0} - \overline{\beta}(\xi,\eta)e_{u_0}^2 + \frac{1}{\eta_1}\tilde{\theta}^T\dot{\theta}^* \\ & + \varepsilon(z)\beta(\xi,\eta)e_{u_0} + \sigma\tilde{\theta}^T\theta + \sigma\sum_{j=1}^{m}\tilde{\kappa}_j^T\kappa_j + \frac{1}{\eta_2}\sum_{j=1}^{m}\tilde{\kappa}_j^T\dot{\kappa}_j^*\end{aligned} \tag{2.78}$$

By virtue of (2.61) and using the following inequalities

$$\sigma\tilde{\theta}^T\theta \le -\frac{\sigma}{2}\left\|\tilde{\theta}\right\|^2 + \frac{\sigma}{2}\left\|\theta^*\right\|^2, \sigma\tilde{\kappa}_j^T\kappa_j \le -\frac{\sigma}{2}\left\|\tilde{\kappa}_j\right\|^2 + \frac{\sigma}{2}\left\|\kappa_j^*\right\|^2 \tag{2.79}$$

$$\frac{1}{\eta_1}\tilde{\theta}^T\dot{\theta}^* \le \frac{\sigma}{4}\left\|\tilde{\theta}\right\|^2 + \frac{1}{\sigma\eta_1^2}\left\|\dot{\theta}^*\right\|^2, \frac{1}{\eta_2}\tilde{\kappa}_j^T\dot{\kappa}_j^* \le \frac{\sigma}{4}\left\|\tilde{\kappa}_j\right\|^2 + \frac{1}{\sigma\eta_2^2}\left\|\dot{\kappa}_j^*\right\|^2 \tag{2.80}$$

$$\varepsilon(z)e_{u_0} \le \frac{1}{4}e_{u_0}^2 + \varepsilon^2(z) \le \frac{1}{4}e_{u_0}^2 + \left(\overline{\varepsilon}_0 s^2 + \overline{\varepsilon}_1\right) \tag{2.81}$$

$$s\overline{\beta}(\xi,\eta)e_{u_0} \le \frac{1}{4}\overline{\beta}(\xi,\eta)e_{u_0}^2 + \overline{\beta}(\xi,\eta)s^2 \tag{2.82}$$

Equation (2.78) can be bounded as follows

$$\dot{V}_i \le -\frac{1}{2}\bar{\beta}(\xi,\eta)e_{u_0}^2 - k_0 s\tanh\left(\frac{s}{\varepsilon_0}\right) - \left(k - \bar{\beta}(\xi,\eta)(1+\bar{\varepsilon}_0)\right)s^2 - \frac{\sigma}{4}\sum_{j=1}^{m}\left\|\tilde{\kappa}_j\right\|^2$$
$$-\frac{\sigma}{4}\left\|\tilde{\theta}\right\|^2 + \frac{\sigma}{2}\left\|\theta^*\right\|^2 + \frac{1}{\sigma\eta_1^2}\left\|\dot{\theta}^*\right\|^2 + \frac{\sigma}{2}\sum_{j=1}^{m}\left\|\kappa_j^*\right\|^2 + \frac{1}{\sigma\eta_2^2}\sum_{j=1}^{m}\left\|\dot{\kappa}_j^*\right\|^2 + \bar{\beta}(\xi,\eta)\bar{\varepsilon}_1 \quad (2.83)$$

Since the parameter vectors $\theta^*, \kappa_j^*, \dot{\theta}^*, \dot{\kappa}_j^*$, $j = 1,2,\ldots,m$ and $\bar{\beta}(\xi,\eta)$ are assumed bounded inside the interval $[t_i, t_{i+1})$. A positive bound ψ_i can be defined over that interval such that

$$\psi_i = \sup_{t\in[t_i,t_{i+1})}\left(\frac{\sigma}{2}\left\|\theta^*\right\|^2 + \frac{1}{\sigma\eta_1^2}\left\|\dot{\theta}^*\right\|^2 + \frac{\sigma}{2}\sum_{j=1}^{m}\left\|\kappa_j^*\right\|^2 + \frac{1}{\sigma\eta_2^2}\sum_{j=1}^{m}\left\|\dot{\kappa}_j^*\right\|^2 + \bar{\beta}(\xi,\eta)\bar{\varepsilon}_1\right) \quad (2.84)$$

Then (2.83) simplifies to the following

$$\dot{V}_i \le -\frac{1}{2}\bar{\beta}(\xi,\eta)e_{u_0}^2 - \left(k - (1+\bar{\varepsilon}_0)\bar{\beta}(\xi,\eta)\right)s^2 - \frac{\sigma}{4}\left\|\tilde{\theta}\right\|^2 - \frac{\sigma}{4}\sum_{j=1}^{m}\left\|\tilde{\kappa}_j\right\|^2 + \psi_i \quad (2.85)$$

Assuming that the design parameter k is chosen such that $k > (1+\bar{\varepsilon}_0)\delta_1$, $\delta_1 > \left|\bar{\beta}(\xi,\eta)\right|$, and define $\gamma = \min\left(2\times\left(k-(1+\bar{\varepsilon}_0)\delta_1\right), 0.5\sigma\eta_1, 0.5\sigma\eta_2\right)$, then (2.85) can be further simplified as

$$\dot{V}_i \le -\frac{1}{2}\bar{\beta}(\xi,\eta)e_{u_0}^2 - \frac{\gamma}{2}s^2 - \frac{\gamma}{2\eta_1}\left\|\tilde{\theta}\right\|^2 - \frac{\gamma}{2\eta_2}\sum_{j=1}^{m}\left\|\tilde{\kappa}_j\right\|^2 + \psi_i \quad (2.86)$$

From the definition of V_i in (2.74), $\dot{V}_i$ can be bounded as follows

$$\dot{V}_i \le -\gamma V_i + \psi_i \quad (2.87)$$

Now we can prove the following theorem on the boundedness of all closed-loop signals.

Theorem 2.3: Consider the system (2.1) subject to type (2.6) and (2.7) actuator failures which are parameterized as (2.8), using the actuation scheme (2.10), the adaptive controller (2.62) with parameter update laws (2.72) and (2.73) guarantees that the closed-loop system is uniformly ultimately bounded (UUB) stable and that the output tracking error converges to a small neighborhood of the origin.

Proof:

Equation (2.87) implies that, over the time interval $[t_i, t_{i+1})$, where the failure pattern is fixed, if $V_i \ge \psi_i/\gamma$ we have $\dot{V}_i < 0$. Now, by integrating (2.87) over the interval $t \in [t_i, t_{i+1})$, we get

$$V_i(t) \le V(t_i^+)e^{-\gamma(t-t_i)} + \frac{\psi_i}{\gamma} \tag{2.88}$$

Now, let us denote $V(t)$ the extension of $V_i(t)$ over the whole time domain. Due to abrupt actuator failures $\tilde{\theta},\ \tilde{\kappa}_j,\ j=1,2,\ldots,m$ and consequently $V(t)$ will exhibit finite jumps at time instants $t_i,\ i=1,2,\ldots,N$, with $t_{N+1}=\infty$, i.e. there are actuator failures after t_N. Let us also denote ΔV_i the jump on $V(t)$ caused by jumps on $\tilde{\theta},\tilde{\kappa}_j,\ j=1,2,\ldots,m$ at time t_i, and t_i^-, t_i^+ the time instants just before and after the jump respectively. Starting from t_1, we can write

$$V(t_1^+) = V(t_1^-) + \Delta V_1 \tag{2.89}$$

And from (2.88) we have

$$V(t_1^-) \le V(t_0)e^{-\gamma(t_1-t_0)} + \frac{\psi_1}{\gamma} \tag{2.90}$$

From (2.89) and (2.90) we can write

$$V(t_1^+) \le V(t_0)e^{-\gamma(t_1-t_0)} + \frac{\psi_1}{\gamma} + \Delta V_1 \tag{2.91}$$

Likewise, we proceed for $t_2,\ldots,t_N$, we end up by the following inequality

$$V(t_N^+) \le V(t_0) + \sum_{i=1}^{N}\frac{\psi_i}{\gamma} + \sum_{i=1}^{N}\Delta V_i \tag{2.92}$$

After t_N, there are no further failures. By integrating (2.87) over $t\in[t_N,\infty)$, one can write

$$V(t) \le V(t_N^+)e^{-\gamma(t-t_N)} + \frac{\psi_{N+1}}{\gamma} \tag{2.93}$$

By substituting (2.92) into (2.93) we obtain

$$V(t) \le \left(V(t_0) + \sum_{i=1}^{N}\frac{\psi_i}{\gamma} + \sum_{i=1}^{N}\Delta V_i\right)e^{-\gamma(t-t_N)} + \frac{\psi_{N+1}}{\gamma} \tag{2.94}$$

It can be concluded that $s(t),\tilde{\theta}(t),\tilde{\kappa}_j(t),\ j=1,2,\ldots,m$ and consequently $u(t)$ are bounded. Regarding (2.74) and (2.94), after time t_N, when there are no further failures, we have

$$|s(t)| \le \sqrt{\left(V(t_0) + \sum_{i=1}^{N}\frac{\psi_i}{\gamma} + \sum_{i=1}^{N}\Delta V_i\right)}e^{-0.5\gamma(t-t_N)} + \sqrt{\frac{\psi_{N+1}}{\gamma}} \tag{2.95}$$

from (2.95) it follows that $V(t)$ is exponentially bounded and converges to a residual set defined by: $\left|s(t)\right| \le \sqrt{\psi_{N+1}/\gamma}$. This means that $e(t)$ and its time derivatives up to order $r-1$ converge to residual sets defined as $\left|e(t)\right| \le \sqrt{2\psi_{N+1}/\gamma}/\lambda$, $\left|\dot{e}(t)\right| \le 2\sqrt{2\psi_{N+1}/\gamma}$. These sets can be made smaller by an adequate choice of the design parameters λ, σ, η_1 and η_2.

2.4.4 Simulation study: Application to wing rock motion control

In this subsection, the proposed adaptive actuator failure compensation control design is applied to a wing rock system to access its effectiveness.

2.4.4.1 Bibliographical analysis on wing rock control

Wing rock describes the oscillatory motion of the roll dynamics in an aircraft which occurs at high angles of attack. This oscillatory motion can be seen in high-speed civil transport and combat aircraft. The main aerodynamic parameters of wing rock are (i) Angle of attack (ii) angle of sweep (iii) Leading edge extensions (iv) Slender forebody. The control of wing rock is a relevant issue as high-speed civil transport and combat aircraft can encounter it in their flight envelope, seriously compromising handling qualities and maneuvering capabilities. Many nonlinear and adaptive control schemes have been proposed for wing rock suppression [96], [113]–[115]. In this subsection, an augmented model of the wing rock motion is considered, and the proposed controller is applied to suppress to roll angle and rate oscillations

2.4.4.2 Modeling of the wing rock motion

Consider the wing rock model studied in [113], [116]. The non-dimensional differential equation (single DOF roll dynamics) describing the free motion of the roll angle ϕ is given by

$$\ddot{\phi}(t) + \hat{a}_0\phi(t) + \hat{a}_1\dot{\phi}(t) + \hat{a}_2\left|\dot{\phi}(t)\right|\dot{\phi}(t) + \hat{a}_3\phi^3(t) + \hat{a}_4\phi^2(t)\dot{\phi}(t) = 0 \tag{2.96}$$

where $\hat{a}_0, \hat{a}_2, \ldots, \hat{a}_4$ are parameters related to the aircraft's operating conditions (i.e. the angle of attack, Reynold number, and wing characteristics). By introducing the reference time $t_s = b/2V$ where V is the air speed and b is the wing span, equation (2.96) becomes [113]

$$\ddot{\phi}(t) + \frac{\hat{a}_0}{t_s^2}\phi(t) + \frac{\hat{a}_1}{t_s}\dot{\phi}(t) + \hat{a}_2\left|\dot{\phi}(t)\right|\dot{\phi}(t) + \frac{\hat{a}_3}{t_s^2}\phi^3(t) + \frac{\hat{a}_4}{t_s}\phi^2(t)\dot{\phi}(t) = 0 \tag{2.97}$$

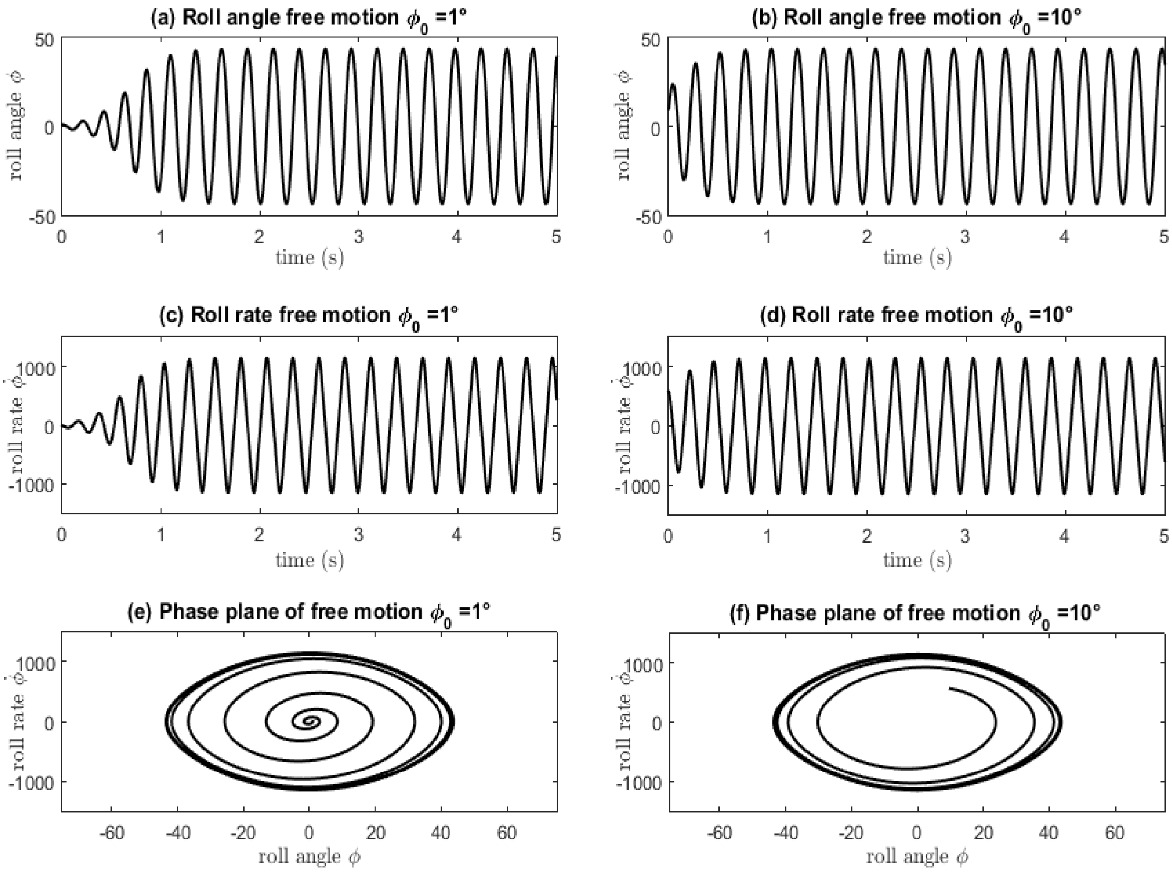

Figure 2.5: Free to roll motion for different release angles.

To illustrate the wing rock phenomenon, let us consider an analytical wing rock model based on parameter identification of wind tunnel based experimental data. The model has a wingspan $b = 0.169\text{m}$, root chord $c_r = 0.479$ m, sweep $\Lambda = 80°$, angle of attack $\alpha = 32.5°$, and air speed $V = 30$ m/s $(\text{Re} = 950000)$, the corresponding coefficients are given as $\hat{a}_0 = 0.00723$, $\hat{a}_1 = -0.03104$, $\hat{a}_2 = 0.53884$, $\hat{a}_3 = 0.00623$ and $\hat{a}_4 = 0.04189$ [113]. The free to roll motion for an initial roll angle $\phi(0) = 1°$ and roll rate $\dot{\phi}(t) = 0$ °/s in the first case and $\phi(0) = 10°$, $\dot{\phi}(t) = 0$ °/s in the second case are shown in Figure 2.5. The free motion starts to oscillate and settles at a limit cycle. The controller must suppress these oscillations.

Now, by including the reference time t_s in the coefficients $\hat{a}_i, i = 0,...,4$, the controlled wing rock model equation with dimensional derivatives can be written as follows

$$\ddot{\phi}(t) + a_0\phi(t) + a_1\dot{\phi}(t) + a_2\left|\dot{\phi}(t)\right|\dot{\phi}(t) + a_3\phi^3(t) + a_4\phi^2(t)\dot{\phi}(t) = u \tag{2.98}$$

with $a_0 = \hat{a}_0/t_s^2$, $a_1 = \hat{a}_1/t_s$, $a_2 = \hat{a}_2$, $a_3 = \hat{a}_3/t_s^2$, $a_4 = \hat{a}_4/t_s$. The equivalent control input u is designed to suppress oscillations, it is related to the controlling torque T as follows

$$u = -T/I_{xx} \tag{2.99}$$

where I_{xx} is the inertia of the model, and T is the controlling torque. In the simulation, it is assumed that the controlling torque is provided by three aileron segments as follows [19]

$$T = \frac{1}{2}\rho V^2 Sb\left(Cl_{da1}\delta_{a1} + Cl_{da2}\delta_{a2} + Cl_{da3}\delta_{a3}\right) \tag{2.100}$$

where ρ is the air density, S is the wing surface and Cl_{dai}, $i = 1,2,3$ are the derivatives of the roll moment coefficient with respect to the aileron's deflection angles δ_{ai}.

Denoting $x = \left[x_1, x_2\right]^T = \left[\phi, \dot{\phi}\right]^T$ and $u = \left[u_1, u_2, u_3\right]^T = \left[\delta_{a1}, \delta_{a2}, \delta_{a3}\right]^T$, the state space model of the wing rock system with augmented actuation system can be written as follows

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -a_0 x_1 - a_1 x_2 - a_2 \left|x_2\right| x_2 - a_3 x_1^2 - a_4 x_1^2 x_2 + b_1 u_1 + b_2 u_2 + b_3 u_3 \end{aligned} \tag{2.101}$$

with $b_i = \rho V^2 SbCl_{dai}/2I_{xx}$, $i = 1,2,3$. The state model (2.101) can be written as follows

$$y^{(2)} = f\left(x\right) + g^T\left(x\right)u \tag{2.102}$$

where $f\left(x\right) = -a_0 x_1 - a_1 x_2 - a_2\left|x_2\right| x_1 - a_3 x_1^2 - a_4 x_1^2 x_2$, $g\left(x\right) = \left[b_1, b_2, b_3\right]^T$ and $y\left(t\right) = \phi\left(t\right)$.

2.4.4.3 Adaptive actuator failure compensation for the wing rock control

The proposed actuator failure compensation control strategy is applied on the analytical wind rock model. The triangular wing shape configuration is considered with a wingspan $b = 0.169\ m$, root chord $c_r = 0.479\ m$, sweep $\Lambda = 80°$, airspeed $V = 30$ m/s ($\text{Re} = 950000$) and angle of attack $\alpha = 32.5°$. The other parameters of the model are computed from the physical variables of the wing, the coefficients a_i are computed from $\hat{a}_i$ and t_s, which gives $a_0 = 922.6568$, $a_1 = -11.0201$, $a_2 = 0.5388$, $a_3 = -785.2666$, and $a_4 = 14.8722$, while the coefficients b_i are computed using the equation: $b_i = \rho V^2 SbCl_{dai}/2I_{xx}$, $i = 1,2,3$ which gives $b_1 = -0.149.1759$, $b_2 = -111.8819$ and $b_3 = -111.8819$. Recall that the control objective is to suppress the roll angle and rate oscillations. Therefore, the roll angle should track as close as possible the reference angle $y_d\left(t\right) = 0°$.

The free design parameters are selected as: $\lambda = 5$, $\eta_1 = \eta_2 = 20$, $\sigma = 0.1$, $\varepsilon_0 = 0.005$, $k = 20$ and $k_0 = 10$. The initial values of the controller parameters θ and κ_j, $j = 1,2,3$ are taken all zero. The actuation proportional gains are chosen $b_1(t) = 2,\ b_2(t) = 2,\ b_3(t) = 1$. Moreover, in this study, the regressor vector is chosen as $\Pi(z) = [1, e, \dot{e}]$.

For the simulation, we consider the following two actuator failure scenarios:

First failure scenario: With initial release angle and rate $\phi(0) = 1°,\ \dot{\phi}(0) = 0°/\text{s}$ respectively, we assume that the first segment remains intact, i.e. $u_1^f(t) = u_1(t)$. At the time $t = 3\ s$, the third aileron segment is stuck at a value $\bar{u}_3(t) = 10$. At the time $t = 6\ s$, the second segment is only 30% effective, i.e. $u_2^f(t) = 0.3u_2(t)$, and at the time $t = 8\ s$, the third segment recovers from its failure, i.e. $u_3^f(t) = u_3(t)$.

The numerical simulation is carried out for $10\ s$, which is a sufficient time given that the model under study is derived from a wind tunnel experiment. The simulation results are shown in Figure 2.6, Figure 2.7, and Figure 2.8, it can be seen that, when an actuator failure occurs, the control effort is redistributed among the healthy actuators and the tracking performance is established after a transient. Besides, it can be seen from the simulation results that when an actuator recovers from a failure it will be exploited automatically by the control scheme and used to control the system. Notice also that the time history of the failure compensator u_f gives a rough information about the failure occurrence, this is interesting as this signal can be used as a fault alarm or indicator.

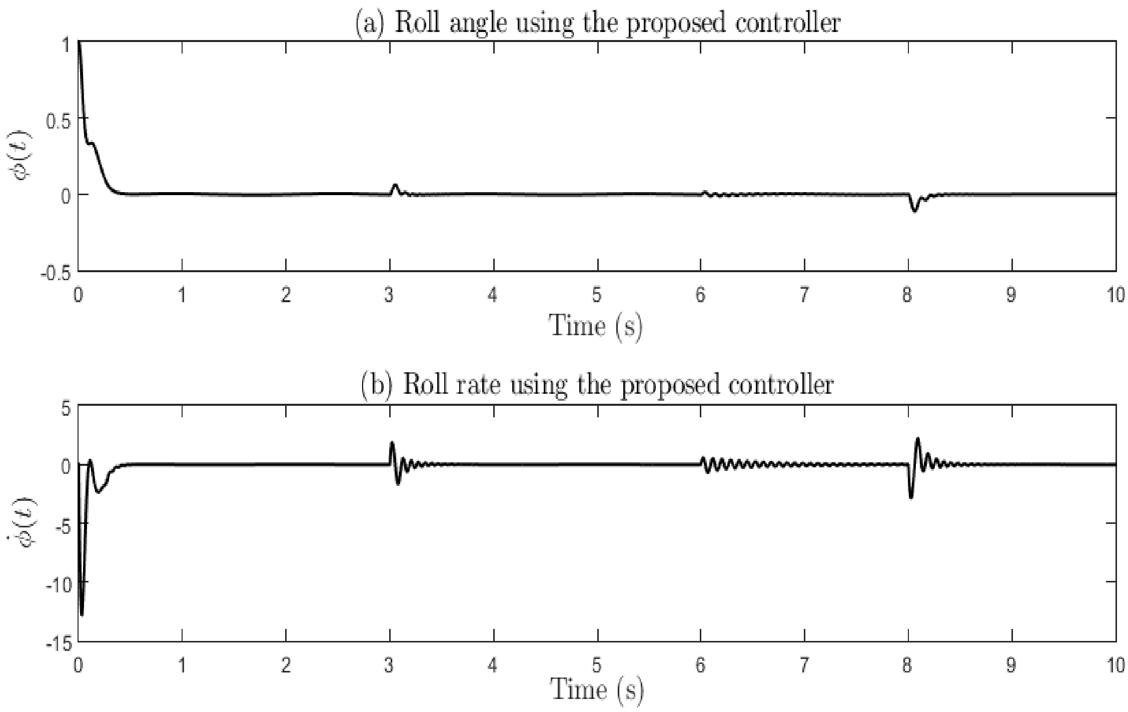

Figure 2.6: Roll angle and roll rate evolution (scenario 1)

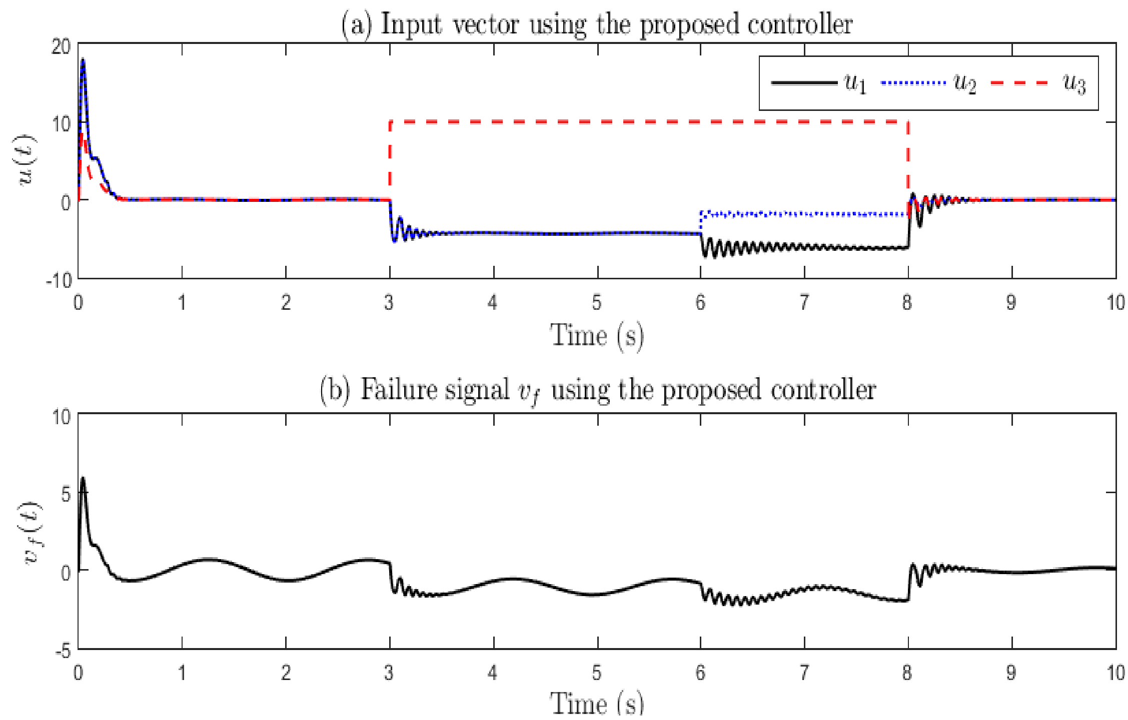

Figure 2.7: Control signals evolution (scenario 1)

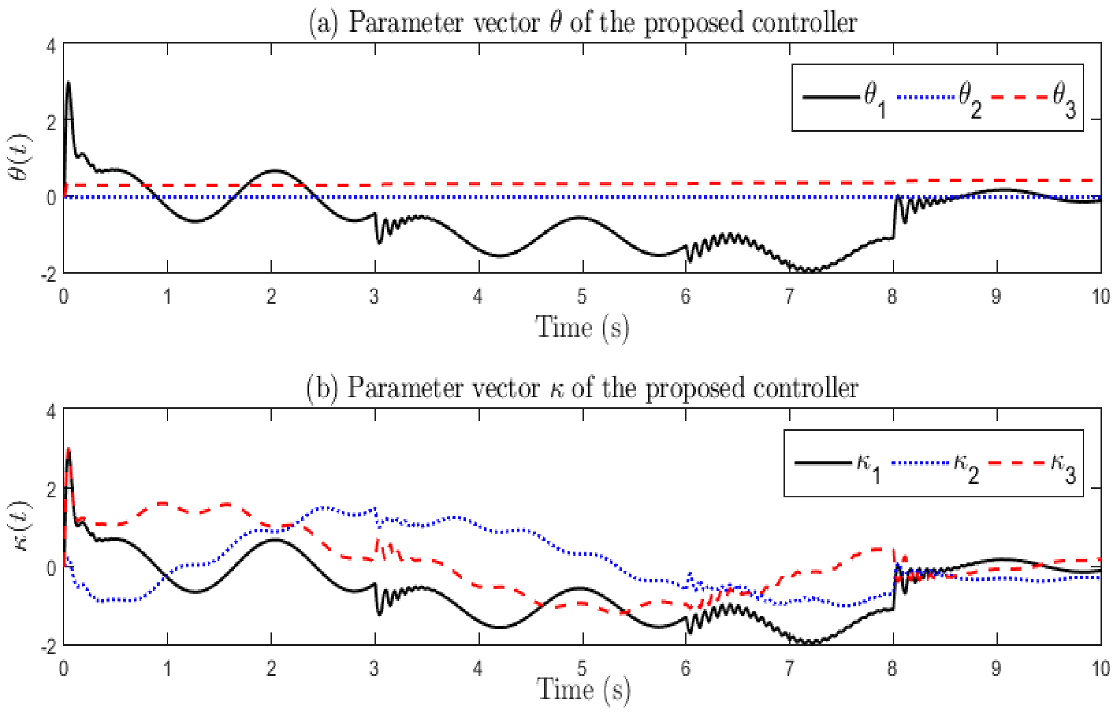

Figure 2.8: Controller parameters evolution (scenario 1)

Second failure scenario: Consider the worst-case failure scenario, with initial release angle and rate $\phi(0)=10°$, $\dot{\phi}(0)=0°/s$ respectively. Assume that at the time $t=5\ s$, the first segment is only 70% effective. At time instant $t=4\,s$, the second segment locks at a value $\bar{u}_2(t)=20$ and then it recovers at the time $t=8\,s$. For the third segment, we assume that at the time $t=3\,s$, it starts oscillating according to the pattern $u_3(t)=1-30\sin(5t)+20\cos(5t)$. The simulation results are shown in Figure 2.9, Figure 2.10, and Figure 2.11, it can be seen that the proposed controller shows superiority in dealing with such failures, the roll rate and roll angle were regulated around the origin. Once again, we see that the control effort is intelligently redistributed among healthy actuators if an actuator fails. Besides, if an actuator recovers from a failure, it is automatically exploited to control the system. In summary, we conclude that the simulation results confirm the theoretical claims

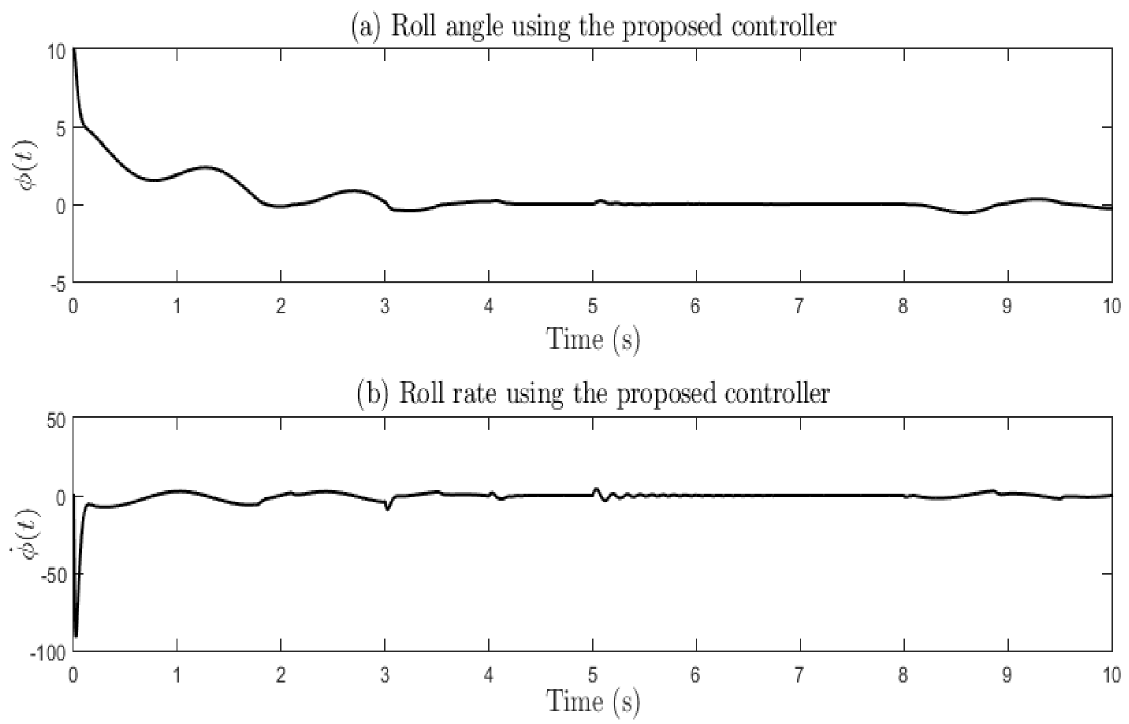

Figure 2.9: Roll angle and roll rate evolution (scenario 2)

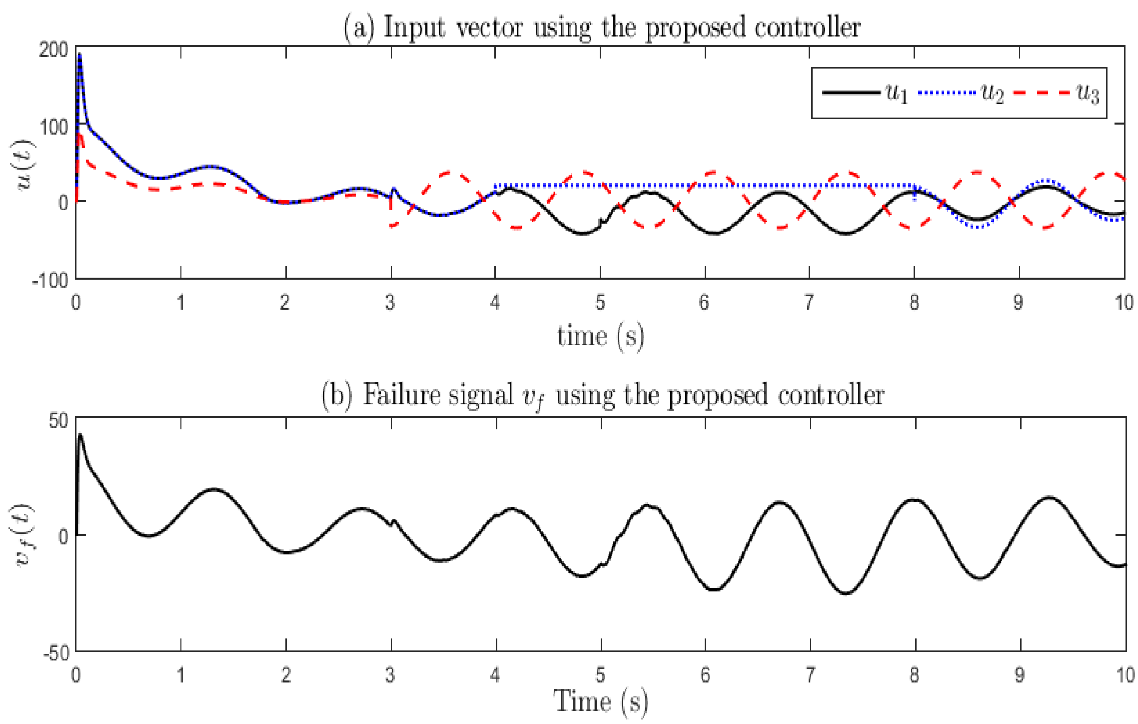

Figure 2.10: Control signals evolution (scenario 2)

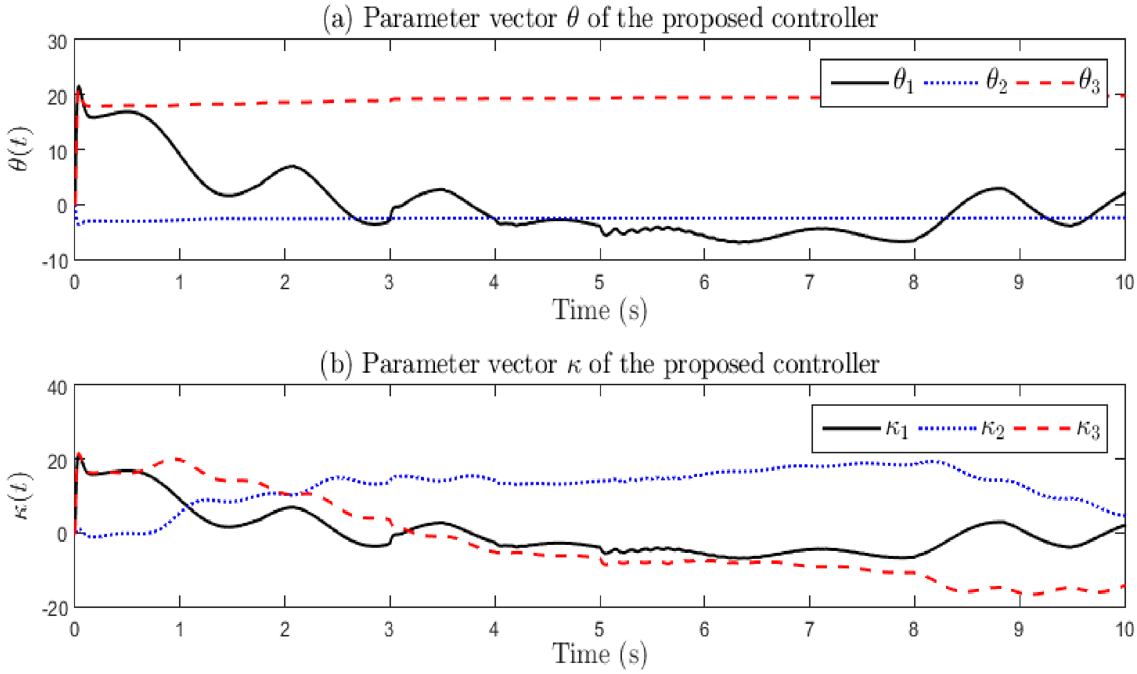

Figure 2.11: Controller parameters evolution (scenario 2)

2.5 Conclusion

In this chapter, the problem of actuator failure compensation control for affine multi-input single-output systems is investigated. Two types of actuator failures are considered. Two designs were proposed, the first design assumes no failure pattern parameterization and the second design assumes that the failure pattern is completely parameterized where the parameters are estimated online and compensated for. In both designs, the parameter update laws are derived based on the control prediction error. Stability and tracking are proved rigorously using Lyapunov theory and piecewise analysis to deal with parameter jumps caused by abrupt failures. The first design assumes no information of the failure pattern and can compensate for the failure by considering it as part of the system dynamics. The second design assumes that the failure pattern is parameterized and estimates its parameters separately then feeds them to the plant controller. Simulation is carried out for two realistic examples; the first design is applied to the angle of attack of a hypersonic aircraft with fuzzy logic systems as regressor, the second design is applied to wing rock control with a proportional derivative (PD) plus an independent term regressor. The simulation results for both designs show the effectiveness and feasibility of the proposed designs and confirm our theoretical claims. In the next chapter, the problem of actuator failure compensation for multi-input multi-output nonlinear systems will be investigated, we will try also to widen the set of compensable failures.

Chapter 3

3 Adaptive control of multi-output nonlinear systems with actuator failures

Chapter Contents

3.1 Introduction

Most practical engineering systems are multi-input multi-output (MIMO) and the design of adaptive controllers for such systems is even more delicate. Particularly for nonlinear systems with uncertain dynamics, where uncertain nonlinearities and interactions are present. Furthermore, the presence of uncertain actuator failures during the course of operation renders the problem even more challenging, and an effective design should manage the existing redundancy to ensure acceptable performance levels (possibly with graceful degradation) despite the presence of actuator failures. In the research literature, the problem of adaptive actuator failure compensation for uncertain MIMO nonlinear systems has been investigated from different perspectives depending on available information on system and failures. In some works, only actuator failures were supposed to be uncertain, while system parameters are supposed to be certain [77]. The case of uncertain systems with uncertain actuator failures has been addressed from two perspectives, the first perspective assumes that system functions can be expressed as a product of uncertain parameters with some known functions [22], [23]. this problem has been solved for parameterized actuator failures using adaptive backstepping designs [117] and for non-parameterized actuator failures using adaptive feedback linearization control [70]. The second perspective assumes no knowledge of system parameters and actuator failures, in this case, function approximators are invoked for the online construction of some ideal controllers. Many works fall into this context, for example, in [80] fuzzy adaptive backstepping design and in [81] using neural dynamic surface control. In these works, the faulty system dynamics functions are approximated online and then plugged into an ideal controller in the framework of indirect adaptive control. Notice also that in most existing works only partial or total loss of effectiveness failure types were addressed, input affine failures are not considered. Under the light of these observations, the aim of this chapter is to solve the problem of actuator failure compensation for a class of uncertain MIMO nonlinear systems with affine actuator failures using a direct adaptive control scheme. Therefore, the remaining of the chapter is thus organized as follows, in section 3.2, the system is described, the actuator failure problem is presented and a mathematical framework is presented. In section 3.3, the adaptive actuator failure estimation and compensation controller is presented; proofs of stability and tracking are also presented. In section 3.4, a full case study on a robot manipulator with redundant joints and a flexible spacecraft system with redundant reaction wheels is presented. Finally, section 3.5 provides a conclusion for the chapter.

3.2 Problem statement and preliminary analysis

3.2.1 System description

In this chapter, we consider the class of multi-input multi-output (MIMO) nonlinear systems with redundant actuators described by the following set of differential equations

$$\begin{aligned} y_1^{(r_1)} &= f_1(x) + \sum_{j=1}^{q} g_{1j}(x) u_j + d_1(t) \\ &\vdots \\ y_p^{(r_p)} &= f_p(x) + \sum_{j=1}^{q} g_{pj}(x) u_j + d_p(t) \end{aligned} \tag{3.1}$$

where $x = \left[y_1, \dots, y_1^{(r_1-1)}, \dots, y_p, \dots, y_p^{(r_p-1)} \right]^T \in \mathbb{R}^n$, represents the state vector of the system which is supposed available for feedback, with $n = r_1 + r_2 + \cdots + r_p$. In addition, $u = \left[u_1, u_2, \dots, u_q \right]^T \in \mathbb{R}^q$ is the control input vector whose actuators may fail during system operation and $y = \left[y_1, y_2, \dots, y_p \right]^T \in \mathbb{R}^p$ is the output of the system. Besides, $f_i(x) \in \mathbb{R}$ and $g_{ij}(x) \in \mathbb{R}$, $i = 1, \dots, p,\ j = 1, \dots, q,\ q > p$ are uncertain but sufficiently smooth nonlinear functions. The terms $d_i(t),\ i = 1, 2, \dots, p$ account for external disturbances. Let us denote

$$y^{(r)} = \left[y_1^{(r_1)}, y_2^{(r_2)}, \dots, y_p^{(r_p)} \right]^T \tag{3.2}$$

Then, the system (3.1) can be written in the following compact form

$$y^{(r)} = F(x) + G(x) u + D(t) \tag{3.3}$$

with $F(x) = \begin{pmatrix} f_1(x) \\ \vdots \\ f_p(x) \end{pmatrix}$, $G(x) = \begin{pmatrix} g_{11}(x) & \dots & g_{1q}(x) \\ \vdots & \ddots & \vdots \\ g_{p1}(x) & \dots & g_{pq}(x) \end{pmatrix}$, and $D(t) = \begin{pmatrix} d_1(t) \\ \vdots \\ d_p(t) \end{pmatrix}$.

3.2.2 Actuator failures model

In this chapter, a more general integrated model describing time-varying, state-dependent actuator failures is considered; this is an input affine failure model described as follows [17]

$$u_j^f(t) = \rho_j(x,t) u_j(t) + \bar{u}_j(x,t),\ t \ge t_j \tag{3.4}$$

for some unknown time $t_j \geq 0$ and actuator index $j \in \{1, 2, \ldots, m\}$. $u_j(t)$ and $u_j^f(t)$ are respectively the input and output of the j^{th} actuator. Besides, $\rho_j(x,t)$ is a multiplicative coefficient that characterizes the actuator effectiveness and $\bar{u}_j(x,t)$ is an additive term that characterizes the actuator bias. Depending on $\bar{u}_j(x,t)$ and $\rho_j(x,t)$, a large set of failure types can be described using the model (3.4), for instance, if $\rho_j(x,t) = 1$ the actuator is totally effective, if $\rho_j(x,t) = 0$ the actuator has totally lost effectiveness and if $\bar{u}_j(x,t)$ is constant then this is the lock in place case, i.e. the actuator provides a constant signal.

Now, in the presence of actuator failures described by (3.4), the faulty control input vector applied to the system (3.1) will be written as follows

$$u^f(t) = \rho(x,t)u(t) + \bar{u}(x,t) \tag{3.5}$$

here $\rho(t) = diag\{\rho_1, \rho_2, \ldots, \rho_q\}$ is the effectiveness matrix, $\bar{u}(x,t) = [\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_q]^T$ is the bias vector and $u(t) = [u_1, u_2, \ldots, u_q]^T$ is the input to the actuation system.

The control objective is to design an adaptive actuator failure compensation control scheme for the system (3.1) such that, in addition to the inherent disturbances and uncertainties in the system, in the presence of actuator failures modeled by (3.4), which are assumed unknown (in terms of pattern, time, and value), the output of the system $y(t) = [y_1(t), y_2(t), \ldots, y_p(t)]^T$ will track as close as possible a desired output vector $y_d(t) = [y_{d1}(t), y_{d2}(t), \ldots, y_{dp}(t)]^T$ while all closed-loop signals are kept within certain bounds. It is assumed that all the signals $y_{di}(t)$, $i = 1, 2, \ldots, p$ and their time derivatives up to the order $r_i - 1$ are available and bounded.

Remark 3.1: In most related works, only loss of effectiveness failure types are considered, [118], [119], while in other works, bias faults are considered alone [79], [87]. In some works, they considered both bias and loss of effectiveness to happen separately [74], [75], [120]. However, in practice, the actuator may lose effectiveness while undergoing a bias. Besides, the actuator failures can be state-dependent which is rarely considered in the research literature. All these aspects are taken into account in the failure model (3.4).

Regarding the redundant structure of the system and the controllability requirements in the presence of actuator failures, the following general assumption on the system (3.1) is stated

Assumption 3.1: It is assumed that the system described in (3.1) is constructed so that for up to $q-p$ actuator failures (totally lost actuators), the remaining actuators can still be used to achieve the desired behavior with acceptable performance levels.

Remark 3.2: Assumption 3.1 implies that when some actuators fail, the remaining effective actuators should have the sufficient effect and power to drive the system and achieve the control objective under all compensable failure scenarios. In other words, assumption 3.1 implies that: *(i)* There is some actuation redundancy, i.e. there are actuators with the same effect. It is worth emphasizing that there should be at least p effective actuators in order to keep the system controllable, otherwise the system will be under-actuated, and new control schemes will be necessary, *(ii)* Actuators with the same effects are arranged into p groups with $q_i,\ i=1,\dots,p$ actuators in each group. For each group, there should be at least one remaining effective actuator. A fixed actuator grouping is considered [77], [86].

3.2.3 Actuator redundancy and compensability conditions

Actuator redundancy is necessary to compensate for actuator failures, without redundancy, if one or more actuators are totally lost, then the system will become under-actuated [87], the controller would not guarantee tracking control due to the lack of necessary hardware redundancy in the system for achieving fault-tolerant control. However, redundancy alone is not enough, some constraints and conditions should be imposed on the system.

With regard to the adaptive control design for nonlinear MIMO systems and the system's redundancy, the following assumption on the control gain matrix $G(x)$ is made:

Assumption 3.2: It is assumed that the control gain matrix $G(x)$ in (3.3) can be decomposed as: $G(x)=G_0(x)R$, where $R\in\mathbb{R}^{p\times q}$ is a known matrix and $G_0(x)\in\mathbb{R}^{p\times p}$ is an unknown symmetric positive definite (SPD) matrix.

Remark 3.3: Assumption 3.2 is reasonable, as for some systems, the control gain matrix in the non-redundant case is symmetric positive definite. Besides, the redundancy (distribution of actuators) is generally known a priori during the system design, this gives information about the matrix R, which can be considered as the actuators distribution matrix. Typical situations include spacecraft system with redundant reaction wheels [78] and redundant joint manipulators [22]. These two examples will be studied in more detail later in the simulation section.

Remark 3.4: The condition on the matrix $G_0(x)$ (positive definiteness) is a sufficient condition to ensure that the control gain matrix is always regular so that the system described in (3.1) is feedback linearizable. Although it seems somehow restrictive, many practical multivariable systems, such as robotic systems, spacecraft systems, etc., fulfill this condition. For mechanical systems, the control gain matrix is directly related to the system's inertia elements, which are usually positive. Besides, it should be stressed that the developments hold if $G_0(x)$ is negative definite with slight modifications.

To ensure that the system remains controllable in the presence of actuator failures, the following condition on the system (3.1) should hold for every compensable actuator failure pattern

$$rank\left(\rho(x,t)R^T\right) = p \tag{3.6}$$

From condition (3.6), it can be concluded that the matrix $R\rho(x,t)R^T$ is symmetric positive definite (SPD). In fact, since $\rho(x,t) = \sqrt{\rho(x,t)}^T \sqrt{\rho(x,t)}$ then: $\forall z \in \mathbb{R}^p$, one has

$$z^T\left(R\rho(x,t)R^T\right)z = z^T\left(\left(R\sqrt{\rho(x,t)}^T\right)\left(\sqrt{\rho(x,t)}R^T\right)\right)z \tag{3.7}$$

It can be seen from (3.7) that if $\left(\sqrt{\rho(x,t)}R^T\right)z = 0$ then the last term of (3.7) equals zero. From condition (3.6), $\rho(x,t)R^T$ is full rank, which means also that $\sqrt{\rho(x,t)}R^T$ is full rank, in other words, its null space is limited to the trivial solution $z = 0$. Thus $z^T\left(R\rho(x,t)R^T\right)z > 0$ for all $z \neq 0$, which means that $R\rho(x,t)R$ is a symmetric positive definite matrix.

3.2.4 Case of interconnected MIMO systems

In the nonlinear MIMO system (3.1), it is assumed that the actuators are common and can affect all the system outputs. There is a special case when a defined set of redundant actuators affect only an output, this situation can be seen in the case of nonlinear multivariable systems which can be described by a set of p interconnected subsystems Σ_i, $i = 1, 2, \ldots, p$ defined as follows

$$\Sigma_i : \begin{cases} \dot{x}_{i1} = x_{i2}, \dot{x}_{i2} = x_{i3}, \ldots, \dot{x}_{ir_i-1} = x_{ir_i} \\ \dot{x}_{ir_i} = f_i(x) + \sum_{j=1}^{q_i} g_{ij}(x)u_{ij} + d_i(t) \\ y_i = x_{i1} \end{cases} \tag{3.8}$$

In this case, it is obvious that every set of actuators affect directly the subsystem $\sum_i$. Then every subsystem $\sum_i$ can be considered as a redundant single variable system. Then, for each subsystem $\sum_i$, actuator redundancy implies that the functions $g_{ij}(x), j=1,\ldots,q_i$ must be linearly dependent, i.e. $g_{ij}(x)=a_{ij}g_{i0}(x)$ for some constants

$$V(t)\le\frac{\psi_0}{\rho}+\left(V(t_0)-\frac{\psi_0}{\rho}\right)e^{-\rho(t-t_0)}+e^{-\rho(t-t_0)}\int_{t_0}^{t}\left(h(\zeta)N(\tau(\zeta))+c_1\right)\dot{\tau}e^{\rho(\zeta-t_0)}d\zeta$$
$$\le a_0+e^{-\rho(t-t_0)}\int_{0}^{t}\left(h(\zeta)N(\tau(\zeta))+c_1\right)\dot{\tau}e^{\rho(\zeta-t_0)}d\zeta$$

and functions $g_{i0}(x)$, for $j=1,\ldots,q_i$ and $i=1,2,\ldots,p$. The condition on compensability translates to the following

$$\exists j\in\{1,2,\cdots,q_i\}:\ \rho_{ij}\neq 0,\ i=1,2,\ldots,p \tag{3.9}$$

By an adequate rearrangement of the inputs, the matrix $G(x)$ will be expressed as follows:

$$G(x)=\begin{bmatrix} g_{11} & \cdots & g_{1q_1} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \cdots & 0 & g_{21} & \cdots & g_{2q_2} & 0 & \cdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \ddots & \vdots & 0 & \cdots & 0 & g_{i1} & \cdots & g_{iq_i} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & g_{p1} & \cdots & g_{pq_p} \end{bmatrix} \tag{3.10}$$

Taking into account the actuation redundancy, $G(x)$ can be written as follows

$$G(x)=G_0(x)R \tag{3.11}$$

where $G_0(x)=diag\{g_1(x),g_2(x),\ldots,g_p(x)\}$ and

$$R=\begin{bmatrix} a_{11} & \cdots & a_{1q_1} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \cdots & 0 & a_{21} & \cdots & a_{2q_2} & 0 & \cdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \ddots & \vdots & 0 & \cdots & 0 & a_{i1} & \cdots & a_{iq_i} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & a_{p1} & \cdots & a_{pq_p} \end{bmatrix} \tag{3.12}$$

In this case, by considering (3.9), the matrix $R\rho(x,t)R^T$ can be obtained as follows

$$R\rho(x,t)R=\begin{bmatrix}\sum_{j=1}^{q_1}\rho_{1_j}a_{1j}^2 & 0 & \cdots & 0\\ 0 & \sum_{j=1}^{q_2}\rho_{2_j}a_{2j}^2 & \ddots & \vdots\\ \vdots & \ddots & \ddots & 0\\ 0 & \cdots & 0 & \sum_{j=1}^{q_p}\rho_{p_j}a_{pj}^2\end{bmatrix} \tag{3.13}$$

From (3.9), it can be concluded that the matrix $R\rho(x,t)R^T$ is diagonal with strictly positive entries and therefore it is a SPD matrix. Moreover, given that $G_0(x)$ is diagonal with strictly positive diagonal elements. It follows that the product $G_0(x)R\rho(x,t)R^T$ is a diagonal positive definite matrix. This result will be further exploited for the adaptive actuator failure compensation controller design.

3.3 Adaptive controller design

This section outlines the design of an adaptive controller for the MIMO system (3.1) or (3.8) to ensure the control objectives in the presence of system and actuator failure uncertainties.

Regarding the redundant structure of the system, the following actuation scheme is proposed

$$u(t)=R^T u_0(t) \tag{3.14}$$

where $u_0=\left[u_{01},u_{02},\ldots,u_{0p}\right]^T$ is a control vector to be allocated among the actuators. Notice that for the particular case of interconnected systems described by (3.8), for each subsystem Σ_i , the actuation scheme (3.14) is equivalent to the following proportional actuation scheme

$$u_{ij}=a_{ij}u_{0j},\ j=1,2,\ldots,q_i \tag{3.15}$$

In the following subsection, an adaptive control scheme is designed for the faulty system (3.1) or (3.8) to achieve the prescribed control objectives in the presence of actuator failures.

3.3.1 Preliminaries

With the actuation scheme (3.14), in the presence of actuator failures modeled as (3.5), the MIMO nonlinear system (3.3) can be written as follows:

$$y^{(r)}=F(x)+G_0(x)R\rho(x,t)R^T u_0(t)+G_0(x)R\bar{u}(x,t)+D(t) \tag{3.16}$$

Equation (3.16) can be further rewritten as

$$y^{(r)} = F(x) + G_0(x)R\rho(x,t)R^T u_0(t) + \bar{v}(x,t) + D(t) \tag{3.17}$$

with $\bar{v}(x,t) = G_0(x)R\bar{u}(x,t)$ which is a time-varying and state-dependent vector. Let us denote $\bar{G}(x) = G_0(x)R\rho(x,t)R^T$, then (3.17) can be put in the following compact form

$$y^{(r)} = F(x) + \bar{G}(x)u_0 + \bar{v}(x,t) + D(t) \tag{3.18}$$

It can be seen from (3.18) that, the faulty rectangular MIMO system is brought into a square MIMO system. In what follows, an adaptive controller is designed for the faulty system (3.18) to achieve the control objectives.

For later technical developments, the following assumption is imposed:

Assumption 3.3: Since $G_0(x)$ and $R\rho(x,t)R^T$ are positive definite matrices, the matrix $\bar{G}(x) = G_0(x)R\rho(x,t)R^T$ is safely assumed to remain positive definite under actuator failures occurrence, i.e. it is assumed that $N(\tau)$ for all nonzero vectors $z \in \mathbb{R}^p$. Besides, it is assumed that $\underline{\lambda}\|z\|^2 \le z^T\bar{G}(x)z \le \bar{\lambda}\|z\|^2$ where $\underline{\lambda}$ and $\bar{\lambda}$ are positive constants.

3.3.2 Ideal controller structure

In this subsection, an adaptive controller is designed for the system (3.18) to compensate for uncertainties and actuator failures. To start, define the output tracking errors:

$$\begin{aligned} e_1(t) &= y_{d1}(t) - y_1(t) \\ \vdots \quad & \quad \vdots \\ e_p(t) &= y_{dp}(t) - y_p(t) \end{aligned} \tag{3.19}$$

Define also the corresponding filtered tracking errors as

$$\begin{aligned} s_1(t) &= \left(\frac{d}{dt} + \lambda_1\right)^{(r_1-1)} e_1(t),\ \lambda_1 > 0 \\ \vdots \quad & \quad \vdots \\ s_p(t) &= \left(\frac{d}{dt} + \lambda_p\right)^{(r_p-1)} e_p(t),\ \lambda_p > 0 \end{aligned} \tag{3.20}$$

From (3.20), $s_i(t),\ i=1,\ldots,p$ are linear differential equations whose solutions imply that the tracking errors $e_i(t),\ i=1,\ldots,p$ and their time derivatives up to order r_i-1 converge to zero [108]. Thus, the control objective becomes the design of a controller to keep $\tilde{\rho}=\rho^*-\rho$ close to zero. Moreover, if $|s_i(t)|\le\Phi_i$ where $\Phi_i,\ i=1,\ldots,p$ are positive constants, it follows that: $\left|e_i^{(j)}(t)\right|\le 2^j\lambda_i^{j-r_i+1}\Phi_i,\ j=0,\ldots,r_i-1,\ i=1,\ldots,p$, these bounds can be reduced by increasing the design parameters λ_i.

Now, taking the time derivatives of the filtered tracking errors, we obtain

$$\begin{aligned}\dot{s}_1 &= \chi_1-\sum\nolimits_{j=1}^{p}\bar{g}_{1j}(x)u_{0j}(t)\\ &\vdots \quad\quad \vdots\\ \dot{s}_p &= \chi_p-\sum\nolimits_{j=1}^{p}\bar{g}_{pj}(x)u_{0j}(t)\end{aligned} \tag{3.21}$$

where $\chi_1,\chi_2,\ldots,\chi_p$ are defined as follows

$$\begin{aligned}\chi_1 &= y_{d1}^{(r_1)}+\beta_{1,r_1-1}e_1^{(r_1-1)}+\ldots+\beta_{1,1}\dot{e}_1-f_1(x)-\bar{v}_1(x,t)\\ &\vdots \quad\quad\quad\quad \vdots\\ \chi_p &= y_{dp}^{(r_p)}+\beta_{p,r_p-1}e_p^{(r_p-1)}+\ldots+\beta_{p,1}\dot{e}_p-f_p(x)-\bar{v}_p(x,t)\end{aligned} \tag{3.22}$$

with $\beta_{i,j}=C_{r_i-1}^{j-1}\lambda_i^{r_i-j},\ C_{r_i-1}^{j-1}=(r_i-1)!/((r_i-j)!(j-1)!),\ j=1,\ldots,r_i-1,\ i=1,\ldots,p$, $f_i(x)$ and $\bar{g}_{ij}(x)$, $i,j=1,\ldots,p$ are the entries of $F(x)$ and $\bar{G}(x)$ respectively. Let us denote $s=\left[s_1,s_2,\ldots,s_p\right]^T$, $\chi=\left[\chi_1,\chi_2,\ldots,\chi_p\right]^T$, then (3.21) can be put into the following matrix form

$$\dot{s}=\chi-\bar{G}(x)u_0(t) \tag{3.23}$$

Given that the matrix $\bar{G}(x)$ is a regular matrix, we choose the following ideal control law [109]

$$u_0^*=\bar{G}^{-1}(x)\left(\chi+Ks+K_0\tanh\left(\frac{s}{\varepsilon_0}\right)\right) \tag{3.24}$$

where $K=diag\{k_1,\ldots,k_p\},\ K_0=diag\{k_{01},\ldots,k_{0p}\}$, with $k_i,\ k_{0i}>0,\ i=1,\ldots,p$, ε_0 is a small positive constant, $\tanh(.)$ stands for the hyperbolic tangent function defined for the vector s

as $\tanh(s/\varepsilon_0) = \left[\tanh(s_1/\varepsilon_0), \ldots, \tanh(s_p/\varepsilon_0)\right]^T$. In fact, by substituting the ideal control law (3.24) into (3.23) we obtain the following equation

$$\dot{s} = -Ks - K_0 \tanh\left(\frac{s}{\varepsilon_0}\right) \tag{3.25}$$

From (3.25) it can be concluded that $\lim_{t\to\infty} s_i(t) = 0$ and therefore $\lim_{t\to\infty} e_i^{(j)}(t) = 0$ for $j = 0, \ldots, r_i - 1,$ and $i = 1, \ldots, p$.

The control law (3.24) is not implementable if the nonlinear functions $F(x)$, $\bar{G}(x)$ and $\bar{v}(x,t)$ are not available. To overcome this difficulty, an adaptive design is used to construct the control law (3.24) online in the framework of direct adaptive control.

3.3.3 Adaptive controller structure

Similar to the single variable case investigated in the previous chapter, we assume that the ideal control law (3.24) can be approximated by a function approximator. Let us denote $u_0^* = \left[u_{01}^*, u_{02}^*, \ldots, u_{0p}^*\right]^T$ the entries of the ideal control law (3.24). Then, each entry u_{0i}^* of u_0^* can be approximated by a feedback controller defined as

$$u_{0i} = \Pi_{\mathbf{i}}^T(z)\theta_i,\ i = 1, 2, \ldots, p \tag{3.26}$$

where $\Pi_i(z_i)$ is the regressor vector, which can have different structures as early mentioned in Chapter 2, z_i is the input to the i^{th} regressor and θ_i is an adjustable parameter vector.

Assume also that there exist piecewise continuous time-varying parameters θ_i^*, $i = 1, 2, \ldots, p$ with possible jumps when one or more abrupt actuator failures occur and bounded time derivatives inside the interval of continuity, i.e. the time interval between two successive abrupt actuator failures such that u_{0i}^* fulfills

$$u_{0i}^* = \Pi_{\mathbf{i}}^T(z_i)\theta_i^* + \varepsilon_i(z_i),\ i = 1, 2, \ldots, p \tag{3.27}$$

Now, let us denote $\Pi(z) = diag\left\{\Pi_1(z_1), \Pi_2(z_2), \ldots, \Pi_p(z_p)\right\}$, $\theta = \left[\theta_1^{\mathrm{T}}, \theta_2^{\mathrm{T}}, \ldots, \theta_p^{\mathrm{T}}\right]^{\mathrm{T}}$, $\theta^* = \left[\theta_1^{*\mathrm{T}}, \theta_2^{*\mathrm{T}}, \ldots, \theta_p^{*\mathrm{T}}\right]^{\mathrm{T}}$ and $\varepsilon(z) = \left[\varepsilon_1(z_1), \varepsilon_2(z_2), \ldots, \varepsilon_p(z_p)\right]^T$. Then, (3.26) and (3.27) can be respectively written in the following matrix form

$$u_0^* = \Pi^T(z)\theta^* + \varepsilon(z) \tag{3.28}$$

$$u_0 = \Pi^T(z)\theta \tag{3.29}$$

Our task at this point is to derive a parameter update law for the parameter vector θ to ensure the control objectives. To this end, let us define the control prediction error vector as follows

$$e_{u_0} = u_0^* - \Pi^T(z)\theta = \Pi^T(z)\tilde{\theta} + \varepsilon(z) \tag{3.30}$$

with $\tilde{\theta} = \theta^* - \theta$. The control error vector e_{u_0} represents the discrepancy between the unknown functions u_0^* and the control inputs u_0. For further technical developments, the following assumption on the minimum approximation error vector $\varepsilon(z)$ is introduced.

Assumption 3.4: Throughout this chapter, it is assumed that for all compensable actuator failures, the approximation error vector $\varepsilon(z)$ in (3.28) can be bounded as follows

$$\varepsilon^T(z)\bar{G}(x)\varepsilon(z) \le \bar{\varepsilon}_0 s^T \bar{G}(x)s + \bar{\varepsilon}_1 \tag{3.31}$$

where $\bar{\varepsilon}_0$ and $\bar{\varepsilon}_1$ are positive constants.

Now, by adding and subtracting the term $\bar{G}(x)u_0^*$ to the right-hand side of (3.24) and using (3.30) we obtain the following expression

$$\dot{s} = -Ks - K_0 \tanh\left(\frac{s}{\varepsilon_0}\right) + \bar{G}(x)e_{u_0} \tag{3.32}$$

In order to derive parameter update law for the vector θ, we consider a quadratic cost function of the control error vector e_{u_0} defined as follows

$$J(\theta) = \frac{1}{2}e_{u_0}^T\bar{G}(x)e_{u_0} = \frac{1}{2}\left(u_0^* - \Pi^T(z)\theta\right)^T \bar{G}(x)\left(u_0^* - \Pi^T(z)\theta\right) \tag{3.33}$$

Based on gradient descent method, the update law for θ that minimizes $J(\theta)$ is given by

$$\dot{\theta} = -\eta\nabla_\theta J(\theta) \tag{3.34}$$

where η is a positive adaptation gain.

From (3.33), we have $\nabla_\theta J(\theta) = -\Pi(z)\bar{G}(x)e_{u_0}$, therefore (3.34) can be written as

$$\dot{\theta} = \eta \Pi(z) \bar{G}(x) e_{u_0} \tag{3.35}$$

In (3.35) the term $\bar{G}(x)e_{u_0}$ is not at our disposal. However, from (3.32) we can write

$$\bar{G}(x) e_{u_0} = \dot{s} + Ks + K_0 \tanh\left(\frac{s}{\varepsilon_0}\right) \tag{3.36}$$

By substituting (3.36) into (3.35) and introducing a σ-modification to ensure the boundedness of the parameters vector $\tilde{\theta}$ and to improve the robustness of the adaptive laws, the parameter update law for θ is given by

$$\dot{\theta} = \eta \Pi(z) \left(\dot{s} + Ks + K_0 \tanh\left(\frac{s}{\varepsilon_0}\right)\right) - \eta\sigma\theta \tag{3.37}$$

where σ is a small positive constant. Note that the adaptive law (3.36) is modified so that the time derivative of the Lyapunov function used for stability analysis becomes negative in the space of the estimated parameter when these parameters exceed certain bound [121].

3.3.4 Stability and tracking analysis

Suppose that one or more abrupt actuators failures occur at time instants t_i, $i = 1,..., N$, and inside the time intervals $[t_i, t_{i+1})$ there are no abrupt failures occurring. For stability and tracking error convergence analysis, let us consider Lyapunov-like functions V_i defined over the intervals $[t_i, t_{i+1})$, $i = 1,\ldots, N$ with fixed failure pattern as follows

$$V_i = \frac{1}{2} s^T s + \frac{1}{2\eta} \tilde{\theta}^{\mathrm{T}} \tilde{\theta} \tag{3.38}$$

The time derivative of V_i over the time intervals $t \in [t_i, t_{i+1})$ is given as

$$\dot{V}_i = s^T \dot{s} - \frac{1}{\eta} \tilde{\theta}^{\mathrm{T}} \dot{\theta} + \frac{1}{\eta} \tilde{\theta}^{\mathrm{T}} \dot{\theta}^* \tag{3.39}$$

Using (3.30), (3.32) and (3.37), equation (3.39) can be rewritten as follows

$$\dot{V}_i = s^T \left(-Ks - K_0 \tanh\left(\frac{s}{\varepsilon_0}\right) + \bar{G}(x) e_{u_0} \right) - \tilde{\theta}^{\mathrm{T}} \left(\Pi(z) \bar{G}(x) e_{u_0} - \sigma\theta \right) + \frac{1}{\eta} \tilde{\theta}^{\mathrm{T}} \dot{\theta}^* \tag{3.40}$$

Substituting (3.30) into (3.40), we obtain

$$\dot{V}_i = -s^T K s - s^T K_0 \tanh\left(\frac{s}{\varepsilon_0}\right) + s^T \bar{G}(x) e_{u_0} - \left(e_{u_0} - \varepsilon(z)\right)^T \bar{G}(x) e_{u_0} + \sigma\tilde{\theta}^{\mathrm{T}}\theta + \frac{1}{\eta}\tilde{\theta}^{\mathrm{T}}\dot{\theta}^* \quad (3.41)$$

Equation (3.41) can be further expanded as follows

$$\dot{V}_i = -s^T K s - s^T K_0 \tanh\left(\frac{s}{\varepsilon_0}\right) + s^T \bar{G}(x) e_{u_0} - e_{u_0}^T \bar{G}(x) e_{u_0} + \varepsilon^{\mathrm{T}}(z) \bar{G}(x) e_{u_0} + \sigma\tilde{\theta}^{\mathrm{T}}\theta + \frac{1}{\eta}\tilde{\theta}^{\mathrm{T}}\dot{\theta}^*$$

(3.42)

Now, we introduce the following inequalities

$$\sigma\tilde{\theta}^{\mathrm{T}}\theta \le -\frac{\sigma}{2}\left\|\tilde{\theta}\right\|^2 + \frac{\sigma}{2}\left\|\theta^*\right\|^2 \quad (3.43)$$

$$\frac{1}{\eta}\tilde{\theta}^{\mathrm{T}}\dot{\theta}^* \le \frac{\sigma}{4}\left\|\tilde{\theta}\right\|^2 + \frac{1}{\sigma\eta^2}\left\|\dot{\theta}^*\right\|^2 \quad (3.44)$$

$$\varepsilon^T(z)\bar{G}(x)e_{u_0} \le \frac{1}{4}e_{u_0}^T \bar{G}(x) e_{u_0} + \varepsilon^T(z)\bar{G}(x)\varepsilon(z) \quad (3.45)$$

$$s^T\bar{G}(x)e_{u_0} \le \frac{1}{4}e_{u_0}^T \bar{G}(x) e_{u_0} + s^T\bar{G}(x)s \quad (3.46)$$

By invoking (3.31) and using (3.43), (3.44), (3.45) and (3.46), $\dot{V}_i$ can be bounded as follows

$$\dot{V}_i \le -\frac{1}{2}e_{u_0}^T \bar{G}(x) e_{u_0} - s^T\left(K - (\bar{\varepsilon}_0 + 1)\bar{G}(x)\right)s - \frac{\sigma}{4}\left\|\tilde{\theta}\right\|^2 + \frac{\sigma}{2}\left\|\theta^*\right\|^2 + \frac{1}{\sigma\eta^2}\left\|\dot{\theta}^*\right\|^2 + \bar{\varepsilon}_1 \quad (3.47)$$

Since θ^* and $\dot{\theta}^*$ are assumed bounded over the time interval $[t_i, t_{i+1})$ where no abrupt actuator failures occur, a positive bound ψ_i can be defined over each interval as follows

$$\psi_i = \sup_{t\in[t_i, t_{i+1})}\left(\frac{\sigma}{2}\left\|\theta^*\right\|^2 + \frac{1}{\sigma\eta^2}\left\|\dot{\theta}^*\right\|^2 + \bar{\varepsilon}_1\right) \quad (3.48)$$

Then (3.47) simplifies to the following

$$\dot{V}_i \le -\frac{1}{2}e_{u_0}^T \bar{G}(x) e_{u_0} - s^T\left(K - (1 + \bar{\varepsilon}_0)\bar{G}(x)\right)s - \frac{\sigma}{4}\left\|\tilde{\theta}\right\|^2 + \psi_i \quad (3.49)$$

By assuming that the free design parameters k_i, $i = 1, \ldots, p$ are chosen such that $k_i > \delta_1$, and define $\gamma = \min\left(2\lambda_{\min}\left(k_i - (1 + \bar{\varepsilon}_0)\right)\delta_1, 0.5\sigma\eta\right)$, where $\lambda_{\min}$ denotes the smallest eigenvalue of $\bar{G}(x)$, then (3.49) can be further simplified as

$$\dot{V}_i \le -\frac{1}{2}e_{u_0}^T \bar{G}(x) e_{u_0} - \frac{\gamma}{2}s^T s - \frac{\gamma}{2\eta}\left\|\tilde{\theta}\right\|^2 + \psi_i \tag{3.50}$$

From the definition of V_i in (3.38), we can write the inequality (3.50) as follows

$$\dot{V}_i \le -\gamma V_i + \psi_i \tag{3.51}$$

From which the following theorem on the boundedness of closed-loop signals can be proved.

Theorem 3.1: For the system described by the equation (3.1) or (3.8) (interconnected system), and subject to actuator failures described by (3.4). Using the actuation scheme given in (3.14), the adaptive controller (3.29) with parameter update law (3.37) guarantees that the closed-loop system signals are uniformly ultimately bounded (UUB) and that the tracking errors converge to a small neighborhood of the origin.

Proof:

Equation (3.51) implies that over each time interval $\left[t_i, t_{i+1}\right)$ when there are no abrupt failures occurring, if $V_i \ge \psi_i/\gamma$ one has $\dot{V}_i < 0$. This means that the set $V_i \le \psi_i/\gamma$ is a positive invariant set. Now, by integrating (3.51) over a fixed pattern interval $t \in \left[t_i, t_{i+1}\right)$, we obtain

$$0 \le V_i(t) \le \frac{\psi_i}{\gamma} + \left(V\left(t_i^+\right) - \frac{\psi_i}{\gamma}\right)e^{-\gamma(t-t_i)} \tag{3.52}$$

Let us denote $V(t)$ the extension of V_i over the whole time domain. Due to abrupt actuator failures, the parameter estimation error $\tilde{\theta}$ and consequently $V(t)$ will exhibit finite jumps at each time instant $t_i,\ i = 1, 2, \ldots, N$ with $t_{N+1} = \infty$, i.e. there are no further actuator failures after t_N . Let us denote ΔV_i the jumps on $V(t)$ caused by jumps on $\tilde{\theta}$ at time instants $t_i,\ i = 1, 2, \ldots, N$. Let t_i^- and t_i^+ be the time instants just before and after the occurrence of the abrupt failure respectively. Hence, starting from t_1, one can write

$$V\left(t_1^+\right) = V\left(t_1^-\right) + \Delta V_1 \tag{3.53}$$

In addition, from (3.52), inside the time interval $\left[t_0, t_1\right)$ we have

$$0 \le V(t) \le \frac{\psi_0}{\gamma} + \left(V\left(t_0^+\right) - \frac{\psi_0}{\gamma}\right)e^{-\gamma(t-t_0)} \tag{3.54}$$

Now, at time instant $t = t_1^-$ one can write

$$V\left(t_1^-\right) \le \frac{\psi_0}{\gamma} + \left(V\left(t_0^+\right) - \frac{\psi_0}{\gamma}\right) e^{-\gamma(t_1 - t_0)} V\left(t_1^-\right) \le \frac{\psi_0}{\gamma} + \left(V\left(t_0^+\right) - \frac{\psi_0}{\gamma}\right) e^{-\gamma(t_1 - t_0)} \tag{3.55}$$

From (3.53), (3.54) and (3.55), it can be noticed that at each time interval where there are no parameter jumps caused by abrupt actuator failures, depending on $V\left(t_0^+\right)$, the speed of convergence γ, the length of the interval between abrupt failures $t \in [t_0, t_1)$ and on the value of the jump ΔV_1, $V\left(t_1^+\right)$ may reach inside or remains outside the region $V \le \psi_0/\gamma$, in either situation, it remains bounded given that the jumps are bounded.

Likewise, for time $t \in [t_i, t_{i+1})$, the jump of V is expressed as follows

$$V\left(t_{i+1}^+\right) = V\left(t_{i+1}^-\right) + \Delta V_{i+1} \tag{3.56}$$

Again, from (3.52), we can write

$$0 \le V(t) \le \frac{\psi_i}{\gamma} + \left(V\left(t_i^+\right) - \frac{\psi_i}{\gamma}\right) e^{-\gamma(t - t_i)} \tag{3.57}$$

At time instant $t = t_{i+1}^-$, one can write

$$V\left(t_{i+1}^-\right) \le \frac{\psi_i}{\gamma} + \left(V\left(t_i^+\right) - \frac{\psi_i}{\gamma}\right) e^{-\gamma(t_{i+1} - t_i)} \tag{3.58}$$

where the jumps ΔV_i, $i = 1, \dots, N$ are assumed bounded. Besides, it is natural to assume that there is a finite number of actuator failures that can occur during the course of operation.

From (3.56), (3.57) and (3.58), it can be noticed that at each interval $t \in [t_i, t_{i+1})$, depending on $V\left(t_i^+\right)$, the speed of convergence γ, the length of the interval between failures $t \in [t_i, t_{i+1})$ and on the value of the jump ΔV_{i+1}, $V\left(t_{i+1}^-\right)$ may reach inside the region $V \le \psi_i/\gamma$ or remains outside this region but in either case remains bounded since the jumps are bounded and the number of failures is finite.

Now, consider the interval $t \in [t_N, \infty)$ where there are no further failures. By integrating (3.51) over the time interval $t \in [t_N, \infty)$, we can write

$$0 \le V(t) \le \frac{\psi_N}{\gamma} + \left(V\left(t_N^+\right) - \frac{\psi_N}{\gamma} \right) e^{-\gamma(t-t_N)} \tag{3.59}$$

From this last inequality, it can be concluded that after t_N when there are no further failures occur, the filtered tracking error vector $s(t)$, the parameter estimation error $\tilde{\theta}(t)$ and consequently the control vector $u_0(t)$ will end up by being uniformly bounded. Besides, the filtered tracking error $s(t)$ will be uniformly ultimately bounded and converges to a residual set defined as $\left\{ s \in \mathbb{R}^p : \|s(t)\| \le \sqrt{2\psi_N/\gamma} \right\}$. Which implies that the tracking errors $e_i(t)$, $i = 1,..., p$ and their time derivatives $e_i^{(j)}(t)$, $j = 1, 2, \ldots, r_i$, all converge to residual sets defined as $\left|e_i^{(j)}(t)\right| \le 2^j \sqrt{2\psi_N / \gamma}$, $j = 1, 2, \ldots, r_i$, $i = 1, 2, \ldots, p$.

3.4 Simulation examples

In this section, two examples are considered, the first is a redundant joint robot manipulator whose model has the general case given in (3.1), and the second example is a flexible spacecraft with redundant reaction wheels whose dynamic model can be expressed as in (3.8).

3.4.1 Case 1: Robot manipulator with redundant joints

3.4.1.1 System description

In a redundant joint robot manipulator, the torque is provided by concurrently actuated joints. This reduces the loads on individual joints and provides some redundancy making the manipulator more resilient; if a joint fails, the control effort is redistributed among healthy joints. In practical situations, if a joint fails, it will be disengaged by a dedicated mechanism (clutch) in order to counteract the torque applied by healthy actuators, see Figure 3.1 [22].

The dynamic model of a concurrently actuated two-link robot manipulator system can be formulated as follows [22]

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \tag{3.60}$$

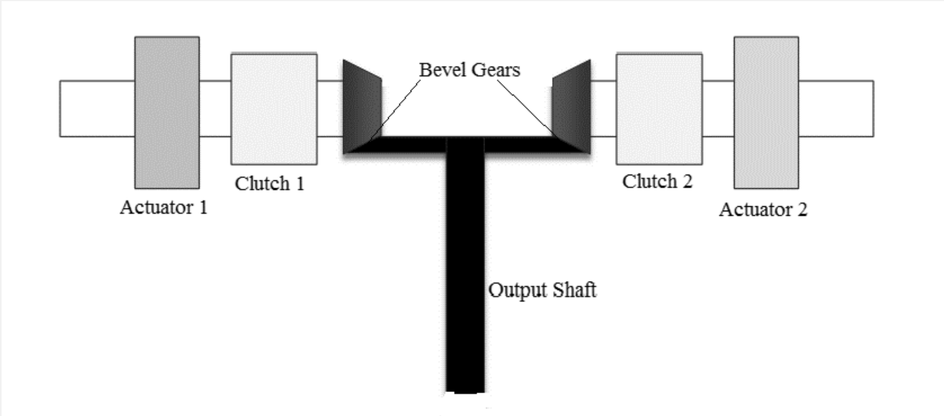

Figure 3.1: Example of a dual actuation system

where $q,\ \dot{q},\ \ddot{q} \in \mathbb{R}^{2\times1}$ are the joint angular position, velocity, and acceleration respectively. Besides $D(q) \in \mathbb{R}^{2\times2}$ is the inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{2\times2}$ is the Coriolis and centrifugal term, $g(q) \in \mathbb{R}^{2\times1}$ is the gravity term, $\tau = \mathbb{R}^{2\times1}$ is the torque vector. For a manipulator with concurrently actuated joints, at the i^{th} joint, $i = 1,2$, there are m_i actuators connected concurrently to provide the required torque. The number of concurrent actuators m_i can be different for each joint. For the i^{th} joint, the applied torque is given by

$$\tau_i = \tau_{i1} + \ldots + \tau_{im_i} \tag{3.61}$$

In the dual actuation case, the corresponding torque vector can be written as

$$\tau = \begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} \tau_{11} \\ \tau_{12} \\ \tau_{21} \\ \tau_{22} \end{pmatrix} \tag{3.62}$$

On the other hand, the dynamic model of an ordinary two-link manipulator (i.e. non-redundant actuation system), is given by [109]

$$\ddot{y} = F(x) + G_0(x)u_0 \tag{3.63}$$

where $x = [q_1, \dot{q}_1, q_2, \dot{q}_2]^T$, $y = [q_1, q_2]^T$, $u_0 = \tau = [\tau_1, \tau_2]^T$, $F(x) = -M^{-1}Q,\ G_0(x) = M^{-1}$, with $M = \begin{pmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \end{pmatrix},\ Q = \begin{pmatrix} -h\dot{q}_2 & -h(q_1 + q_2) \\ h\dot{q}_1 & 0 \end{pmatrix},\ M_{11} = a_1 + 2a_3\cos(q_2) + 2a_4\sin(q_2),\ M_{22} = a$,

$M_{12} = a_2 + a_3\cos(q_2) + a_4\sin(q_2)$ and $h = a_3\sin(q_2) - a_4\cos(q_2)$, with $a_3 = M_e l_1 l_{ce}\cos(\delta_e)$, $a_4 = M_e l_1 l_{ce}\sin(\delta_e)$, $a_2 = I_e + M_e l_{ce}^2$, $a_1 = I_1 + M_1 l_{ce}^2 + I_e + M_e l_{ce}^2 + M_e l_1^2$.

Regarding the torque expression in (3.62) and assuming that each link is actuated by two identical actuators, the dynamic model of the redundant two-link manipulator becomes

$$\ddot{y} = F(x) + G_0(x)Ru \tag{3.64}$$

where $u = [\tau_{11}, \tau_{12}, \tau_{21}, \tau_{22}]^T$ and $R = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$.

From (3.64), it is obvious that the control gain matrix satisfies assumption 3.2, i.e. $G(x) = G_0(x)R$. The actuation scheme is then chosen as

$$u = R^T u_\mathbf{0},\ u_\mathbf{0} = [\tau_{01}, \tau_{02}]^T \tag{3.65}$$

where τ_{01} and τ_{02} are the nominal torques that are to be designed.

3.4.1.2 Simulation results

A numerical simulation is carried out on a dual link manipulator with redundant actuation scheme. For the simulation, the numerical values of the robot manipulator are taken as in [109], the initial state vector is $x(0) = [0.5,\ 0,\ 0.25,\ 0]^T$. The desired trajectories are specified as $y_{d1}(t) = \sin(t)$ and $y_{d2}(t) = \cos(t)$. The free design parameters are selected as: $\lambda_1 = \lambda_2 = 1$, $K = diag\{1,1\}$, $K_0 = diag\{5,5\}$, $\varepsilon_0 = 0.01$, $\eta = 0.001$. The initial value of the parameters vector θ is taken zero. The regressor is based on a FLS with input vector $z = [e_1, \dot{e}_1, e_2, \dot{e}_2]$. The fuzzy logic system consists of three membership functions defined for each input as

$$\mu_{F_j^1}(z_j) = \exp\left(-\frac{1}{2}\left(\frac{z_j + 0.25}{0.6}\right)^2\right),\ \mu_{F_j^2}(z_j) = \exp\left(-\frac{1}{2}\left(\frac{z_j}{0.6}\right)^2\right),\ \mu_{F_j^3}(z_j) = \exp\left(-\frac{1}{2}\left(\frac{z_j + 0.25}{0.6}\right)^2\right)$$

The simulation is carried out for $30\ s$ and the following two failure scenarios are considered:

First failure scenario: In the first scenario, suppose that for the first link, the first actuator is intact, i.e. $\tau_{11}(t) = \tau_{11}(t)$, while at the time $t = 12\ s$, the second actuator has undergone a lock in place failure type, i.e. $\bar{\tau}_{12}(t) = 3.5$. For the second link, we suppose that the first actuator begins to lose effectiveness exponentially starting from the time $t = 25\ s$, i.e.

$\tau_{21}(t)=\rho_{21}(t)\tau_{21}(t),\ for\ 25\text{ s}<t\le 30\ s$ with $\rho_{21}(t)=e^{-(t-25)}$ while the second actuator gets locked in place at the time $t=8\ s$, then it recovers from the failure at the time $t=18\ s$. i.e. $\tau_{22}(t)=\bar{\tau}_{22}(t)=2.5,\ for\ 8\text{ s}<t\le 18\ s$. The simulation is carried out for $30\ s$. The simulation results for the first scenario are shown in Figure 3.2 (first link position and velocity), Figure 3.3 (second link position and velocity) and Figure 3.4 (provided torques). It can be seen that the proposed actuator failure compensation control scheme was able to meet the control objectives (stability and tracking) despite the presence of actuator failures. All signals are bounded; when an actuator fails the effort will be redistributed among healthy actuators and the tracking resumes after a transient regime.

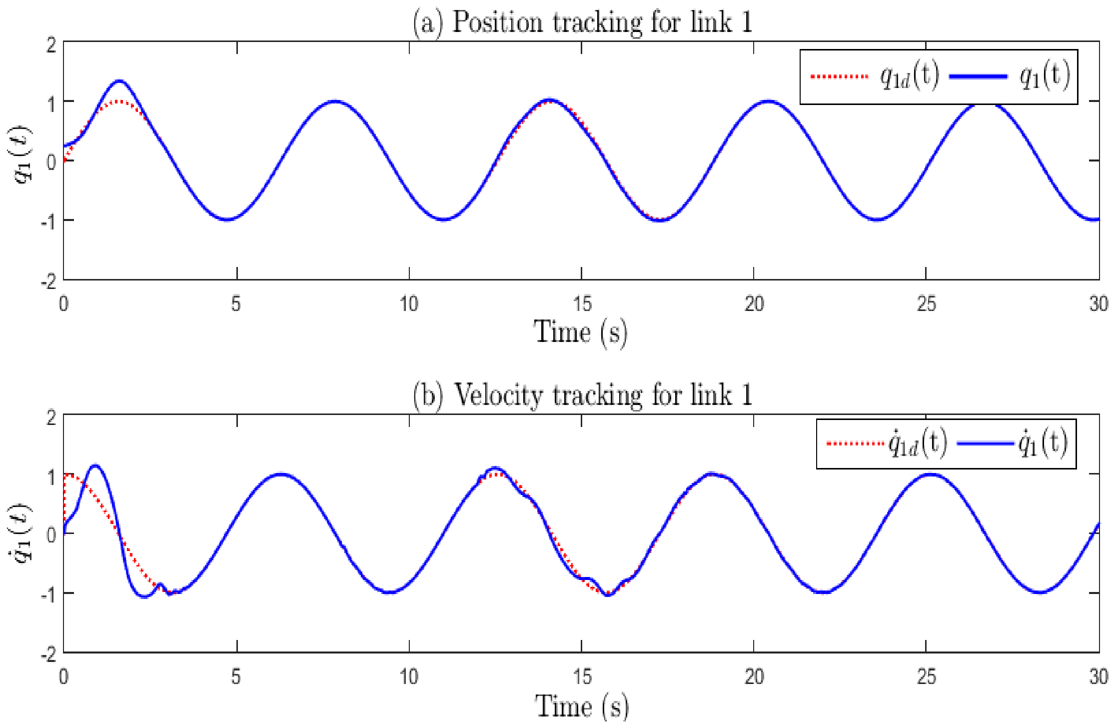

Figure 3.2: Tracking curves for link1 (scenario 1)

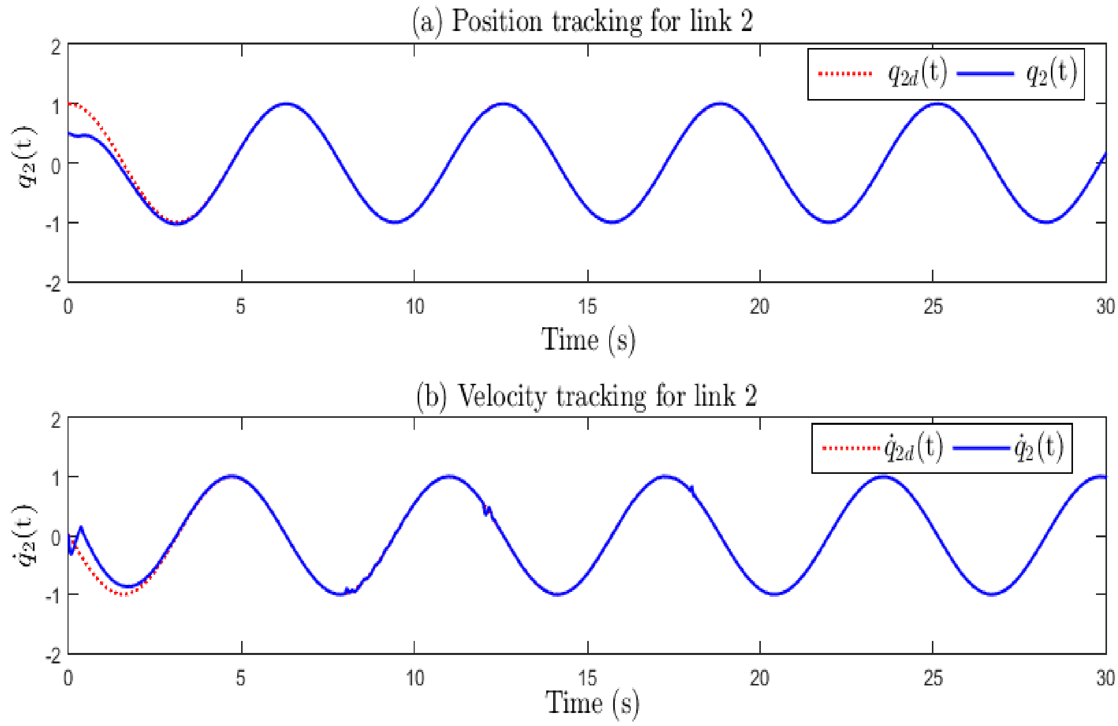

Figure 3.3: Tracking curves for link2 (scenario1)

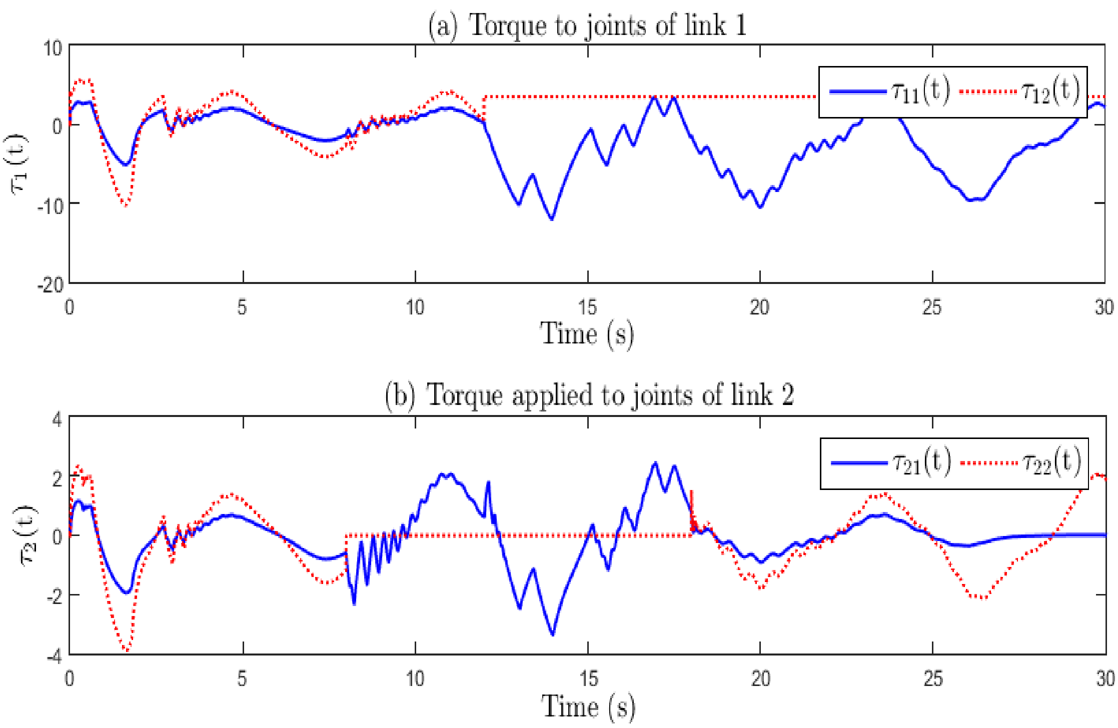

Figure 3.4: Applied torque curves (scenario 1)

Failure scenario 2: In this scenario, we assume that for the first link, the first actuator remains intact, while the second actuator has undergone failure with a time-varying pattern, at the time $t = 15\ s$, $\tau_{12}(t) = \overline{\tau}_{12}(t)$, *for* $15\ s < t \le 30\ s$ with $\overline{\tau}_{12} = 2.5 + 0.1\cos(2t) + 0.3\sin(2t)$. For the second link, we suppose that at the time $t = 20\ s$, the first actuator becomes only 80% effective, i.e. $\tau_{21}(t) = \rho_{21}(t)\tau_{21}(t)$, *for* $20\ s < t \le 30\ s$ with $\rho_{21}(t) = 0.8$ while the second actuator gets locked in place at the time $t = 10\ s$, i.e. $\tau_{22}(t) = \overline{\tau}_{22}(t)$, *for* $10\ s < t \le 30\ s$ with $\overline{\tau}_{22}(t) = 2.5$.

The simulation results are shown in Figure 3.5, Figure 3.6 and Figure 3.7, once again it can be noticed that the results confirm our claims, the proposed controller is effective in dealing with the actuator failures and at each failure, the control effort is redistributed among healthy actuators. Besides, if an actuator recovers from a failure, it will be automatically exploited by the controller to drive the link.

In summary, it can be concluded that the simulation results confirm the theoretical developments and claims presented in this chapter. The proposed adaptive actuator failure compensation control scheme (with fuzzy approximators) was effective in compensating joint fault for a redundant joint dual manipulator.

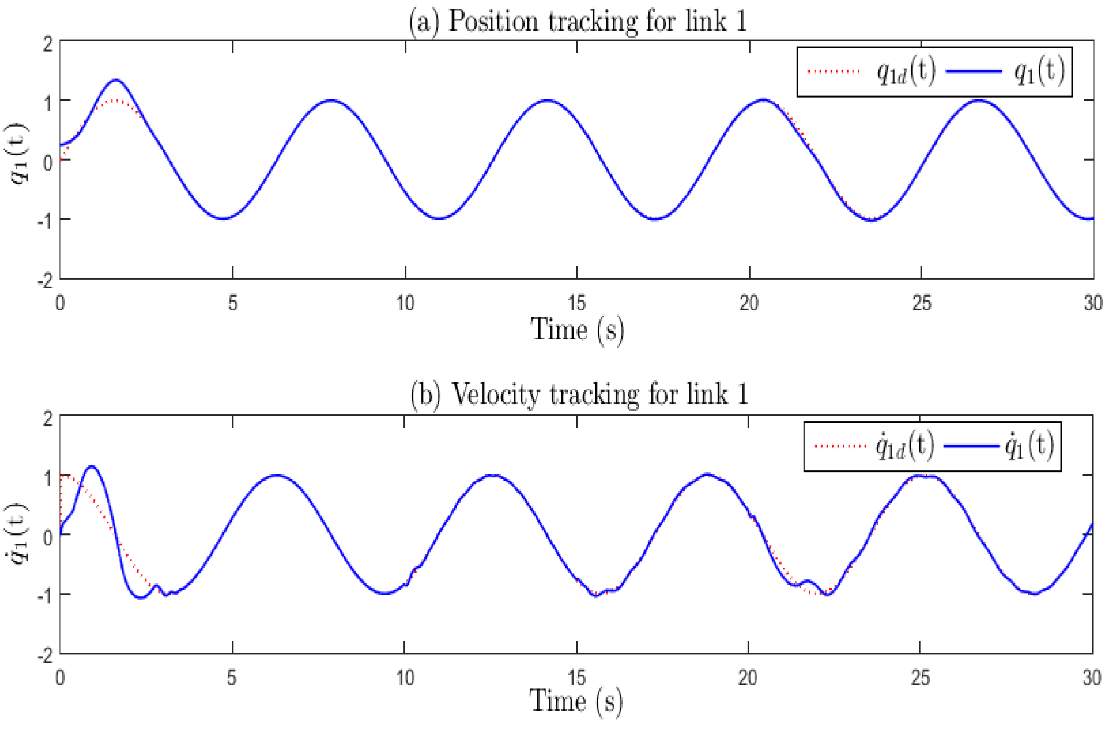

Figure 3.5: Position and velocity tracking for link1 (scenario 2)

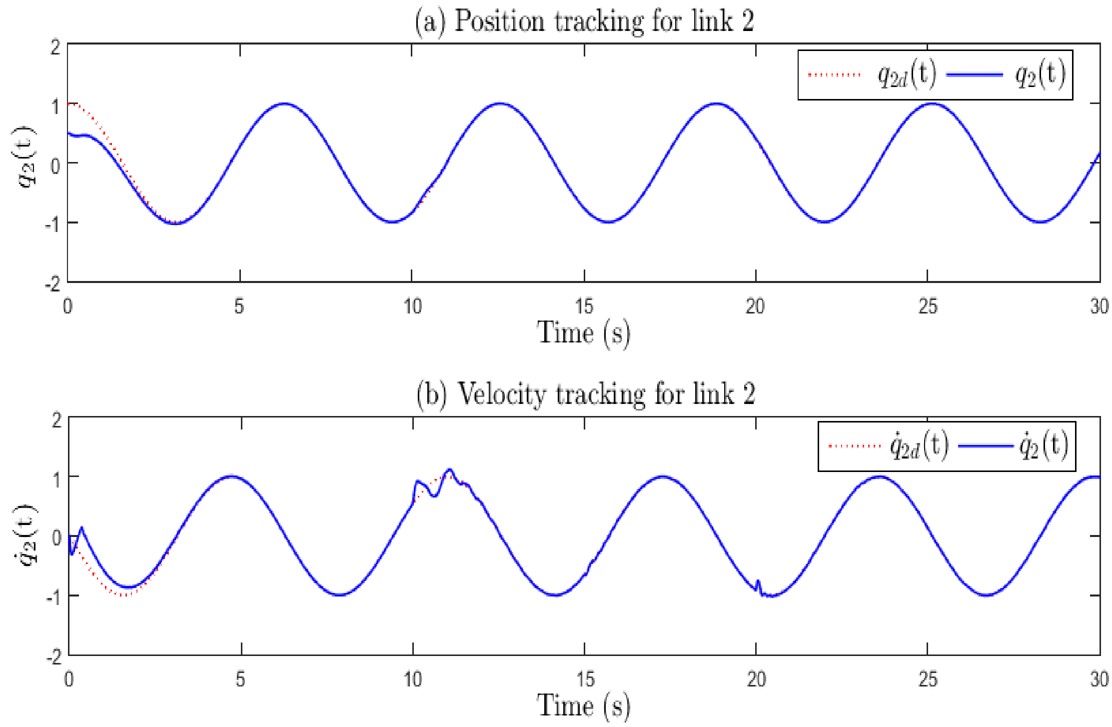

Figure 3.6: Position and velocity tracking for link2 (scenario 2)

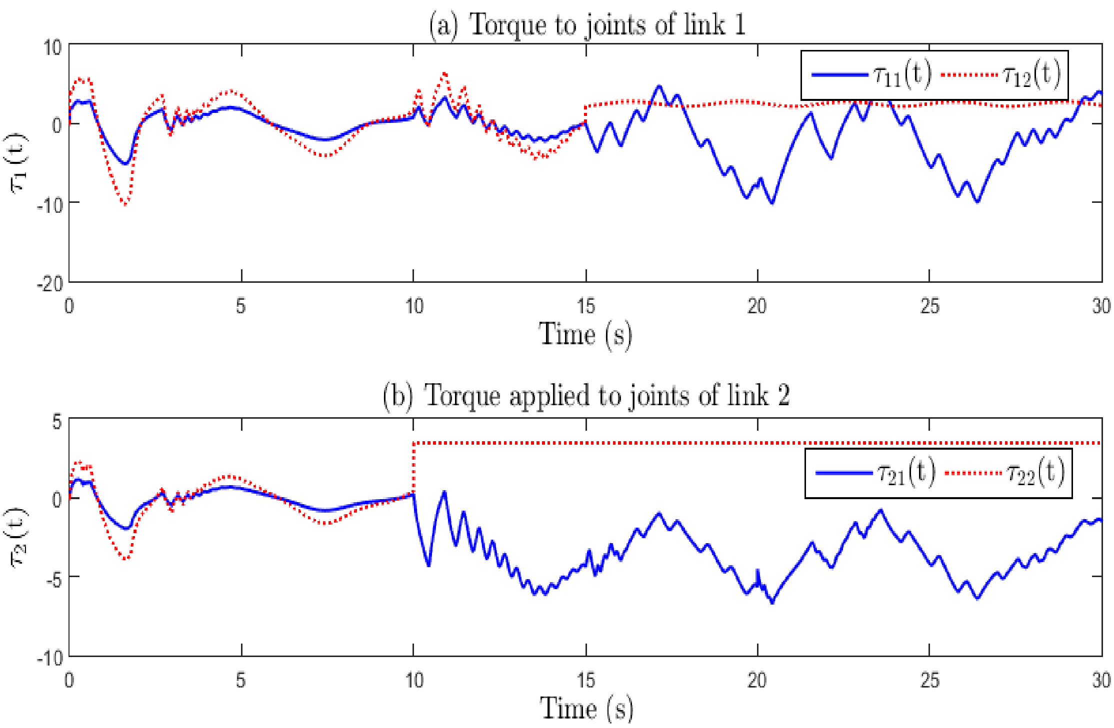

Figure 3.7: Applied torque curves (scenario 2)

3.4.2 Case 2: Flexible spacecraft system with redundant reaction wheels

In this subsection, the proposed adaptive actuator failure compensation control design is applied to an interconnected MIMO system which is the attitude model of a flexible spacecraft with redundant reaction wheels.

3.4.2.1 Kinematic equations

By considering the attitude control problem of spacecraft in steady operation and small Euler angle rotations, the kinematic equations of a flexible spacecraft with a solar array moving in a circular orbit can be approximated as [118], [122], [123]

$$\omega_1 = \dot{\phi} - \omega_0 \psi,\ \omega_2 = \dot{\theta} - \omega_0,\ \omega_3 = \dot{\psi} + \omega_0 \phi \tag{3.66}$$

where $\omega = [\omega_1, \omega_2, \omega_3]^T$ is the angular velocity of the spacecraft with respect to the inertial frame I, ω_0 denotes the orbital rate of the spacecraft, and $[\phi, \theta, \psi]^T$ represents the attitude orientation of the spacecraft in the body frame B with respect to orbital frame O obtained by roll-pitch-yaw rotations ϕ, θ and ψ are the roll, pitch and yaw angles respectively.

3.4.2.2 Dynamic equations of a spacecraft with redundant reaction wheels

The dynamics of a spacecraft with flexible solar array actuated by three reaction wheels distributed along the three axes are governed by the following equations [118], [122]

$$J\dot{\omega} + \omega^{\times}\left(J\omega + J_s\Omega_s\right) + \delta\ddot{\eta} = u + T_d \tag{3.67}$$

$$\ddot{\eta} + 2\xi\Lambda\dot{\eta} + \Lambda^2\eta + \delta^{\mathrm{T}}\dot{\omega} = 0 \tag{3.68}$$

where $\omega^{\times}$ is the skew symmetric matrix of the vector ω which is defined as follows

$$\omega^{\times} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{3.69}$$

The spacecraft inertia matrix is assumed diagonal with $J = diag\left(J_1, J_2, J_3\right)$, besides $J_s\Omega_s = [J_{s1}\Omega_{s1}, J_{s2}\Omega_{s2}, J_{s3}\Omega_{s3}]^T$ is the angular momentum of the reaction wheels, $u = -J_s\dot{\Omega}_s = [u_1, u_2, u_3]^T$ is the control torque applied to the spacecraft, $T_d = [T_{d1}, T_{d2}, T_{d3}]^T$

accounts for the external disturbances undergone by the spacecraft, $2\xi\Lambda = diag\left(2\xi_i\Lambda_i\right)$ and $\Lambda^2 = diag\left(\Lambda_i^2\right)$ are the damping matrices and stiffness matrices of the elastic modes with ξ and Λ are the modal damping and frequency respectively ($i = 1,...,N$, where N is the number of elastic modes considered), $\delta \in \mathbb{R}^{3\times N}$ is the coupling matrix between the elastic structures and rigid body of the spacecraft, and $\eta \in \mathbb{R}^N$ denotes the modal coordinate vector.

Remark 3.5: Recall that the external disturbance torques include the gravity gradient torque, the magnetic disturbance torque, the aerodynamic torque, the solar radiation torque, internal disturbance torque and other environmental torques that can affect the spacecraft during the whole orbital operations [118].

Now, assume that the spacecraft is actuated by redundant reaction wheels distributed according to a redundancy distribution matrix $R \in \mathbb{R}^{3\times m}$, where $m > 3$ denotes the total number of reaction wheels [79], [87], [124], then equation (3.67) can be rewritten as follows

$$J\dot{\omega} + \omega^{\times}\left(J\omega + RJ_s\Omega_s\right) + \delta\ddot{\eta} = Ru + T_d \tag{3.70}$$

where the new angular momentum of the redundant reaction wheels is $J_s\Omega_s = \left[J_{s1}\Omega_{s1},...,J_{sm}\Omega_{sm}\right]^T$, and the corresponding control vector is $u = -J_s\dot{\Omega}_s = \left[u_1,...,u_m\right]^T$.

Remark 3.6: Spacecraft systems can have 3, 4 or 6-reaction wheels configuration [125]. For the purpose of our study, we consider a 6-reaction wheels configuration where the reaction wheels are distributed as shown in Figure 3.8 with the distribution matrix $R \in \mathbb{R}^{3\times 6}$.

Remark 3.7: Obviously, the matrix $R \in \mathbb{R}^{3\times m}$ is known a priori and it can be made full rank by properly placing the actuators at a certain location and direction for the given spacecraft.

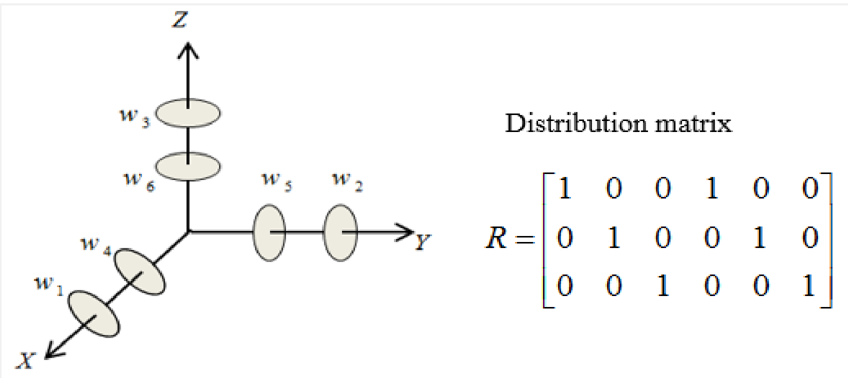

Figure 3.8: Redundant reaction wheels distribution

Define the state vector $x=\left[\phi \;\; \dot{\phi} \;\; \theta \;\; \dot{\theta} \;\; \psi \;\; \dot{\psi}\right]^T=\left[x_1 \;\; x_2 \;\; x_3 \;\; x_4 \;\; x_5 \;\; x_6\right]^T$. Then, under the conditions of small displacements, equations of motion for the flexible spacecraft given in (3.68), (3.69) and (3.70) can be decoupled into the following three subsystems [118], [124].

Roll subsystem:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1(x)+g_1(x)u_1+g_1(x)u_4+d_1 \end{cases} \tag{3.71}$$

with $g_1(x)=1/J_1$ and $f_1(x)=J_2-J_3/J_1\left(x_4x_6+\omega_0x_1x_4-\omega_0x_6-\omega_0^2x_1\right)+\omega_0x_6$, u_1 and u_4 are respectively the control torques provided by the first and fourth reaction wheels $d_1=-\sum_{i=1}^{N}\delta_{1i}\ddot{\eta}_i+T_{d1}/J_1+\left(\left(J_{s2}\Omega_{s2}+J_{s5}\Omega_{s5}\right)/J_1\right)\left(x_6+\omega_0x_1\right)-\left(\left(J_{s3}\Omega_{s3}+J_{s6}\Omega_{s6}\right)/J_1\right)\left(x_4-\omega_0\right)$ is the disturbance for the roll dynamics.

Pitch subsystem:

$$\begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2(x)+g_2(x)u_2+g_2(x)u_5+d_2 \end{cases} \tag{3.72}$$

with $g_2(x)=1/J_2$ and $f_2(x)=J_3-J_1/J_2\left(x_2x_6+\omega_0x_1x_2-\omega_0x_5x_6-\omega_0^2x_1x_5\right)$, u_2 and u_5 denote the control torques provided by the second and fifth wheels. the disturbance to the pitch is

$$d_2=-\sum_{i=1}^{N}\delta_{2i}\ddot{\eta}_i+T_{d2}/J_2+\left(\left(J_{s1}\Omega_{s1}+J_{s4}\Omega_{s4}\right)/J_2\right)\left(x_6+\omega_0x_1\right)+\left(\left(J_{s3}\Omega_{s3}+J_{s6}\Omega_{s6}\right)/J_2\right)\left(x_2-\omega_0x_5\right)$$

Yaw subsystem:

$$\begin{cases} \dot{x}_5 = x_6 \\ \dot{x}_6 = f_3(x)+g_3(x)u_3+g_3(x)u_6+d_3 \end{cases} \tag{3.73}$$

with $g_3(x)=1/J_3$ and $f_3(x)=J_1-J_2/J_3\left(x_2x_4-\omega_0x_4x_5-\omega_0x_2-\omega_0^2x_5\right)-\omega_0x_2$, u_3 and u_6 denote the control torques provided by the third and the sixth wheels. $d_3=-\sum_{i=1}^{N}\delta_{3i}\ddot{\eta}_i+T_{d3}/J_3+\left(\left(J_{s3}\Omega_{s3}+J_{s6}\Omega_{s6}\right)/J_3\right)\left(x_4-\omega_0\right)-\left(\left(J_{s2}\Omega_{s2}+J_{s5}\Omega_{s5}\right)/J_3\right)\left(x_2-\omega_0x_5\right)$ is the disturbance to the yaw subsystem.

The roll, pitch and yaw equations (3.71), (3.72) and (3.73) can be grouped into the following three interconnected subsystems Σ_{roll}, Σ_{pitch}, Σ_{yaw} which can be put into the following form

$$\Sigma_i : \begin{cases} \dot{x}_{2i-1} = x_{2i} \\ \dot{x}_{i2} = f_i(x)+\sum_{j=1}^{2}g_{ij}(x)u_{ij}+d_i(t), \; i=1,2,3 \\ y_i = x_{2i-1} \end{cases} \tag{3.74}$$

The state space model of the spacecraft system can be further put into the following form

$$y^{(2)} = F(x) + G(x)u + D(t) \tag{3.75}$$

with $y = \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix}^T$, $F(x) = \begin{bmatrix} f_1(x) & f_2(x) & f_3(x) \end{bmatrix}^T$, $G(x) = G_0(x)R$, $G_0(x) = diag\{g_1(x) \quad g_2(x) \quad g_3(x)\}$ and $D(t) = \begin{bmatrix} d_1(t) & d_2(t) & d_3(t) \end{bmatrix}^T$.

Reaction wheels are sensitive devices that are vulnerable to different sources of faults: *(i)* failure to respond to the issued control signals; *(ii)* decreased reaction torque or partial loss of effectiveness; *(iii)* increased bias torque; and *(iv)* continuous generation of reaction torque [118]. Here, we assume that these failures can be roughly described by the model (3.4).

3.4.2.3 Simulation results

For the spacecraft parameters, the nominal inertia and coupling matrices are given as in [107]

$$J = \begin{pmatrix} 973.4 & 0 & 0 \\ 0 & 354.8 & 0 \\ 0 & 0 & 808.5 \end{pmatrix} \text{kg.m}^2,\ \delta = \begin{pmatrix} 1 & 0.1 & 0.1 \\ 0.5 & 0.1 & 0.01 \\ -1 & 0.3 & 0.01 \end{pmatrix} \text{kg}^{1/2}.\text{m} \tag{3.76}$$

Assume also that there are three elastic modes at $\Lambda = diag\{0.602\pi, 1.088\pi, 1.846\pi\}$ rad/s, with damping coefficients $\xi_1 = \xi_2 = \xi_3 = 0.01$. The orbital rate is taken $\omega_0 = 0.0011$ rad/s.

The inertia matrix entries are assumed unknown and may change during the operation, but they remain strictly positive.

The desired trajectories for the roll pitch and yaw angles are specified respectively as follows:

$\phi_d(t) = 0.1\sin(0.1\pi t)$, $\theta_d(t) = -0.05 + 0.05\sin(0.1\pi t)$, $\psi_d(t) = 0.1\sin(0.1\pi - \pi/4)$.

The spacecraft is subjected to general time-varying disturbances, which have the same order of magnitude with the actual environmental disturbances; since the gravitation and aerodynamics drag are associated with the angular velocities ω_1, ω_2 and ω_3. The external disturbance torque is modeled as in [123] with

$$T_d = 10^{-3} \begin{bmatrix} -3 + 3\cos(0.01t) - 4.5\sin(0.02t) + 3\omega_1\cos(0.015t) \\ 6 + 4.5\sin(0.01t) - 6\cos(0.02t) - 6\omega_2\sin(0.015t) \\ -6 + 6\sin(0.01t) + 4.5\cos(0.02t) - 3\omega_3\sin(0.015t) \end{bmatrix} \text{N/m} \tag{77}$$

The initial values of the angular velocities are taken as $\omega_1(0)=0$, $\omega_2(0)=0$ and $\omega_3(0)=0.15/2\pi$ rad/s. Therefore, the corresponding initial state vector can be computed as follows: $x(0)=\begin{bmatrix}0 & 0 & 0 & 0.0011 & 0 & 0.0239\end{bmatrix}^T$. The initial values of the controller parameters $\theta(0)$ are set equal to 0.5. The free design parameters are selected as: $\eta=20,\ \varepsilon_0=0.001,\ \sigma=0.1,\ \lambda_1=\lambda_2=\lambda_3=0.5,\ K=diag\{5 \quad 5 \quad 5\}$ and $K_0=2diag\{2 \quad 10 \quad 2\}$ The regressors are chosen as $\Pi_i(z_i)=\left[1, s_i, x^T\right]$. For the simulation, the following two failure scenarios are considered:

First failure scenario: Suppose that at the time $t=90\ s$, the first reaction wheel is only 50% effective, at the time $t=50\ s$, the second reaction wheel becomes only 75% effective, the third reaction wheel is 100% effective but at the time $t=20\ s$, a bias torque adds to the reaction wheel such that $u_3^f=u_3+2$. At the time $t=70\ s$ the fourth reaction wheel issue an oscillating torque $u_4^f=\overline{u}_4=4+\cos(3t)-2\sin(3t)$, at the time $t=60\ s$, the fifth reaction wheel starts providing a constant torque $u_5^f=\overline{u}_5=7$. The sixth reaction wheel undergoes a loss of effectiveness and a negative bias torque such that $u_6^f=0.5u_6-7$. The simulation results are shown in Figure 3.9, Figure 3.10, Figure 3.11, Figure 3.12 and Figure 3.13. It can be seen that the proposed controller ensures stability and tracking despite the presence of actuator failures, it can be also seen that the modal displacements decay towards the origin. Which shows the effectiveness of the proposed adaptive actuator failure compensation controller.

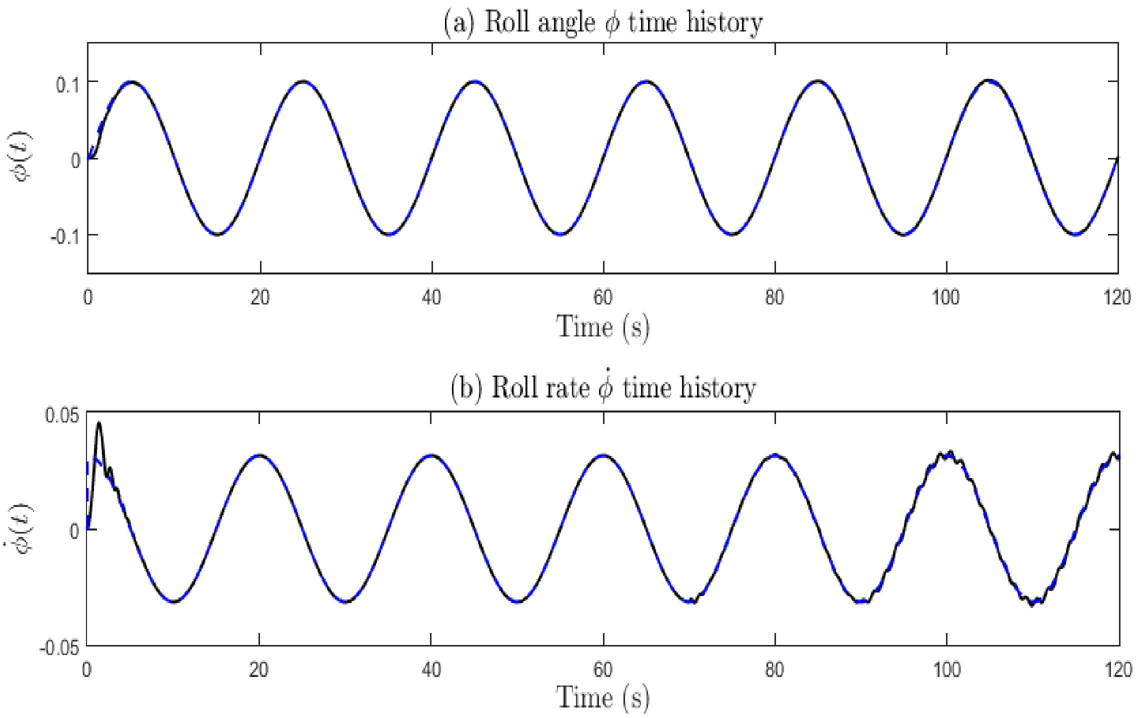

Figure 3.9: Roll angle and rate time evolution (scenario 1)

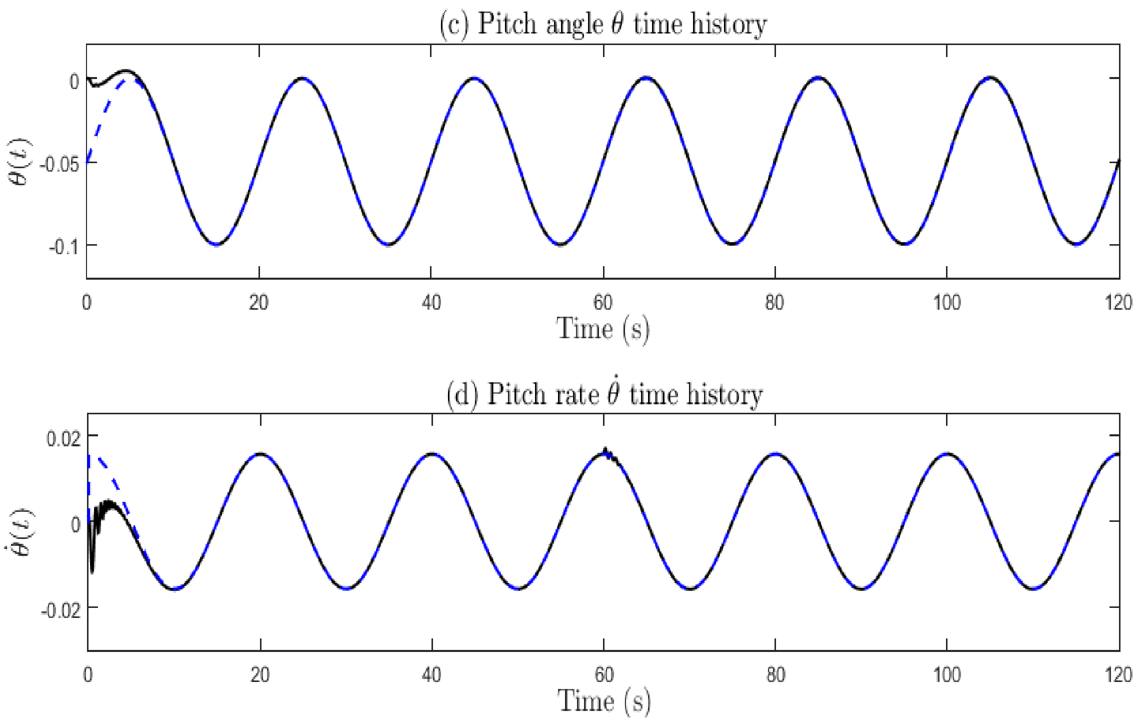

Figure 3.10: Pitch angle and rate time evolution (scenario 1)

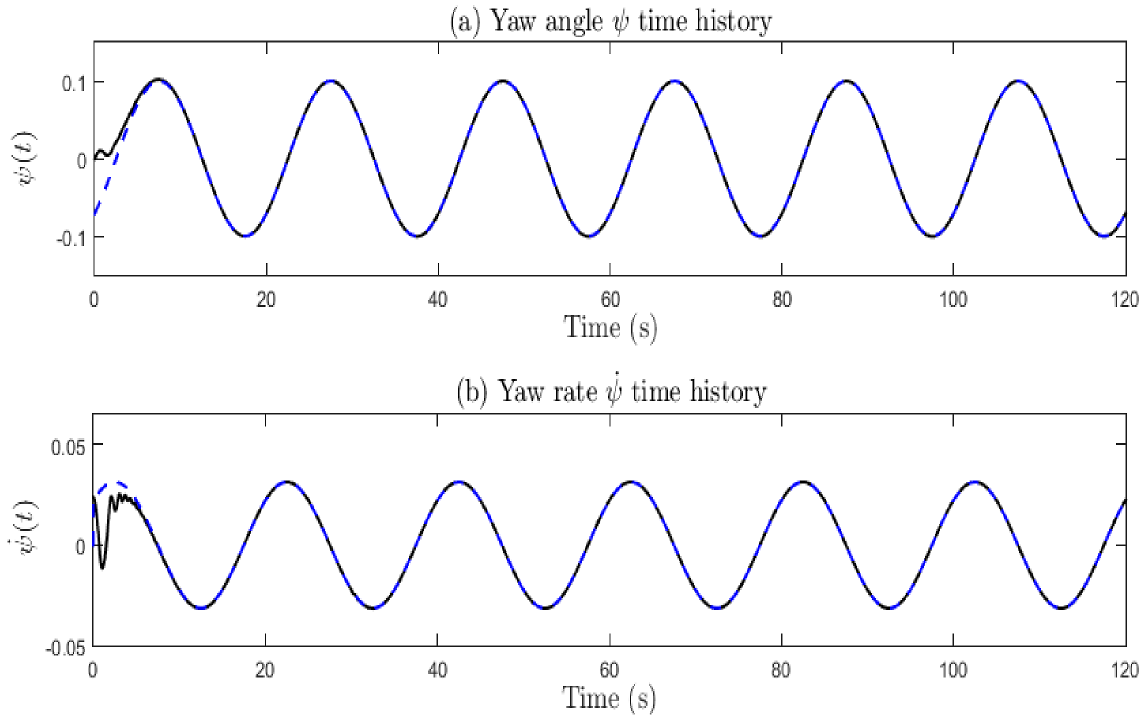

Figure 3.11: Yaw angle and rate time evolution (scenario 1)

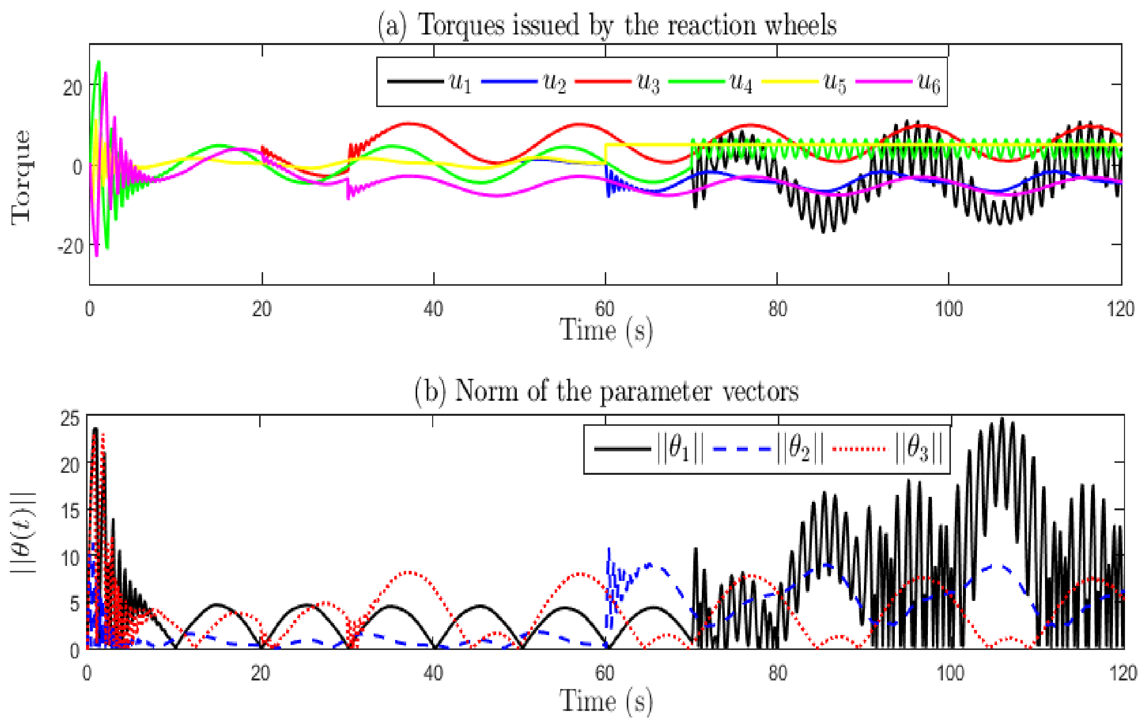

Figure 3.12: Control signals and controller parameters (scenario 1)

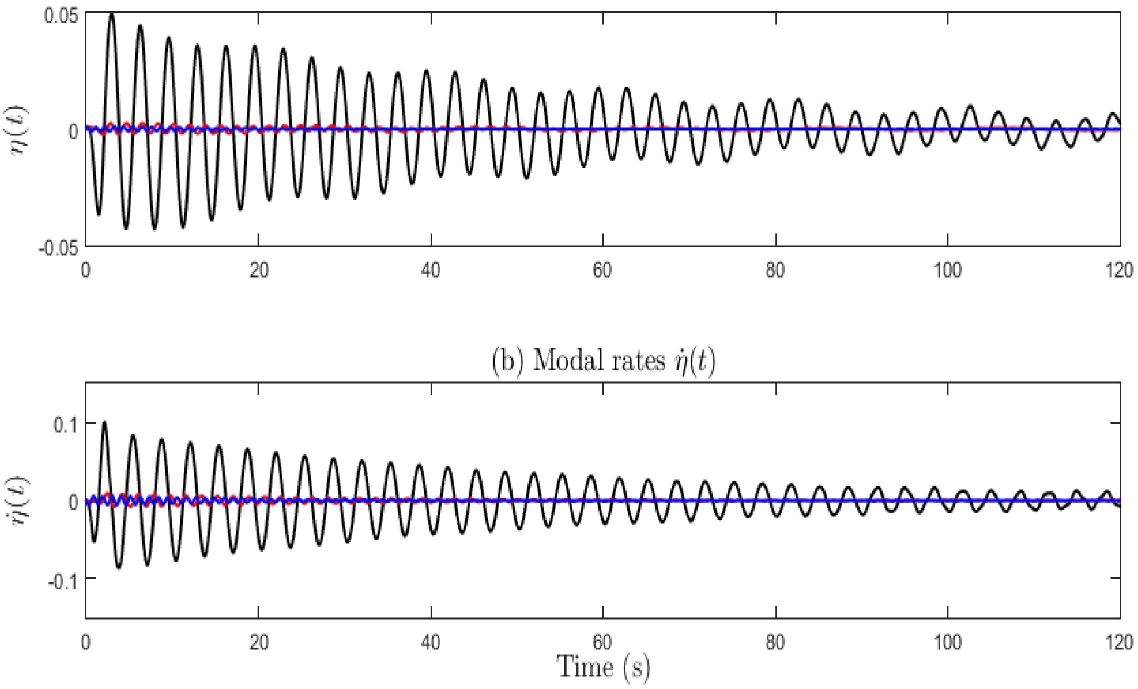

Figure 3.13: Modal displacements and rates time evolution (scenario 1)

Second failure scenario: In the second situation, we consider the presence of more general incipient, abrupt, time-varying, and state-dependent actuator failures. Thus we assume that at the time $t = 20\ s$, the first reaction wheel fails according to the pattern $u_1^f = 0.6u_1 + 8$, at the time $t = 80\ s$, the second reaction wheel undergoes an abrupt loss of effectiveness and becomes only 40% effective. Starting from $t = 20\ s$, the third reaction wheel starts losing effectiveness smoothly according to the pattern $u_3^f = \rho_3(t)u_3$, with $\rho_3(t) = \left(3 + \tanh\left(-t + 50\right)/10\right)/4$, so that after some elapsed time, the effectiveness settles at 50%. Suppose also that, at the time $t = 60\ s$, the fourth reaction wheel undergoes a time-varying and state-dependent bias fault defined as $u_4^f = (4 + x_3)\cos(2t) - (2 + x_4)\sin(2t)$. At the time $t = 100\ s$, the fifth reaction wheel fails according to the pattern $u_5^f = 5 + 0.1x_4\cos(3t)$ and at the time $t = 30\ s$, the sixth reaction wheel fails according to the pattern $u_6^f = x_5 + x_6 - 7$. The simulation results are shown in Figure 3.14, Figure 3.15, Figure 3.16, Figure 3.17 and Figure 3.18. It can be seen that again, in the presence of failures the proposed controller deal effectively with those failures, the tracking requirements are kept with graceful degrading despite this worst failure scenario. The simulation results indeed confirm the effectiveness of the proposed actuator failure compensation control design and confirm our theoretical claims.

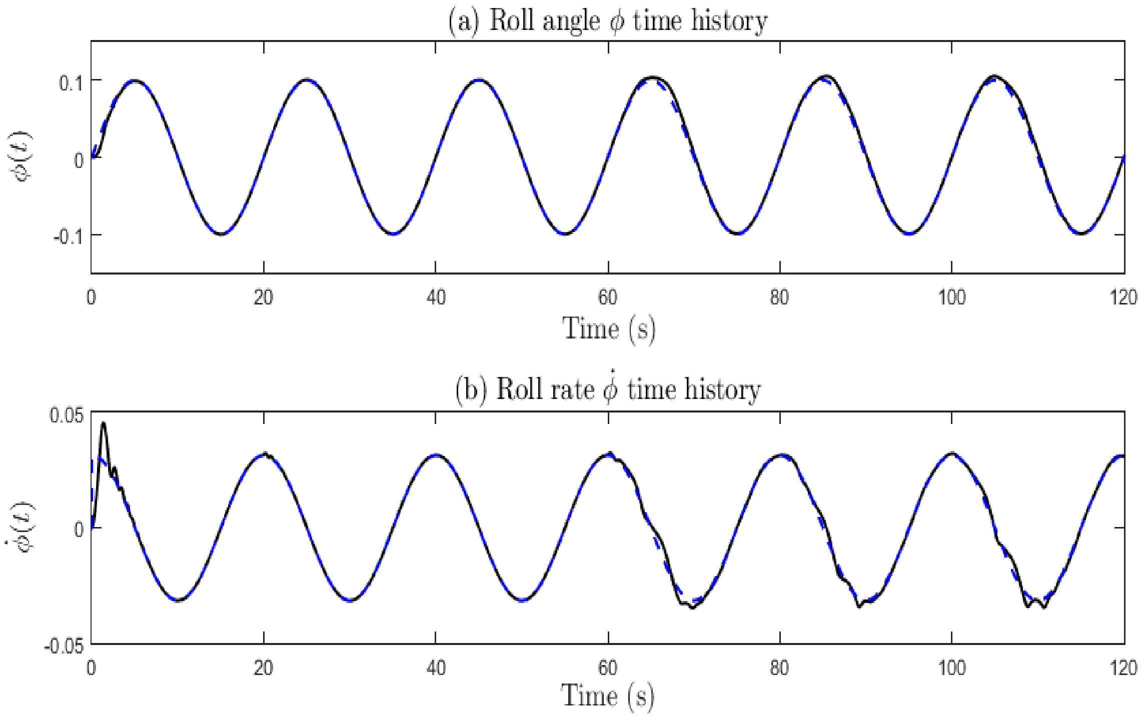

Figure 3.14: Roll angle and rate time evolution (scenario 2)

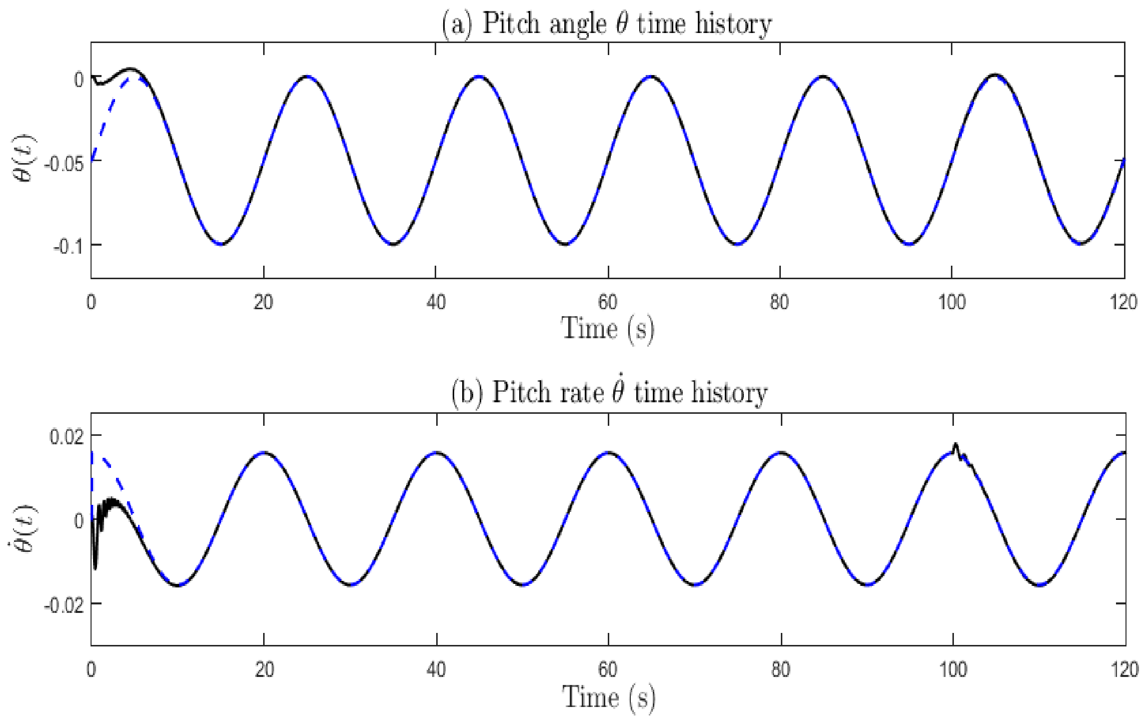

Figure 3.15: Pitch angle and rate time evolution (scenario 2)

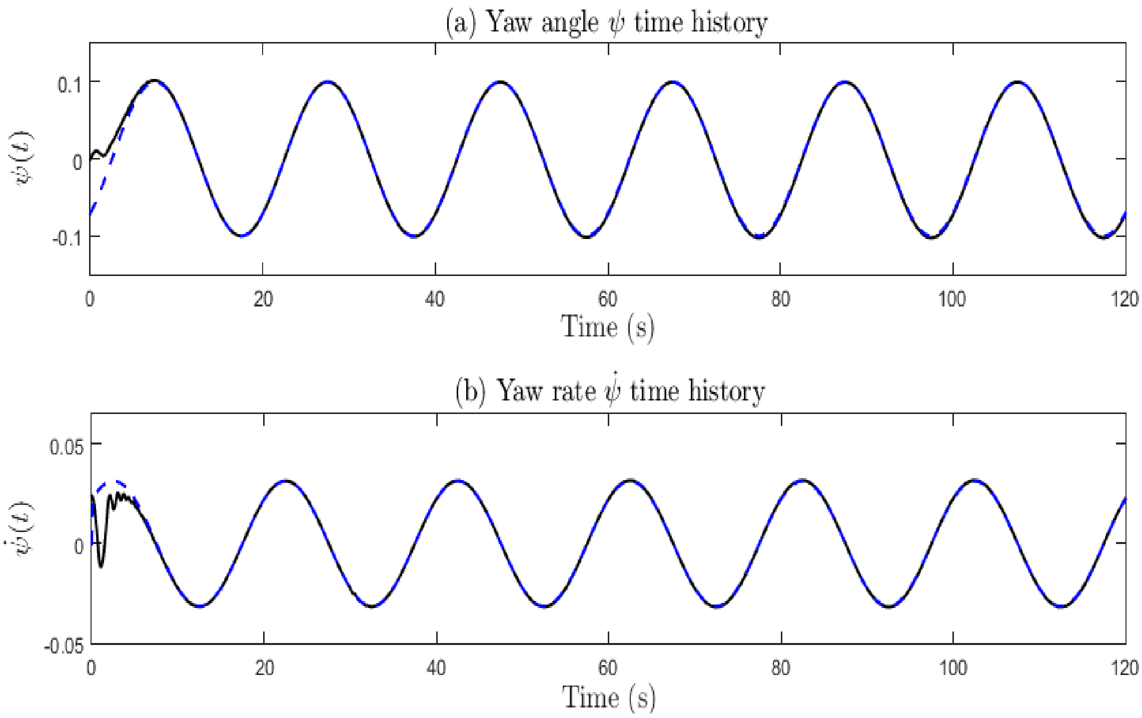

Figure 3.16: Yaw angle and rate time evolution (scenario 2)

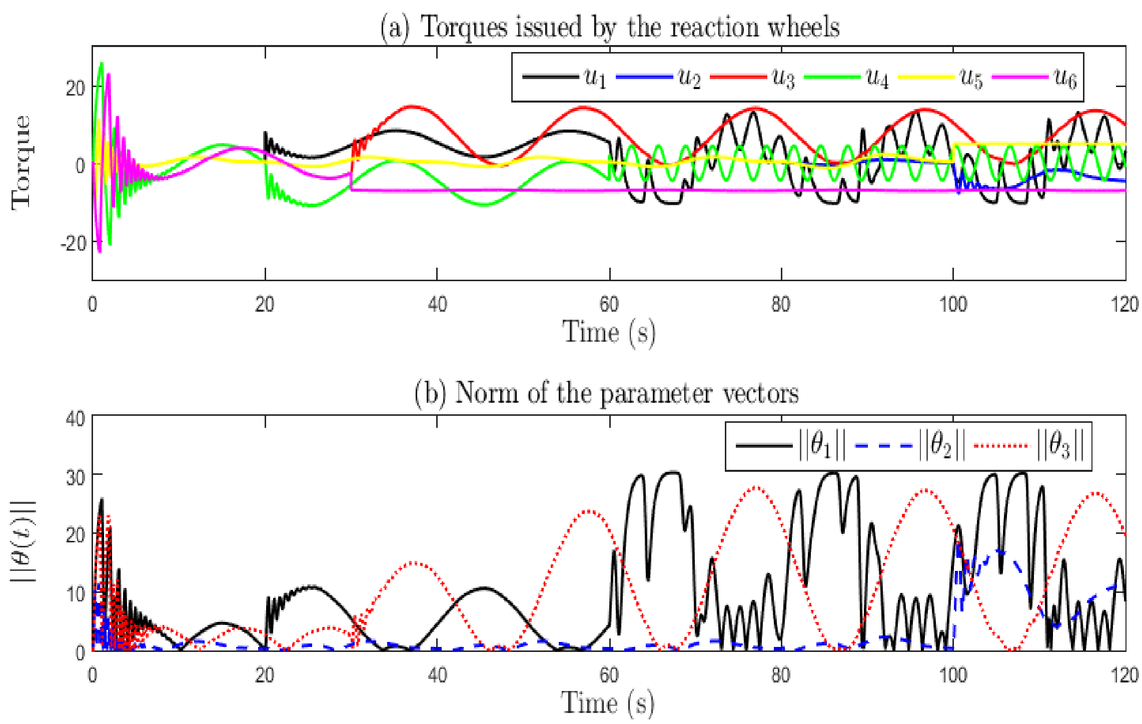

Figure 3.17: Control signals and controller parameters (scenario 2)

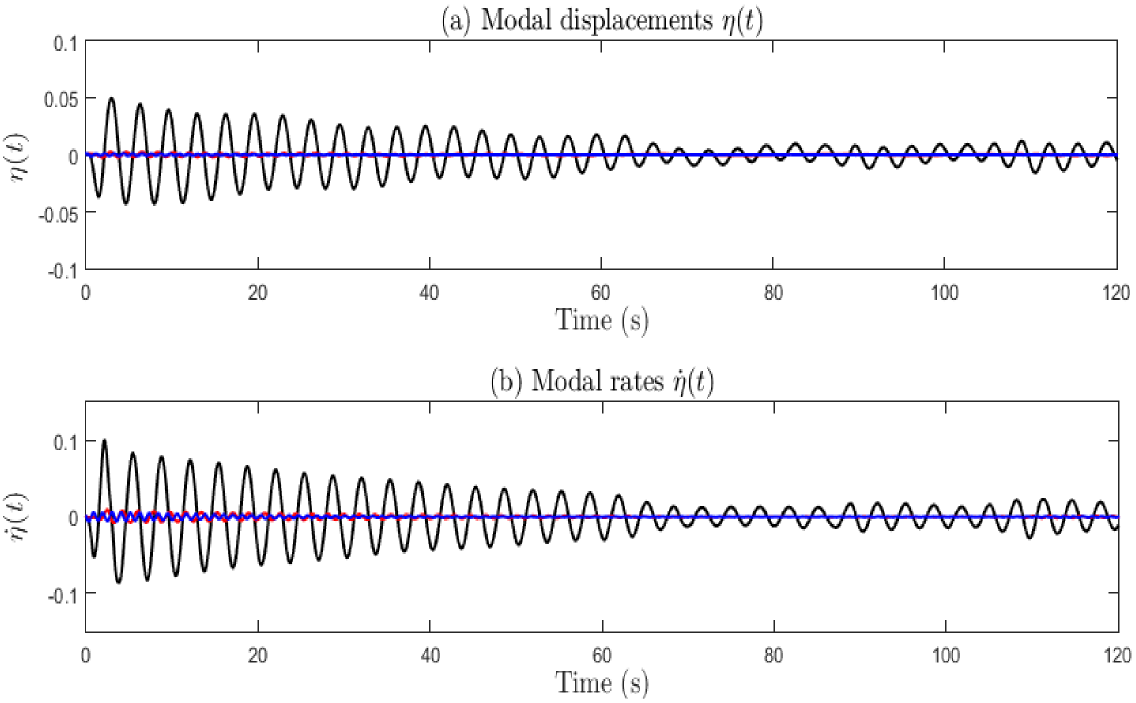

Figure 3.18: Modal displacements and rates time evolution (scenario 2)

In summary, the simulation results were in accordance with the theoretical claims. The proposed adaptive controller was able to ensure the control objectives in the normal case, when one or more actuator failures occur, the controller is updated so that the control effort is redistributed among healthy actuators, the proposed controller can deal with a large set of time-varying and state-dependent failures as demonstrated by the many failure scenarios considered.

3.5 Conclusion

In this chapter, the problem of actuator failure compensation for a class of uncertain nonlinear MIMO systems is investigated by considering a pretty wider set of actuator failures, namely, the affine time-varying and state-dependant failures. Under some structural assumptions on the control gain matrix and the actuator failures, a direct approximation based adaptive controller is designed that can compensate for abrupt and incipient actuator failures. In addition, a special consideration is given for the class of interconnected systems, for those systems the assumptions are somehow less restrictive. Simulation studies on a redundant joint manipulator and a flexible spacecraft actuated by redundant reaction wheels confirm the theoretical claims. In the next chapter, we try to widen the class of actuator failures to non-affine models and we try also to deal with the problem of unknown control direction.

Chapter 4

4 Adaptive control of nonlinear systems with general actuator failures and unknown control direction

Chapter Contents

4.1 Introduction

In the previous chapters, the problem of actuator failure compensation control has been investigated by assuming a rather limited set of compensable actuator failures. In chapter 2, it was assumed that the actuator failures can be either partial loss of effectiveness or bias while in chapter 3 it was assumed that the failures are affine functions of the input. Besides, in both chapters it was assumed that the control direction (control gain sign) is known ahead of controller design, in the second chapter, this assumption was made explicit while is the third chapter, implicit assumptions that lead to the positive definiteness of the control gain matrix were assumed. The proposed actuator failure compensation strategies are interesting and effective in dealing with uncertain nonlinear systems with uncertain actuator failures that fall into the prescribed class given that the control direction is known. In many situations, however, the actuator failure pattern is even more complicated to be described as a bias, loss of effectiveness or as an input affine actuator failure. For instance, if a flight control surface is damaged or broken, its response to controls may be nonlinear and complicated. Furthermore, in the presence of uncertain actuator failures, the knowledge of the control direction can be difficult, added to this difficulty the transformation of the system dynamics from an input affine system to a non-affine system for which a new control design methodology should be sought. Motivated by these observations, the purpose of this chapter is to extend the class of compensable actuator failures to the more general nonaffine failures and to solve the problem of unknown control direction. Two designs will be proposed in this regard, the first design used the known Nussbaum -type function technique and the second uses a direct adaptive estimation of a sign-like term that gives information of the control direction. The remaining of the chapter is organized as follows: In section 4.2 the system under study is presented and the non-affine failure model is described, assumptions for the controller design and the existence of such controller are also given in this section. In section 4.3, a first actuator failure compensation control design based on the Nussbaum- type function is provided with the application to the dynamic model describing the angle of attack of a hypersonic aircraft with the use of fuzzy logic systems (FLS) as function approxixmators. In section 4.4, a second control design using a new technique that directly estimated a sign-like term is developed and applied to the same dynamic model of the hypersonic aircraft angle of attack. Finally, in section 4.5, a conclusion of the chapter is provided.

4.2 Problem statement and preliminaries

4.2.1 System description

In this chapter, we consider the class of nonlinear control affine multiple-input single-output (MISO) systems described as follows

$$\begin{cases} \dot{x}_i = x_{i+1},\ i = 1,\ldots,n-1 \\ \dot{x}_n = f(x) + g^T(x)u \\ y = x_1 \end{cases} \tag{4.1}$$

which can be alternatively expressed in the following input-output form

$$y^{(n)} = f(x) + g^T(x)u \tag{4.2}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$ is the state vector which is supposed available for feedback, $y \in \mathbb{R}$ is the output of the system, $u = [u_1, u_2, \ldots, u_m]^T \in \mathbb{R}^m$ is the control input vector acting directly on the system and whose actuators may fail during system's operation. Besides $f(x)$ and $g(x) = [g_1(x), g_2(x), ..., g_m(x)]^T$ are nonlinear sufficiently smooth unknown function and vector function respectively. It is assumed that the entries $g_i(x), i = 1,...,m$ have the same sign which is unknown.

Remark 4.1: Notice that only the class of nonlinear systems in the canonical form (4.1) will be considered in this chapter. However, all the developments and results can be easily applied to the general case of nonlinear affine systems with stable internal dynamics and which satisfy the assumptions stated in Chapter 2.

4.2.2 Actuator failure modeling

In this paper, we consider a more general class of non-affine actuator failures that can be state-dependent and time-varying, this can be expressed as follows

$$u_j^f = f_j(x, u_j, t),\ t \geq t_j \tag{4.3}$$

where $f_j(x, u_j, t)$ is an unknown nonlinear smooth function and t_j is the failure time which is also assumed unknown, u_j and u_j^f denote the input and output of the j^{th} actuator respectively.

Remark 4.2: Equation (4.3) describes the most general class of actuator failures. Most existing works on actuator failure compensation deal only with a predefined model of actuator failures, such as loss of effectiveness [118], [126], bias or offset [19], [74], [102]. In some cases, the actuator failures are assumed completely parameterized [87], [120]. In these works, failures are generally described using one of the following models

$$u_j^f = \rho_j u_j,\ u_j^f = \bar{u}_j(t),\ u_j^f = \rho_j u_j + \bar{u}_j(t) \tag{4.4}$$

Failure models described in (4.4) can represent many practical failures. However, in some cases, the fault cannot always be described in the above affine form, actuators can have a more delicate behavior and the existing actuator failure models cannot be enough to describe the behavior of such failed actuators. Therefore, it becomes very necessary to investigate the more general case of non-modeled (non-affine) actuator failures.

Obviously, redundancy is required to ensure actuator failure compensability; actuators should have the same characteristic so that in case one or more actuator are lost, the remaining healthy actuators can compensate for its effect. Regarding this similarity of the actuators, their inputs will be designed using a proportional actuation scheme as follows [74]

$$u_j(t) = b_j(t)u_0,\ j = 1,\ldots,m \tag{4.5}$$

where $u_0(t)$ is a signal to be designed and $b_j(t),\ j = 1,\ldots,m$ are control allocation coefficients that describe the contribution of each actuator.

Remark 4.3: The scheme (4.5) is reasonable, for instance, elevator segments in an aircraft can be controlled via a common control signal, which is allocated to different segments.

The control objective is to design an adaptive controller with a parameter update law that generates u_0 such that the output $y(t)$ tracks any desired trajectory $y_d(t)$ with bounded derivatives as precise as possible, despite the presence of actuator failures.

Regarding the actuator failure compensation control design, the following assumption is stated:

Assumption 4.1: The system (4.1) is structured so that, in the presence of $m-1$ actuator failures, the remaining actuators can be controlled effectively.

Assumption 4.1 can be translated to the following condition: For the failures to be compensable, the following condition should be maintained during all the course of operation

$$\sum_{j=1}^{m} g_j(x) b_j(t) \frac{\partial f_j(x, b_j(t) u_0, t)}{\partial u_0} \neq 0 \tag{4.6}$$

As will be further apparent, the term in (4.6) is the control gain counterpart for the non-affine case. In this chapter, its sign is assumed either positive or negative but do not change sign, besides, it is assumed unknown.

Remark 4.4: Condition (4.6) implies that the effect of the control signal $u_0(t)$ should be present for actuator failure scenario to be compensable, in other words, there should be some actuation effect and the system should be always effected by some control actions.

4.2.3 Ideal controller existence

With the proportional actuation scheme (4.5), in the presence of actuator failures described by (4.3), the system (4.2) can be written as follows

$$y^{(n)} = f(x) + \sum_{j=1}^{m} g_j(x) f_j(x, b_j(t) u_0, t) \tag{4.7}$$

It can be seen from (4.7) that the presence of actuator failures do not only lead to actuation deficiency, but also bring extra uncertainties and complexity to the system. In this case, the situation is even more complicated, as the resulting system is no more affine in control. This makes the controller design task more challenging. The controller should compensate system uncertainties as well as uncertain dynamics induced by possible actuator failures, besides it should handle the problem of unknown control direction.

Now, let us define the output tracking error $e(t)$ as follows

$$e(t) = y_d(t) - y(t) \tag{4.8}$$

and the filtered tracking error $s(t)$ as follows

$$s(t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} e(t), \lambda > 0 \tag{4.9}$$

From (4.9), $s(t) = 0$ represents a linear differential equation whose solution implies that the tracking error and its derivatives $e^{(k)}(t)$, $k = 1, \ldots, n-1$ converge to zero [108], henceforth, the control objective will be to bring $s(t)$ close to zero. Moreover, if $s(t)$ is bounded, the tracking error and its derivatives will be also bounded, more specifically, if one has $|s(t)| \le \Phi$ where Φ is a positive constant, it can be concluded that [108]: $|e^{(k)}(t)| \le 2^k \lambda^{k-n-1}$, $k = 1, \ldots n-1$. These bounds can be reduced by increasing the free design parameter λ.

Now, by taking the time derivative of $s(t)$ along system trajectories we obtain

$$\dot{s} = \gamma_0 - f(x) - \sum_{j=1}^{m} g_j(x) f_j(x, b_j(t) u_0, t) \tag{4.10}$$

with $\gamma_0 = y_d^{(n)} + \sum_{i=1}^{n-1} k_i e^{(i)}$ and $k_i = C_{n-1}^{i-1} \lambda^{n-i}$.

By adding and subtracting the term $ks + k_0 \tanh(s/\varepsilon_0)$ to the right-hand term of (4.10) we obtain the following expression

$$\dot{s} = -ks - k_0 \tanh\left(\frac{s}{\varepsilon_0}\right) + \left(\gamma - f(x) - \sum_{j=1}^{m} g_j(x) f_j(x, b_j(t) u_0, t)\right) \tag{4.11}$$

with $\gamma = ks + k_0 \tanh(s/\varepsilon_0) + \gamma_0$.

From (4.11), given the fact that γ, $f(x)$, $g_j(x)$ and $b_j(t)$ do not depend explicitly on u_0, since $\sum_{j=1}^{m} g_j(x) b_j(t) \left(\partial f_j(x, b_j(t) u_0, t) / \partial u_0\right) \ne 0$, it follows that

$$\frac{\partial \left(\gamma - f(x) - \sum_{j=1}^{m} g_j(x) f_j(x, b_j(t) u_0, t)\right)}{\partial u_0} \ne 0 \tag{4.12}$$

Then, by invoking the implicit function theorem [127], [128], it follows that there exists an implicit local control signal u_0^* that satisfies the following equation

$$\gamma - f(x) - \sum_{j=1}^{m} g_j(x) f_j(x, b_j(t) u_0^*, t) = 0 \tag{4.13}$$

By combining (4.11) and (4.13), it follows that the control signal u_0^* satisfies the following

$$\dot{s} = -ks - k_0 \tanh\left(\frac{s}{\varepsilon_0}\right) \tag{4.14}$$

From which it can be concluded that $s(t)$ and $e^{(i)}(t)$, $i=1,\ldots,n-1$, all converge to zero. The problem is that the control signal u_0^* cannot be implemented as it is inherently implicit, besides system functions and actuator failures are assumed unknown, and the control gain sign is unknown. In the following, this ideal controller is constructed online using two adaptive control schemes. In the first scheme, the unknown control gain sign is using the Nussbaum function technique while in the second scheme another new approach is proposed.

4.3 Design using the Nussbaum-type function technique

4.3.1 Adaptive controller design

In this section, the problem of unknown control direction is addressed by using the Nussbaum-type function technique [129]. This technique was first proposed in by Nussbaum in 1983 [129], since then, it has proved its effectiveness in dealing the problem of unknown control directions [85], [128], [130], [131]. In what follows, the Nussbaum-type function is defined and a fundamental lemma is given for further stability analysis.

Definition 4.1: A Nussbaum-type function can be thought of as an estimator of the control gain sign, it swings between positive and negative values according to the error dynamics. It has an increasing amplitude that provides an opportunity for a selected control direction to correct inappropriate deviation caused by erroneous previous control [132]. Recall that a function $N(\tau)$ is called a Nussbaum-type function if it satisfies the following properties [129], [133]

$$\limsup_{v\to\infty}\left(\frac{1}{v}\int_0^v N(\tau)\,d\tau\right)=+\infty \tag{4.15}$$

$$\liminf_{v\to\infty}\left(\frac{1}{v}\int_0^v N(\tau)\,d\tau\right)=-\infty \tag{4.16}$$

Notice that the Nussbaum-type function should have infinite gains and infinite switching frequencies. There are many functions that satisfy these properties, for example, the continuous functions $\tau^2\sin(\tau)$, $\tau^2\cos(\tau)$, $e^{\tau^2}\cos(\tau)$. In this section, the even Nussbaum-type function: $N(\tau)=\tau^2\cos(\tau)$ will be used.

For further technical developments, the following lemma regarding the Nussbaum-type function is introduced [133]–[135], for the detailed proof, one can refer to [134].

Lemma: 4.1: Let $V(\cdot)$ and $\tau(\cdot)$ be two smooth functions defined on the interval $\left[t_i, t_f\right)$ with $V(t) \geq 0,\ \forall t \in \left[t_i, t_f\right)$, let $N(\cdot)$ be an even smooth Nussbaum-type function. If the following inequality holds: $0 \leq V(t) \leq a_i + e^{-\rho(t-t_i)} \int_{t_i}^{t} \left(h(x(\zeta)) N(\tau(\zeta)) + c_1\right) \dot{\tau}(\zeta) e^{\rho(\zeta - t_i)} d\zeta$ for all $t \in \left[t_i, t_f\right)$, where $h(\cdot)$ is a piecewise continuous function which takes values in the unknown closed interval $I : \left[l^-, l^+\right]$ with $0 \notin I$, a_i is a positive number and c_1 is a constant, then $V(t)$, $\tau(t)$ and $\int_{t_i}^{t} \left(g(\zeta) N(\tau(\zeta)) + c_1\right) \dot{\tau}(\zeta) d\zeta$ are bounded on $\left[t_i, t_f\right)$.

Now, assume that the implicit ideal control law u_0^* can be approximated using a function approximator which is linear in the parameters as follows

$$u_0 = \Pi^T(z)\theta(t) \tag{4.17}$$

where $\Pi(z)$ is a regressor vector with input z and $\theta(t)$ is an adjustable parameter vector. Assume also that there exists a piecewise continuous time-varying parameter vector $\theta^*(t)$ with possible jumps when one or more abrupt actuator failures occur with bounded time derivative inside the interval of continuity, such that

$$u_0^* = \Pi^T(z)\theta^*(t) \tag{4.18}$$

Define also the control prediction error as follows

$$e_{u_0} = u_0^* - u_0 = \Pi^T(z)\tilde{\theta} \tag{4.19}$$

with $\tilde{\theta} = \theta^* - \theta$. Let us denote $b_a(x, u_0, t) = \sum_{j=1}^{m} g_j(x) b_j(t) \partial f_j\left(x, b_j(t) u_{0a}, t\right) / \partial u_{0a}$ and $b(x, u_0, t) = \sum_{j=1}^{m} g_j(x) f_j\left(x, b_j(t) u_0, t\right)$, with $u_{0a} = u_0^* + a\left(u_0 - u_0^*\right)$ and $a \in [0,1]$.

Now, by virtue of the mean value theorem [85], there exists $a \in [0,1]$ that satisfies the following

$$b(x, u_0, t) = b\left(x, u_0^*, t\right) + b_a(x, u_a, t)\left(u_0 - u_0^*\right) \tag{4.20}$$

Substituting (4.20) into (4.11) and taking into account (4.13), one can write the following

$$\dot{s} = -ks - k_0 \tanh\left(\frac{s}{\varepsilon_0}\right) + b_a(x, u_a, t) e_{u_0} \tag{4.21}$$

with $e_{u_0} = u_0^* - u_0$. Equation (4.21) can be further reformulated as follows

$$b_a(x, u_a, t) e_{u_0} = \dot{s} + ks + k_0 \tanh\left(\frac{s}{\varepsilon_0}\right) \tag{4.22}$$

Based on the gradient descent method, the parameter update law for the vector θ that minimizes the control prediction quadrature error $J(\theta) = (1/2) e_{u_0}^2$ is given by [109]

$$\dot{\theta} = -\alpha(t) \nabla_\theta J(\theta) \tag{4.23}$$

Given that $\nabla_\theta J(\theta) = -\Pi(z) e_{u_0}$, by choosing $\alpha(t) = \alpha_0 b_a(x, u_a, t)$ and introducing a $\sigma-$modification, and denote $\rho^* = \operatorname{sign}(b_a(x, u_a, t))$. The parameter update law (4.23) becomes

$$\dot{\theta} = \alpha_0 \rho^* \Pi(z) \left(\dot{s} + ks + k_0 \tanh\left(\frac{s}{\varepsilon_0}\right) \right) - \alpha_0 \sigma \theta \tag{4.24}$$

where $\rho^* = \operatorname{sign}(b_a(x, u_a, t))$ represents the control direction which is assumed unknown.

To deal with the problem of unknown control direction, we use the Nussbaum-type function technique, then, the new parameter update laws is modified as follows [135]

$$\dot{\theta} = \alpha_0 N(\tau) \Pi(z) \left(\dot{s} + ks + k_0 \tanh\left(\frac{s}{\varepsilon_0}\right) \right) - \alpha_0 \sigma \theta \tag{4.25}$$

$$\dot{\tau} = \alpha_1 \left(\dot{s} + ks + k_0 \tanh\left(\frac{s}{\varepsilon_0}\right) \right)^2 \tag{4.26}$$

$$N(\tau) = \tau^2 \cos(\tau) \tag{4.27}$$

where α_1 is a positive adaptation gain.

4.3.2 Stability and tracking analysis

For closed-loop stability and tracking analysis, suppose that at time instants t_i, $i = 1, 2, \ldots, N$, one or more actuators fail. Consider the following Lyapunov function candidate defined over a time interval $[t_i, t_{i+1})$ where there are no jumps caused by abrupt actuator failures

$$V_i = \frac{1}{2}s^2 + \frac{1}{2\alpha_0}\tilde{\theta}^T\tilde{\theta},\ t \in [t_i, t_{i+1}) \tag{4.28}$$

The time derivative of V_i over the time interval $t \in [t_i, t_{i+1})$ is

$$\dot{V}_i = s\dot{s} - \frac{1}{\alpha_0}\tilde{\theta}^T\dot{\theta} + \frac{1}{\alpha_0}\tilde{\theta}^T\dot{\theta}^* \tag{4.29}$$

Substituting (4.21) and (4.25) into (4.29), one can obtain

$$\begin{aligned}\dot{V}_i = & -ks^2 - k_0 s\tanh\left(\frac{s}{\varepsilon_0}\right) + sb_a(x,u_a,t)e_{u_0} + \frac{1}{\alpha_0}\tilde{\theta}^T\dot{\theta}^* \\ & -\frac{1}{\alpha_0}\tilde{\theta}^T\left(\alpha_0 N(\tau)\Pi(z)\left(\dot{s} + ks + k_0\tanh\left(\frac{s}{\varepsilon_0}\right)\right) - \alpha_0\sigma\theta\right)\end{aligned} \tag{4.30}$$

By using (4.22) and (4.26), and noticing that $\tilde{\theta}^T\Pi(z) = \Pi^T(z)\tilde{\theta} = e_{u_0}$, $\dot{V}_i$ becomes

$$\begin{aligned}\dot{V}_i = & -ks^2 - k_0 s\tanh\left(\frac{s}{\varepsilon_0}\right) + sb_a(x,u_a,t)e_{u_0} + \sigma\tilde{\theta}^T\theta \\ & -\frac{N(\tau)}{b_a(x,u_a,t)}\left(\dot{s} + ks + k_0\tanh\left(\frac{s}{\varepsilon_0}\right)\right)^2 + \frac{1}{\alpha_0}\tilde{\theta}^T\dot{\theta}^*\end{aligned} \tag{4.31}$$

Now, let us introduce the following inequality

$$\begin{aligned}sb_a(x,u_a,t)e_{u_0} & \le \frac{1}{4\alpha_1}s^2 + \alpha_1\left(b_a(x,u_a,t)e_{u_0}\right)^2 \\ & \le \frac{1}{4\alpha_1}s^2 + \alpha_1\left(\dot{s} + ks + k_0\tanh\left(\frac{s}{\varepsilon_0}\right)\right)^2\end{aligned} \tag{4.32}$$

Using (4.32), and noticing that $k_0 s\tanh(s/\varepsilon_0) \ge 0$, (4.31) can be bounded as follows

$$\begin{aligned}\dot{V}_i \le & -ks^2 - k_0 s\tanh\left(\frac{s}{\varepsilon_0}\right) + \frac{1}{4\alpha_1}s^2 + \alpha_1\left(\dot{s} + ks + k_0\tanh\left(\frac{s}{\varepsilon_0}\right)\right)^2 \\ & -\frac{N(\tau)}{b_a(x,u_a,t)}\left(\dot{s} + ks + k_0\tanh\left(\frac{s}{\varepsilon_0}\right)\right)^2 + \sigma\tilde{\theta}^T\theta + \frac{1}{\alpha_0}\tilde{\theta}^T\dot{\theta}^*\end{aligned} \tag{4.33}$$

Regarding (4.26), (4.33) can be further bounded as follows

$$\dot{V}_i \le -\left(k - \frac{1}{4\alpha_1}\right)s^2 + \left(1 - \frac{N(\tau)}{\alpha_1 b_a(x,u_a,t)}\right)\dot{\tau} + \sigma\tilde{\theta}^T\theta + \frac{1}{\alpha_0}\tilde{\theta}^T\dot{\theta}^* \tag{4.34}$$

Furthermore, we consider the following two inequalities

$$\sigma\tilde{\theta}^T\theta \le -\frac{\sigma}{2}\left\|\tilde{\theta}\right\|^2 + \frac{\sigma}{2}\left\|\theta^*\right\|^2 \tag{4.35}$$

$$\frac{1}{\alpha_0}\tilde{\theta}^T\dot{\theta}^* \le \frac{\sigma}{4}\left\|\tilde{\theta}\right\|^2 + \frac{1}{\sigma\alpha_0^2}\left\|\dot{\theta}^*\right\|^2 \tag{4.36}$$

Then (4.34) can be simplified as follows

$$\dot{V}_i \le -\left(k - \frac{1}{4\alpha_1}\right)s^2 - \frac{\sigma}{4}\left\|\tilde{\theta}\right\|^2 + \left(1 - \frac{N(\tau)}{\alpha_1 b_a(x,u_a,t)}\right)\dot{\tau} + \frac{\sigma}{2}\left\|\theta^*\right\|^2 + \frac{1}{\sigma\alpha_0^2}\left\|\dot{\theta}^*\right\|^2 \tag{4.37}$$

If the free design parameter k is selected such that $k > 1/(1/4\alpha_1 + \kappa_1)$, then (4.37) simplifies to the following

$$\dot{V}_i \le -\rho V_i + \left(c_1 + h(x)N(\tau)\right)\dot{\tau} + \psi_i \tag{4.38}$$

with $c_1 = 1$, $\rho = \min\left(2/(1/4\alpha_1 + \kappa_1), \sigma\alpha_0/2\right)$ and $h(x) = -1/\alpha_1 b_a(x,u_a,t)$. Besides ψ_i is defined for every interval $[t_i, t_{i+1})$ as: $\psi_i = \sup_{t=[t_i,t_{i+1})}\left(\frac{\sigma}{2}\left\|\theta^*\right\|^2 + \frac{1}{\sigma\alpha_0^2}\left\|\dot{\theta}^*\right\|^2\right)$.

Now, we can prove the following theorem on the stability and convergence

Theorem 4.1: For the system (4.1) subject to actuator failures (4.3) and whose control direction is unknown, if the condition (4.6) is satisfied, then the adaptive control scheme (4.5), (4.17) equipped with parameter update laws (4.25)-(4.27) ensure closed-loop stability and asymptotic convergence of the tracking error and its derivatives up to order $n-1$.

Proof:

For convenience, let us denote $V(t)$ the extension of V_i, $i = 1,...,N$ over the whole time domain. Starting with the interval $[0, t_1)$, multiplying (4.38) by $e^{\rho(t-t_0)}$ yields

$$\frac{d}{dt}\left(V(t)e^{\rho(t-t_0)}\right) \le \psi_0 e^{\rho(t-t_0)} + \left(h(x)N(\tau) + c_1\right)\dot{\tau}e^{\rho(t-t_0)} \tag{4.39}$$

Now, by integrating (4.39) inside the time interval $t \in [0, t_1)$, one obtain the following inequality

$$V(t) \le \frac{\psi_0}{\rho} + \left(V(t_0) - \frac{\psi_0}{\rho} \right) e^{-\rho(t-t_0)} + e^{-\rho(t-t_0)} \int_{t_0}^{t} \Big(h(\zeta) N\big(\tau(\zeta)\big) + c_1 \Big) \dot{\tau} e^{\rho(\zeta - t_0)} d\zeta$$
$$\le a_0 + e^{-\rho(t-t_0)} \int_{0}^{t} \Big(h(\zeta) N\big(\tau(\zeta)\big) + c_1 \Big) \dot{\tau} e^{\rho(\zeta - t_0)} d\zeta \tag{4.39}$$

where $a_0 = \psi_0/\rho + V(t_0)$.

Given that $b_a(x, u_a, t) \ne 0$, $h(x)$ will have values in the interval $I = \left[-(\alpha_1 b)^{-1}, -(\alpha_1 b)^{-1} \right]$ with $0 \notin I$. By virtue of Lemma 4.1, it can be concluded from (4.39) the signals $V(t)$, $N(\tau)$, τ and $\int_0^t \Big(h(\zeta) N\big(\tau(\zeta)\big) + c_1 \Big) \tau(\zeta) d\zeta$ are bounded on the interval $[0, t_1)$. Let us also denote $\Phi_0 = \left| \int_{t_0}^{t_1^-} \Big(h(\zeta) N\big(\tau(\zeta)\big) + c_1 \Big) \tau(\zeta) d\zeta \right|$. Suppose that $V(t)$ exhibits finite jumps due to discontinuities in θ^* caused by abrupt actuator failures, denote $\Delta V(t_i)$ these jumps such that

(4.40)

where t_i^- and t_i^+ are the time instants just before and after the jump.

Regarding (4.39) and (4.40), one can write

$$V\left(t_1^+\right) \le a_0 + \Phi_0 + \Delta V(t_1) \tag{4.41}$$

Now consider the time interval $[t_1, t_2)$, likewise, by multiplying (4.38) by $e^{\rho(t-t_1)}$ and integrating the resulting inequality in the interval $[t_1, t_2)$, we end up with the following inequality

$$V(t) \le a_1 + e^{-\rho(t-t_1)} \int_{t_1^+}^{t} \Big(h(\zeta) N\big(\tau(\zeta)\big) + c_1 \Big) \dot{l} e^{\rho(\zeta - t_1)} d\zeta \tag{4.42}$$

where $a_1 = \psi_1/\rho + V\left(t_1^+\right)$.

By virtue of Lemma 4.1, it can be concluded from (4.39) that the signals $V(t)$, $N(\tau)$, τ and $\int_{t_1^+}^t \Big(h(\zeta) N\big(\tau(\zeta)\big) + c \Big) \tau(\zeta) d\zeta$ are bounded on the interval $[t_1, t_2)$. Let us also denote $\left| \Phi_2 = \int_{t_1^+}^{t_2^-} \Big(h(\zeta) N\big(\tau(\zeta)\big) + c \Big) \tau(\zeta) d\zeta \right|$. Regarding (4.41) and (4.42), we can write

$$V\left(t_2^+\right) \le a_1 + \Phi_1 + \Delta V\left(t_2\right) \le V\left(t_0\right) + \Phi_0 + \Phi_1 + \frac{\psi_0}{\rho} + \frac{\psi_0}{\rho} + \Delta V\left(t_1\right) + \Delta V\left(t_2\right) \tag{4.43}$$

By following the same procedure for $t_2, ..., t_N$, at time t_N where no further actuator failures occur, we can conclude that $V\left(t_N^+\right)$ is bounded by a finite quantity as follows

$$V\left(t_N^+\right) \le V\left(t_0\right) + \sum_{k=0}^{N-1}\left(\frac{\psi_k}{\rho} + \Phi_k + \Delta V\left(t_{k+1}\right)\right) \tag{4.44}$$

Consider now the interval $t \in \left[t_N, t_f\right)$, we proceed in the same way, multiplying (4.38) by $e^{\rho(t-t_N)}$ and integrating over this interval we end up with the following inequality

$$V\left(t\right) \le a_N + e^{-\rho(t-t_N)} \int_{t_N^+}^{t} \left(h\left(\zeta\right) N\left(\tau\left(\zeta\right)\right) + c_1\right) \dot{I} e^{\rho(\zeta - t_N)} d\zeta \tag{4.45}$$

where $a_N = V\left(t_N^+\right) + \psi_N / \rho$, which is a finite quantity. Given the fact that there is a finite number of actuator failures accruing and there are finite jumps caused by abrupt failures, it can be concluded that a_N is finite. By invoking Lemma 4.1, it can be concluded from (4.39) that the singals $V\left(t\right)$, $N\left(\tau\right)$, τ and $\int_{t_N^+}^{t} \left(h\left(\zeta\right) N\left(\tau\left(\zeta\right)\right) + b\right) \tau\left(\zeta\right) d\zeta$ are bounded on the interval $\left[t_N^+, t_f\right)$. Since there are no parameter jumps after t_N and no time-escape phenomena, t_f can extend to infinity. Therefore, it can be concluded that the signals $s\left(t\right)$, $\tilde{\theta}\left(t\right)$, $x\left(t\right)$, and $u_0\left(t\right)$ remain bounded. Consequently, it follows that $s\left(t\right) \in L^2$, and $\dot{s}\left(t\right) \in L^\infty$. By virtue of the Barbalat's lemma, it can be concluded that the signals $s\left(t\right)$, $e\left(t\right)$ and its derivatives up to order $n-1$ converge asymptotically.

4.3.3 Simulation results

To access its effectiveness, the proposed adaptive control scheme is applied to the dynamic model describing the angle of attack of a hypersonic aircraft. The model under study was already presented in Chapter 2. We choose a proportional actuation scheme with $b_1\left(t\right) = 1,\ b_2\left(t\right) = 2$ in a first case, then we choose $b_1\left(t\right) = -1,\ b_2\left(t\right) = -2$ in a second step The free design parameters are chosen as: $k = 5,\ k_0 = 2,\ \lambda = 2,\ \varepsilon = 0.01,\ \alpha_0 = 5,\ \alpha_1 = 0.01$. Each fuzzy logic system (FLS) input is composed of five Gaussian membership functions given as in Chapter 2. The initial values of the controller parameters $\theta_i(0) = 0.01$, $i = 1,\ldots,25$ and the initial value of τ is $\tau(0) = 1$. The initial values of the angle of attack and the pitch rate are chosen as $\alpha(0) = 0.03$ rad and $q(0) = 0$ rad/s. The desired angle of attack is $\alpha_d = \pi/36\sin(0.2t)$. For the simulation, we consider the following two failure scenarios:

First failure scenario: Consider the following scenario: Suppose that at the time $t = 40\ s$, the first elevator segment undergoes a failure according to the pattern $u_1^f = f_1(x, u_1, t) = 0.75u_1^3 - 0.2$. At the time $t = 80\ s$, the second elevator segment fails according to the following pattern: $u_2^f = (0.5 + 0.5\sin^2(t))u_2 + 0.08\sin(0.5t)$. It can be easily checked that for all time history, the failure pattern fulfills the prescribed compensability assumption. The simulation is carried out for $120\ s$, the results for a positive control direction are shown in Figure 4.1 (system response) and Figure 4.2 (control signal and Nussbaum function), and those for negative control gain sign i.e. $b_1(t) = -1$, $b_2(t) = -2$ are shown in Figure 4.3 and Figure 4.4. It can be clearly seen that for both situations the proposed adaptive control scheme ensures stability and tracking despite the presence of actuator failures. Besides, the accurate estimate of the control direction (the same sign as $b(x, u_a, t)$) is provided after a short transient regardless the occurrence of actuator failures.

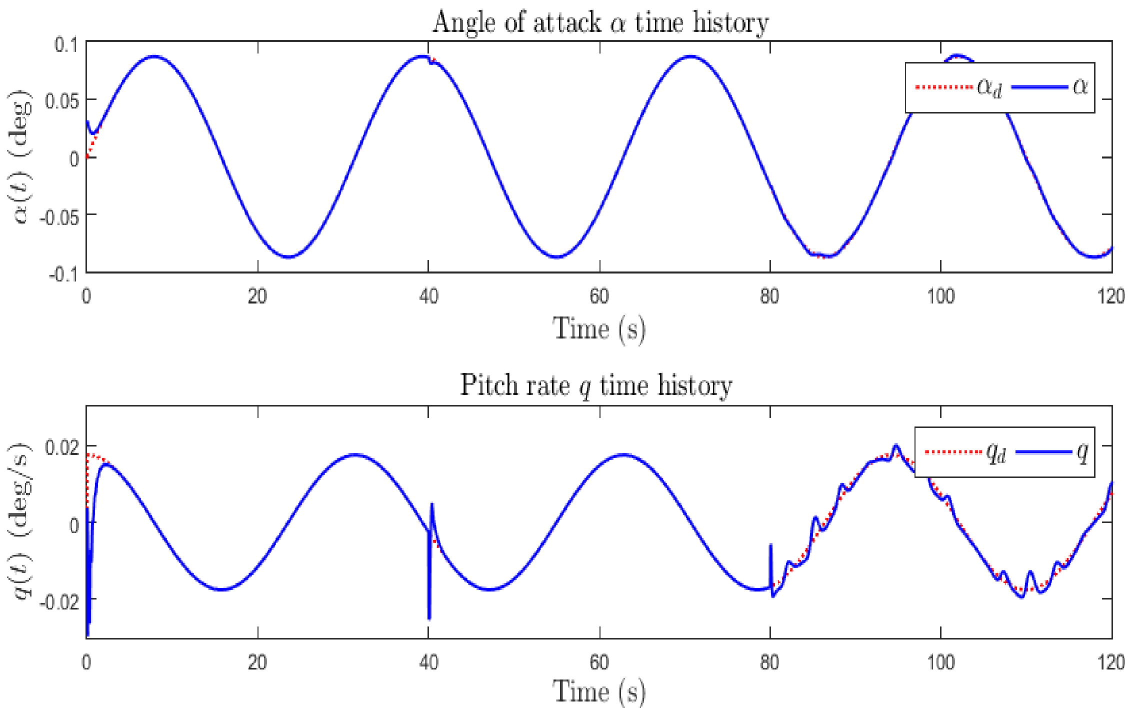

Figure 4.1: Angle of attack and pitch rate (Nussbaum scenario 1, positive)

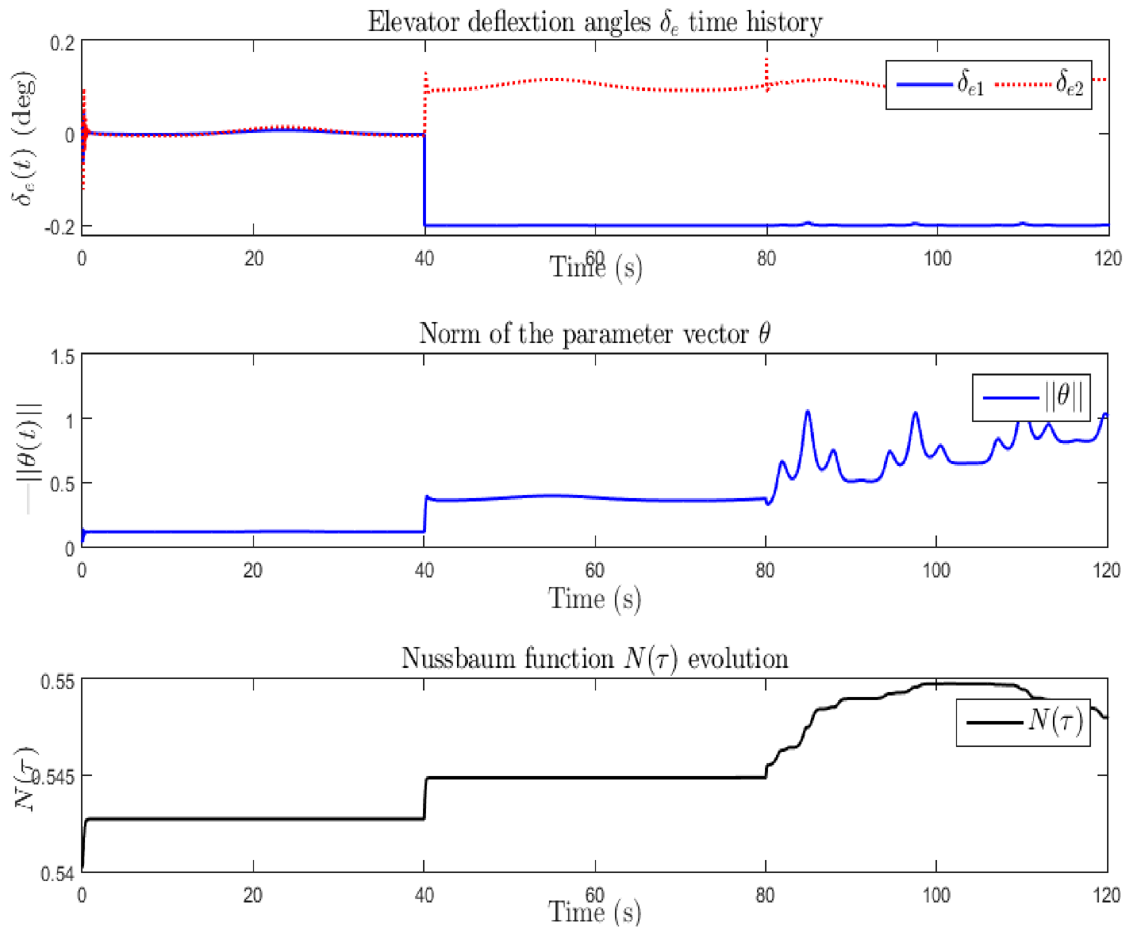

Figure 4.2: Control signals and parameters using Nussbaum (scenario 1, positive)

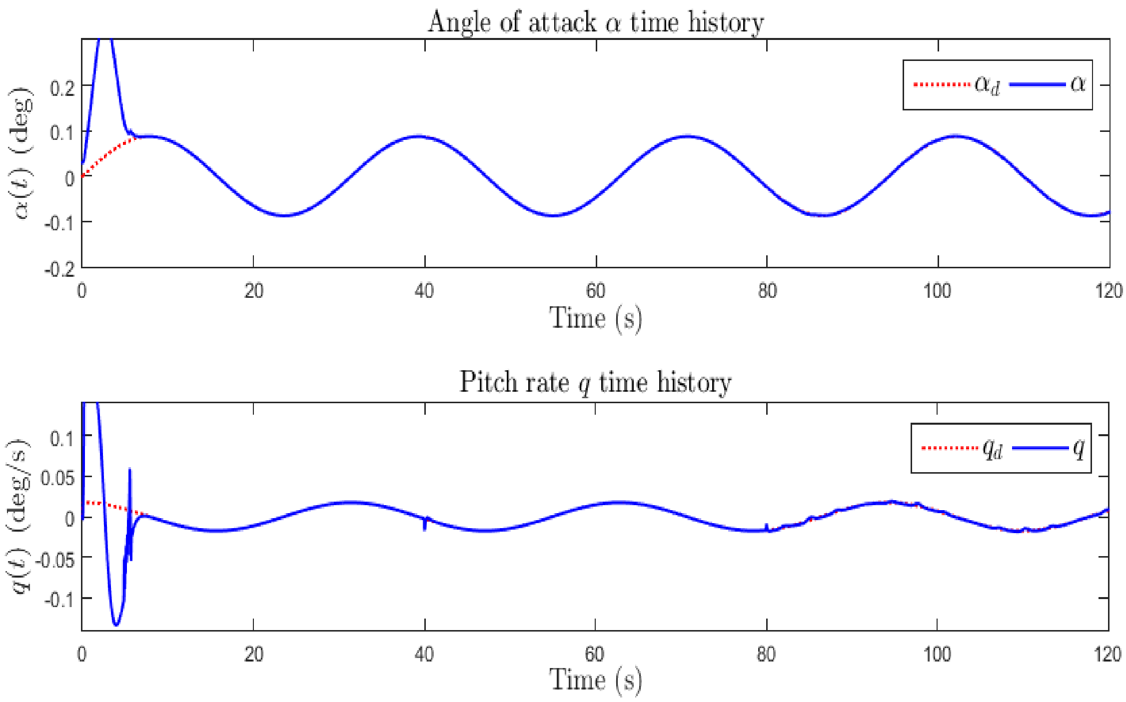

Figure 4.3: Angle of attack and pitch rate (Nussbaum scenario 1, negative)

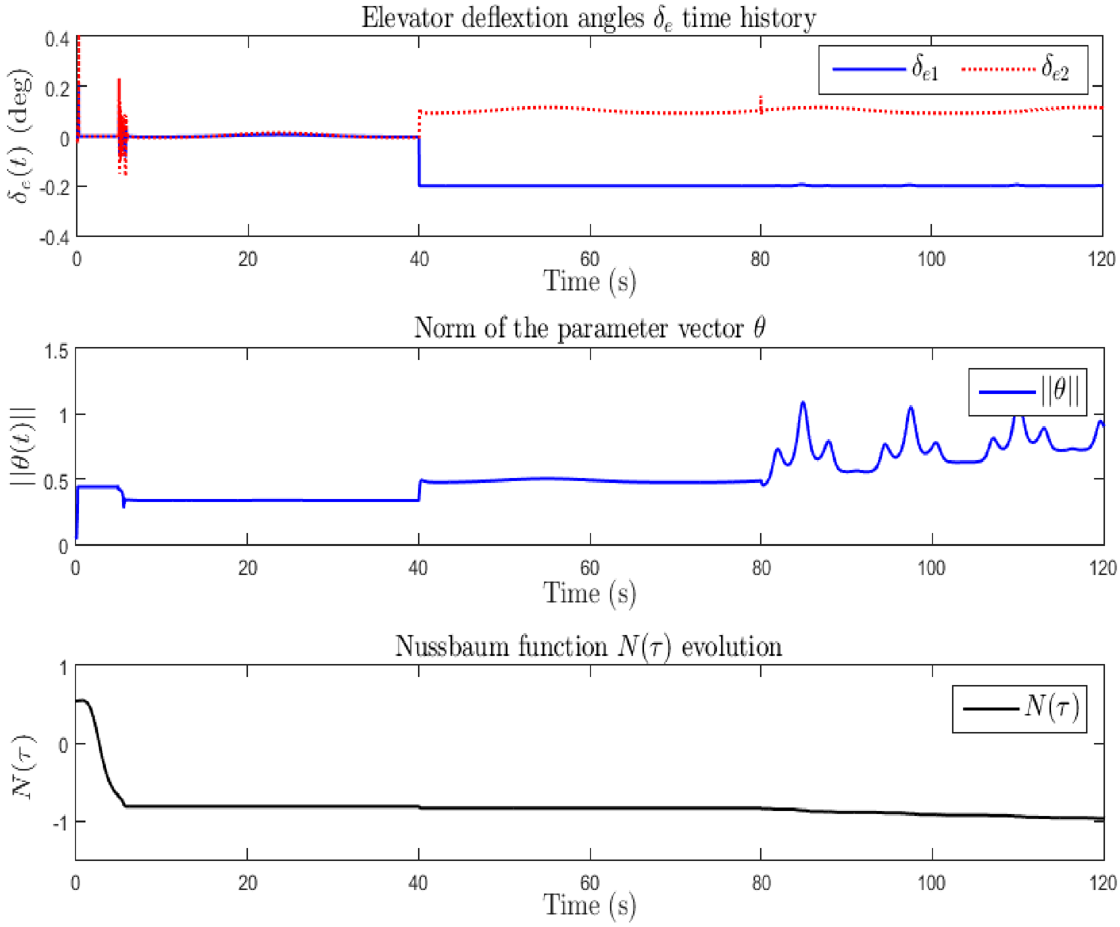

Figure 4.4: Control signals and parameters using Nussbaum (scenario 1, negative)

Second failure scenario: Consider the following scenario, suppose that at the time $t = 40\ s$, the second elevator fails according to the pattern $u_2^f = 0.1x_1 + 0.05u_2 + 0.05\sin(0.5t)$, then at the time $t = 70\ s$ it recovers to normal operation. In addition, we suppose that at the time $t = 90\ s$ the first elevator fails according to the pattern $u_1^f = 0.4u_1^3 + 0.5x_1x_2$. Again, it can be checked that the failure patterns fulfill the prescribed assumptions. The simulation is carried out by assuming a positive control direction, i.e. $b_1(t) = 1,\ b_2(t) = 2$, the simulation results are shown in Figure 4.5 for the output signals and in Figure 4.6 for the control input signals. It can be clear concluded that despite the presence of such a severe failure scenario, the proposed actuator failure compensation controller was able to ensure the tracking and stability. It can be also noticed that the estimate of the Nussbaum function depends on the failure pattern. But the sign of $N(\tau)$ is always kept the same as the control gain sign, i.e. the sign of $b_a(x, u_a, t)$ after short a short transient period of time.

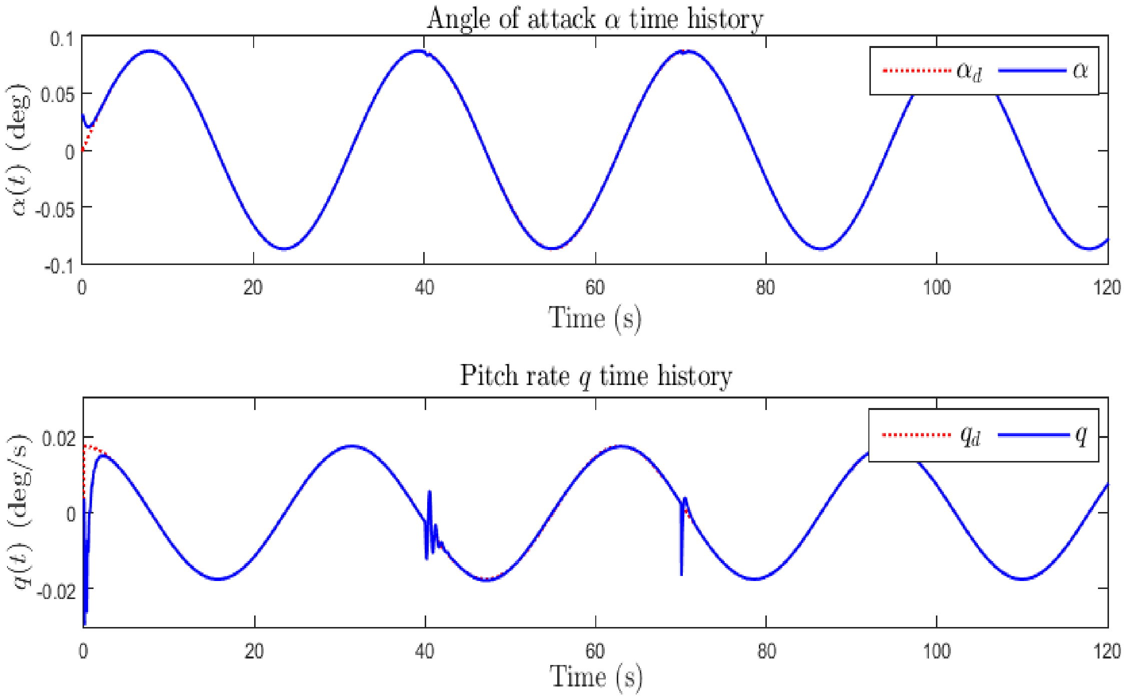

Figure 4.5: Angle of attack and pitch rate (Nussbaum scenario 2, positive)

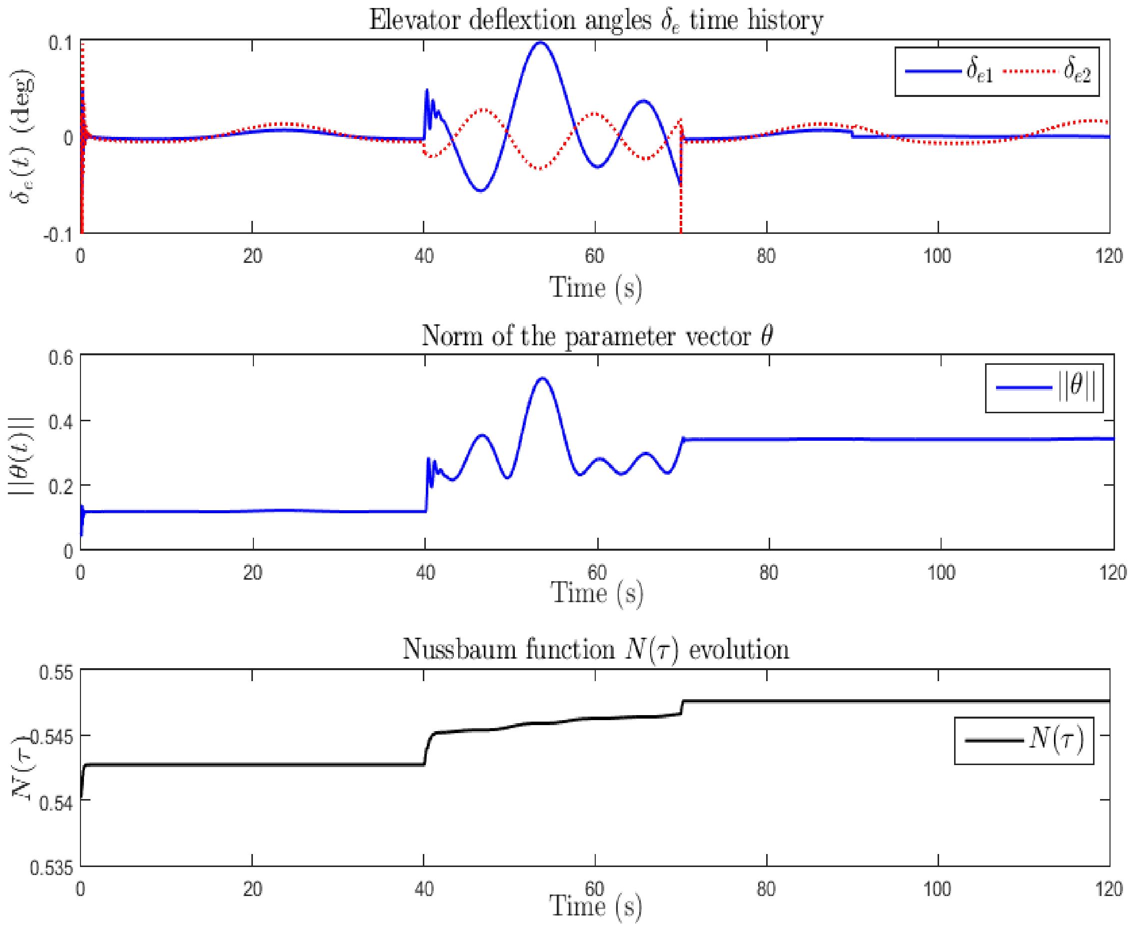

Figure 4.6: Control signals and parameters using Nussbaum (scenario 2, positive)

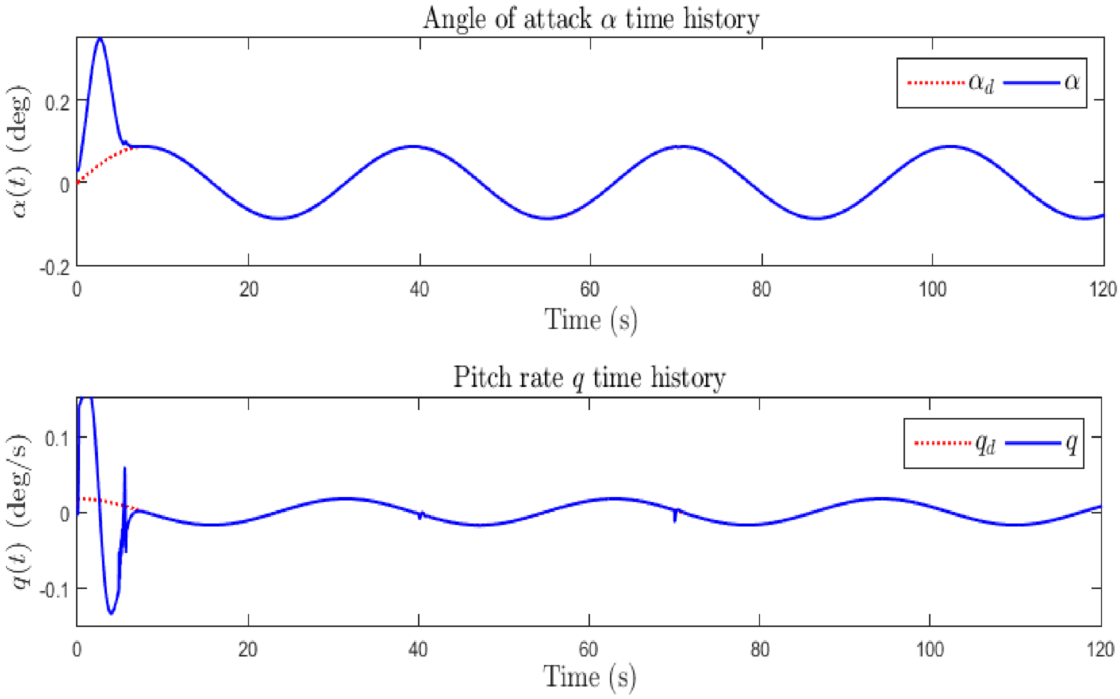

Figure 4.7: Angle of attack and pitch rate (Nussbaum scenario 2, negative)

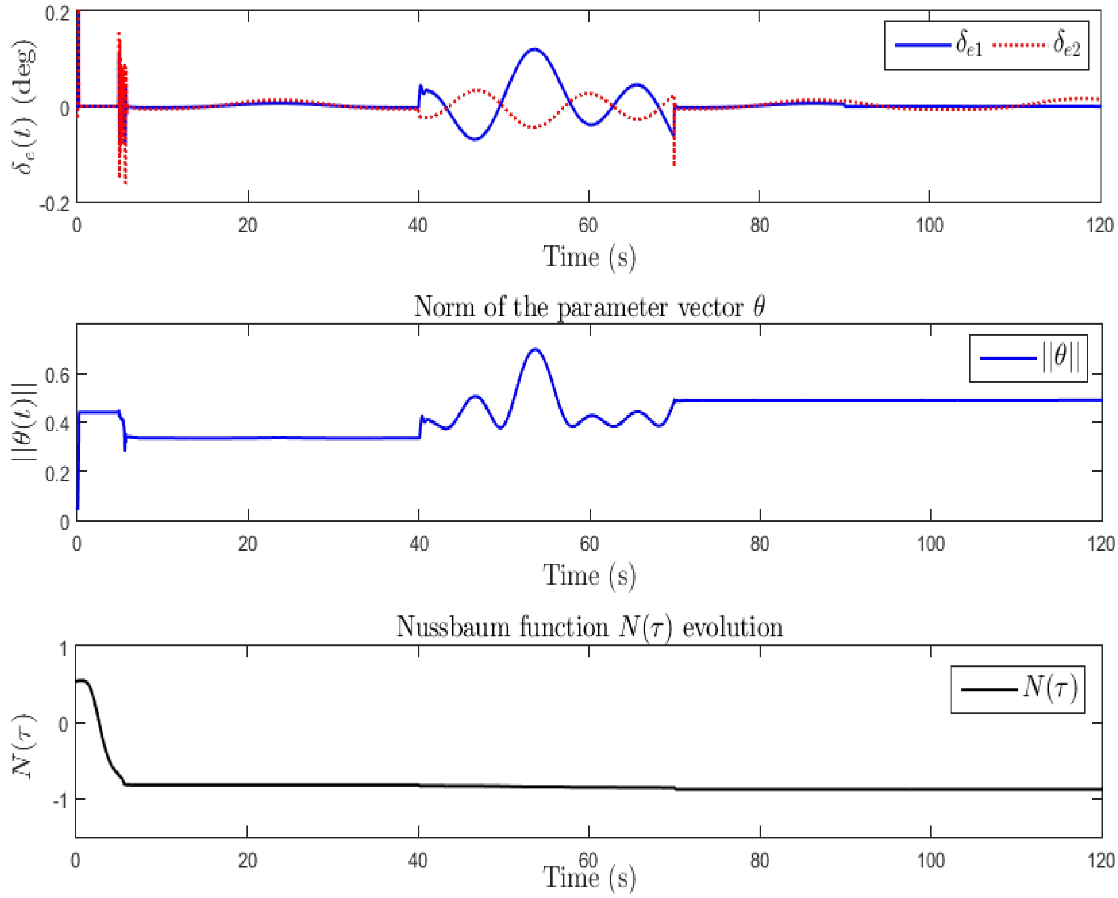

Figure 4.8: Control signals and parameters using Nussbaum (scenario 2, negative)

4.4 Design without Nussbaum-type function

In this section, a new design that does not use Nussbaum functions is proposed, within this design, the control gain sign is estimated directly and plugged into the parameter update law.

4.4.1 Adaptive controller design

In this section, we assume that the implicit ideal controller u_0^* can be approximated as

$$u_0^* = \Pi^T(z)\theta^*(t) + \varepsilon(z) \tag{4.46}$$

The adaptive version of (4.46) is given by

$$u_0 = \Pi^T(z)\theta(t) \tag{4.47}$$

Before proceeding, we assume that the approximation error $\varepsilon(z)$ satisfies the following inequality which will be used later for stability proofs

$$\varepsilon^2(z) \le \bar{\varepsilon}_0 s^2 + \bar{\varepsilon}_1 \tag{4.48}$$

with $\bar{\varepsilon}_0, \bar{\varepsilon}_1 > 0$. Define also the corresponding control prediction error as

$$e_{u_0} = u_0^* - u_0 = \Pi^T(z)\tilde{\theta} + \varepsilon(z) \tag{4.49}$$

where $\tilde{\theta} = \theta^* - \theta$.

By introducing the unknown parameter $\rho^* = \text{sign}(b_a(x, u_a, t))$, (4.21) can be written as

$$\dot{s} = -ks - k_0 \tanh(s/\varepsilon_0) + \rho^* \left|b_a(x, u_a, t)\right| e_{u_0} \tag{4.50}$$

By using (4.49), one can write

$$\rho^* e_{u_0} = \rho^* \left(\Pi^T(z)\tilde{\theta} + \varepsilon(z)\right) \tag{4.51}$$

Let us denote ρ the online estimate of ρ^* and $\tilde{\rho} = \rho^* - \rho$ the estimation error. By noticing that $\rho^* = \tilde{\rho} + \rho$, (4.51) can be expressed as follows

$$\rho^* e_{u_0} = -\tilde{\rho} u_0 + \rho \Pi^T(z)\tilde{\theta} + \tilde{\rho}\Pi^T(z)\tilde{\theta} + \rho^* \varepsilon(z) \tag{4.52}$$

Which can be further written as

$$-\tilde{\rho}u_0 + \rho\Pi^T(z)\tilde{\theta} = \rho^* e_{u_0} - \tilde{\rho}\Pi^T(z)\tilde{\theta} - \rho^*\varepsilon(z) \tag{4.53}$$

The parameter update laws for θ and ρ are chosen to minimize the control prediction quadrature error $J(\theta) = (1/2) b_a(x, u_a, t)\rho^* e_{u_0}^2$, these update laws are designed as [109]

$$\dot{\theta} = \alpha_0 \rho \Pi(z)\left(\dot{s} + ks + k_0 \tanh\left(\frac{s}{\varepsilon_0}\right)\right) - \alpha_0\sigma_0\theta \tag{4.54}$$

$$\dot{\rho} = -\alpha_1 u_0 \left(\dot{s} + ks + k_0 \tanh\left(\frac{s}{\varepsilon_0}\right)\right) - \alpha_1\sigma_1\theta \tag{4.55}$$

where α_0, α_1 are positive adaptation gains, and σ_0, σ_1 are small positive constants.

4.4.2 Closed-loop stability analysis

For stability and tracking error convergence analysis, suppose that, at time instants $t_i,\ i = 1, 2, \ldots, N$, one or more actuators fail. For stability analysis, consider the following Lyapunov-like function defined over a time interval of constant failure pattern

$$V_i = \frac{1}{2}s^2 + \frac{1}{2\alpha_0}\tilde{\theta}^T\tilde{\theta} + \frac{1}{2\alpha_1}\tilde{\rho}^2,\ t \in [t_i, t_{i+1}) \tag{4.56}$$

The time derivative of V_i over the interval $t \in [t_i, t_{i+1})$ is given as

$$\dot{V}_i = s\dot{s} + \frac{1}{\alpha_0}\tilde{\theta}^T\dot{\theta}^* - \frac{1}{\alpha_0}\tilde{\theta}^T\dot{\theta} - \frac{1}{\alpha_1}\tilde{\rho}^T\dot{\rho} \tag{4.57}$$

Substituting (4.22), (4.54) and (4.55) into (4.57) yields

$$\begin{aligned}\dot{V}_i &= -ks^2 - k_0 s \tanh\left(\frac{s}{\varepsilon_0}\right) + sb_a(x, u_a, t)e_{u_0} + \frac{1}{\alpha_0}\tilde{\theta}^T\dot{\theta}^* \\ &-\rho\tilde{\theta}^T\Pi(z)\left(b_a(x, u_a, t)e_{u_0}\right) + \sigma\tilde{\theta}^T\theta + \tilde{\rho}u_0\left(b_a(x, u_a, t)e_{u_0}\right) - \sigma\tilde{\rho}\rho\end{aligned} \tag{4.58}$$

Which can be further written as

$$\begin{aligned}\dot{V}_i &= -ks^2 - k_0 s \tanh\left(\frac{s}{\varepsilon_0}\right) + sb_a(x, u_a, t)e_{u_0} \\ &-\left(\rho\Pi^T(z)\tilde{\theta} - \tilde{\rho}u_0\right)b_a(x, u_a, t)e_{u_0} + \sigma_0\tilde{\theta}^T\theta + \sigma_1\tilde{\rho}\rho + \frac{1}{\alpha_0}\tilde{\theta}^T\dot{\theta}^*\end{aligned} \tag{4.59}$$

Regarding (4.53), (4.59) can be written as

$$\begin{aligned}\dot{V}_i = & -ks^2 - k_0 s \tanh\left(\frac{s}{\varepsilon_0}\right) + sb_a(x,u_a,t)e_{u_0} - \left|b_a(x,u_a,t)\right| e_{u_0}^2 \\ & + \left(\tilde{\rho}\Pi^T(z)\tilde{\theta} + \rho^*\varepsilon(z)\right) b_a(x,u_a,t)e_{u_0} + \sigma_0\tilde{\theta}^T\theta + \sigma_1\tilde{\rho}\rho + \frac{1}{\alpha_0}\tilde{\theta}^T\dot{\theta}^*\end{aligned} \tag{4.60}$$

Using the following inequalities

$$\sigma_0\tilde{\theta}^T\theta \le -\frac{\sigma_0}{2}\left\|\tilde{\theta}\right\|^2 + \frac{\sigma_0}{2}\left\|\theta^*\right\|^2,\ \sigma_1\tilde{\rho}\rho \le -\frac{\sigma_1}{2}\tilde{\rho}^2 + \frac{\sigma_1}{2}\rho^{*2} \tag{4.61}$$

$$sb_a(x,u_a,t)e_{u_0} \le \left|b_a(x,u_a,t)\right| e_{u_0}^2 + \frac{1}{4}b_a(x,u_a,t)s^2 \tag{4.62}$$

$$\frac{1}{\alpha_0}\tilde{\theta}^T\dot{\theta}^* \le \frac{\sigma_0}{4}\left\|\tilde{\theta}\right\|^2 + \frac{1}{\sigma_0\alpha_0^2}\left\|\dot{\theta}^*\right\|^2 \tag{4.63}$$

Equation (26) can be bounded as follows

$$\begin{aligned}\dot{V}_i \le & -ks^2 + \frac{1}{4}b_a(x,u_a,t)s^2 - \frac{\sigma_0}{4}\left\|\tilde{\theta}\right\|^2 + \frac{\sigma_0}{2}\left\|\theta^*\right\|^2 + \frac{1}{\sigma_0\alpha_0^2}\left\|\dot{\theta}^*\right\|^2 \\ & + \left(\tilde{\rho}\Pi^T(z)\tilde{\theta} + \rho^*\varepsilon(z)\right) b_a(x,u_a,t)e_{u_0} - \frac{\sigma_1}{2}\tilde{\rho}^2 + \frac{\sigma_1}{2}\rho^{*2}\end{aligned} \tag{4.64}$$

Now, we assume that the following inequality is satisfied

$$\left(\tilde{\rho}\Pi^T(z)\tilde{\theta} + \rho^*\varepsilon(z)\right) b_a(x,u_a,t)e_{u_0} \le \kappa_1\left\|\tilde{\theta}\right\|^2 + \kappa_2\tilde{\rho}^2 + \kappa_3 e_{u_0}^2 + \delta^2(t) \tag{4.65}$$

where κ_1, κ_2 and κ_3 are positive constants and $\delta(t)$ is a bounded function.

By substituting (4.65) into (4.64), and recalling that $\left|b_a(x,u_a,t)\right| \le \bar{b}_a$ we obtain

$$\begin{aligned}\dot{V}_i \le & -\left(k - \frac{1}{4}\bar{b}_a\right)s^2 - \left(\frac{\sigma_0}{4} - \kappa_1\right)\left\|\tilde{\theta}\right\|^2 - \left(\frac{\sigma_1}{2} - \kappa_2\right)\tilde{\rho}^2 \\ & + \frac{\sigma_0}{2}\left\|\theta^*\right\|^2 + \frac{\sigma_1}{2}\rho^{*2} + \frac{1}{\sigma_0\alpha_0^2}\left\|\dot{\theta}^*\right\|^2 + \kappa_3 e_{u_0}^2 + \delta^2(t)\end{aligned} \tag{4.66}$$

If the free design parameters are selected as $k > \bar{b}_a/4$, $\sigma_0 > 4\kappa_1$, $\sigma_1 > 2\kappa_2$, under the assumption that $\theta^*, \dot{\theta}^*$ and ρ^* are bounded then (4.66) can be simplified as follows

$$\dot{V}_i \le -\gamma V_i + \psi_i \tag{4.67}$$

where γ and ψ_i are defined as follows

$$\gamma = \min\left(2\left(k-\frac{1}{4}\overline{b}_a\right), 2\left(\frac{\sigma_0}{2}-\kappa_1\right), 2\left(\frac{\sigma_1}{2}-\kappa_2\right)\right) \tag{4.68}$$

$$\psi_i = \sup_t\left(\frac{\sigma_0}{2}\left\|\theta^*\right\|^2+\frac{\sigma_1}{2}\rho^{*2}+\frac{1}{\sigma_0\alpha_0^2}\left\|\dot{\theta}^*\right\|^2+\kappa_3 e_{u_0}^2+\delta^2(t)\right) \tag{4.69}$$

Now we can prove the following theorem on stability and tracking performance.

Theorem 4.2: Consider the system (4.1) with unknown control direction and subject to general actuator failures described by (4.3), using the actuation scheme (4.5), the adaptive controller (4.47) equipped with the parameter update laws (4.54) and (4.55) guarantees that the closed-loop system is UUB stable and the tracking error converges to a small residual set.

Proof:

The proof is similar to that presented in the previous chapter. From (4.67) implies that, for $t\in[t_i,t_{i+1})$, where the failure pattern is fixed, given that $V_i \geq \psi_i/\gamma$, we have $\dot{V}_i < 0$. By integrating (4.67) over the interval $t\in[t_i,t_{i+1})$, we can write

$$V_i(t) \leq V\left(t_i^+\right)e^{-\gamma(t-t_i)}+\frac{\psi_i}{\gamma} \tag{4.70}$$

Let us denote $V(t)$ the extension of V_i, $i=1,2,...,N$ over the whole time domain $t\in[0,\infty)$, due to possible abrupt actuator failures θ^* and $V(t)$ will exhibit finite jumps at each time instant t_i, $i=1,...,N$, i.e. there are no further failures after t_N. Let us also denote ΔV_i the jump on V caused by jumps on θ^* at time t_i. Besides, let t_i^- and t_i^+ be the time instants just before and after the occurrence of failure respectively.

Now, starting from t_1, one can write

$$V\left(t_1^+\right)=V\left(t_1^-\right)+\Delta V_1 \tag{4.71}$$

From (4.70) we have

$$V\left(t_1^-\right) \leq V\left(t_0\right)e^{-\gamma(t_1-t_0)}+\frac{\psi_1}{\gamma} \tag{4.72}$$

And from (4.71) and (4.72), we can write

$$V\left(t_1^+\right) \le V\left(t_0\right) e^{-\gamma\left(t_1 - t_0\right)} + \frac{\psi_1}{\gamma} + \Delta V_1 \tag{4.73}$$

Likewise, we proceed the same way for $t_2, \ldots, t_N$. At the end, we obtain

$$V\left(t_N^+\right) \le V\left(t_0\right) + \sum_{i=1}^{N} \frac{\psi_i}{\gamma} + \sum_{i=1}^{N} \Delta V_i \tag{4.74}$$

After time t_N, there are no further actuator failures that occur, by integrating (4.67) over the interval $t \in \left[t_N, \infty\right)$, and using (4.74) we obtain

$$V\left(t\right) \le \left(V\left(t_0\right) + \sum_{i=1}^{N} \frac{\psi_i}{\gamma} + \sum_{i=1}^{N} \Delta V_i\right) e^{-\gamma\left(t - t_N\right)} + \frac{\psi_{N+1}}{\gamma} \tag{4.75}$$

It follows that $s\left(t\right), \tilde{\theta}\left(t\right)$ and $u\left(t\right)$ are bounded. Regarding the definition of $V\left(t\right)$ in (4.56) and (4.75), after time t_N, we have

$$\left|s\left(t\right)\right| \le \left(\sqrt{\left|s\left(0\right)\right|^2 + \frac{1}{\alpha_0}\left\|\theta\left(0\right)\right\|^2 + \frac{1}{\alpha_1}\left\|\rho\left(0\right)\right\|^2 + \sum_{i=1}^{N} \frac{\psi_i}{\gamma} + \sum_{i=1}^{N} \Delta V_i}\right) e^{-0.5\gamma\left(t - t_N\right)} + \sqrt{\frac{2\psi_{N+1}}{\gamma}} \tag{4.76}$$

This implies that $V\left(t\right)$ is exponentially bounded and converges to the residual set $\left|s\left(t\right)\right| \le \sqrt{2\psi_{N+1}/\gamma}$, which means that the tracking error and its derivatives converge to residual sets as follows: $\left|e^{(i)}\left(t\right)\right| \le 2^i \lambda^{i-n} \sqrt{2\psi_{N+1}/\gamma},\ i = 1, \ldots, n-1,$ these sets can be made smaller by an adequate choice of the design parameters k, σ and η_0.

Remark 4.5: It is worth noticing that one main shortcoming of the Nussbaum-type function based actuator failure compensation design, is that the ideal approximation error $\varepsilon\left(z\right)$ was set to zero, this is assumed so that the Lyapunov- function introduced in the analysis can be bounded. In the present design, however, this is not the case.

4.4.3 Simulation results

In this section, the proposed controller is applied to dynamics of the angle of attack of a hypersonic aircraft model. The model under study was given in Chapter 2. The initial values of the controller parameters $\theta_i\left(0\right) = 0.01$, for the control gain sign (control direction) estimate ρ we consider two cases: the first with $\rho\left(0\right) = 1$ and the second with $\rho\left(0\right) = -1$. We choose a proportional actuation scheme with $b_1(x) = -1$, $b_2(x) = -2$. The other free design parameters are selected as: $k = 5,\ k_0 = 2,\ \lambda = 2,\ \varepsilon_0 = 0.001,\ \alpha_0 = 5$ and $\alpha_1 = 10,\ \sigma_0 = \sigma_1 = 0.01$. Each fuzzy input is composed of five Gaussian membership functions given as in Chapter 2.

The initial values of the angle of attack and the pitch rate are chosen as $\alpha(0) = 0.03$ rad and $q(0) = 0$ rad/s. The desired angle of attack is $\alpha_d = \pi / 36 \sin(0.2t)$ rad.

For the simulation, we consider the following scenarios, each with different initial conditions.

First failure scenario: In this scenario, let $b_1(x) = 1$, $b_2(x) = 2$. Suppose that at the time $t = 80\ s$, the first elevator segment undergoes a non-affine failure as $u_1^f = 0.5u_1^3 + 0.2u_1$. In addition, at the time $t = 60\ s$, the second elevator fails as: $u_2^f = \left(0.5 + 0.5\sin^2(t)\right)u_1 + 0.08\sin(0.5t)$, which is an affine time-varying failure. It can be seen that for all time history, the failure pattern fulfills the prescribed assumptions. The simulation is carried out for $120\ s$, results for $\rho(0) = 1$ are shown in Figure 4.9 (angle of attack and pitch rate) and Figure 4.10 (control signal and control direction estimate), the results for $\rho(0) = -1$ are shown in Figure 4.11(angle of attack and pitch rate) and Figure 4.12 (control signals and control direction esitmate), it can be clearly seen that the proposed actuator failure compensation controller indeed ensures the prescribed control objectives despite the presence of state-dependent and time varying-actuator failures modelled by a non-affine function. Besides, it can be seen that regardless the sign of $\rho(0)$, the sign of ρ is the same as that of $b_a(x, u_a, t)$ which is positive in this scenario.

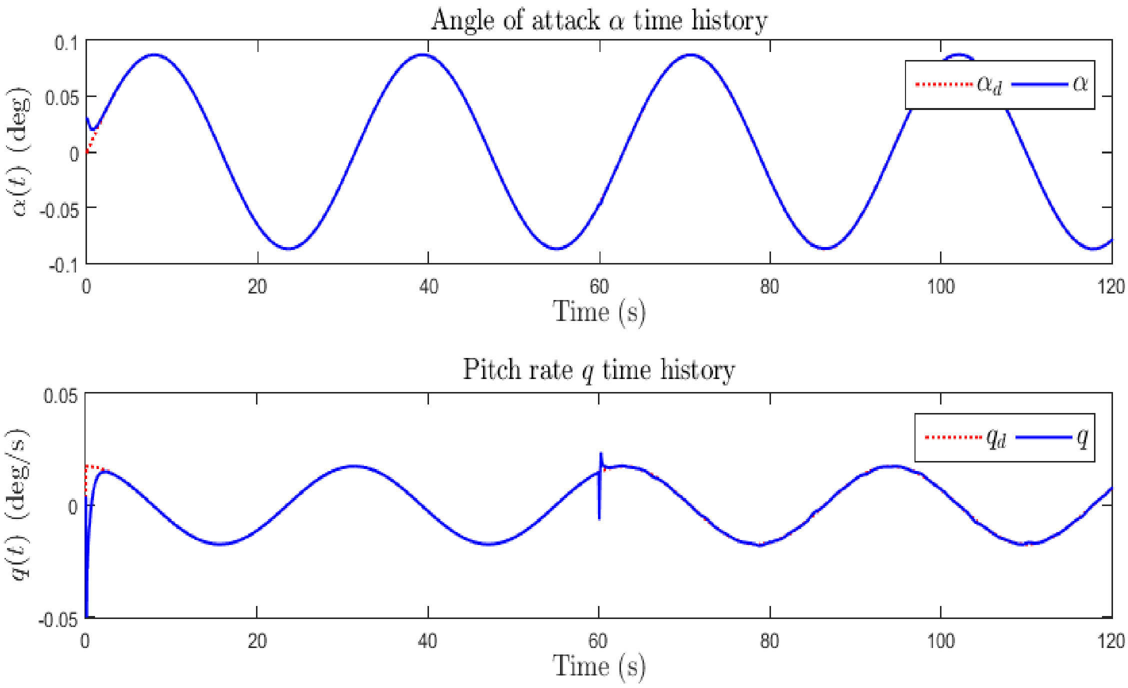

Figure 4.9: Angle of attack and pitch rate without Nussbaum (scenario 1, $\rho(0) = -1$)

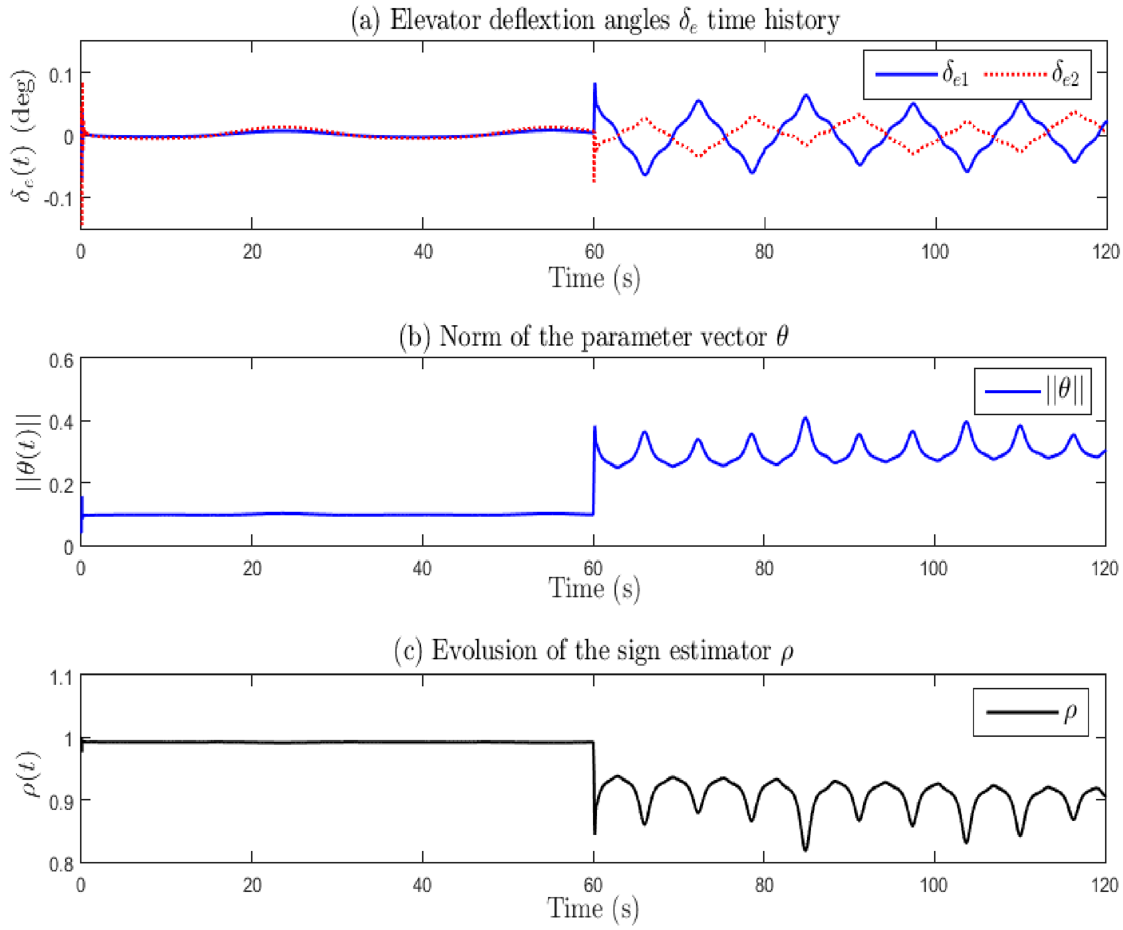

Figure 4.10: Control signals and parameters (scenario 1, $\rho(0) = 1$)

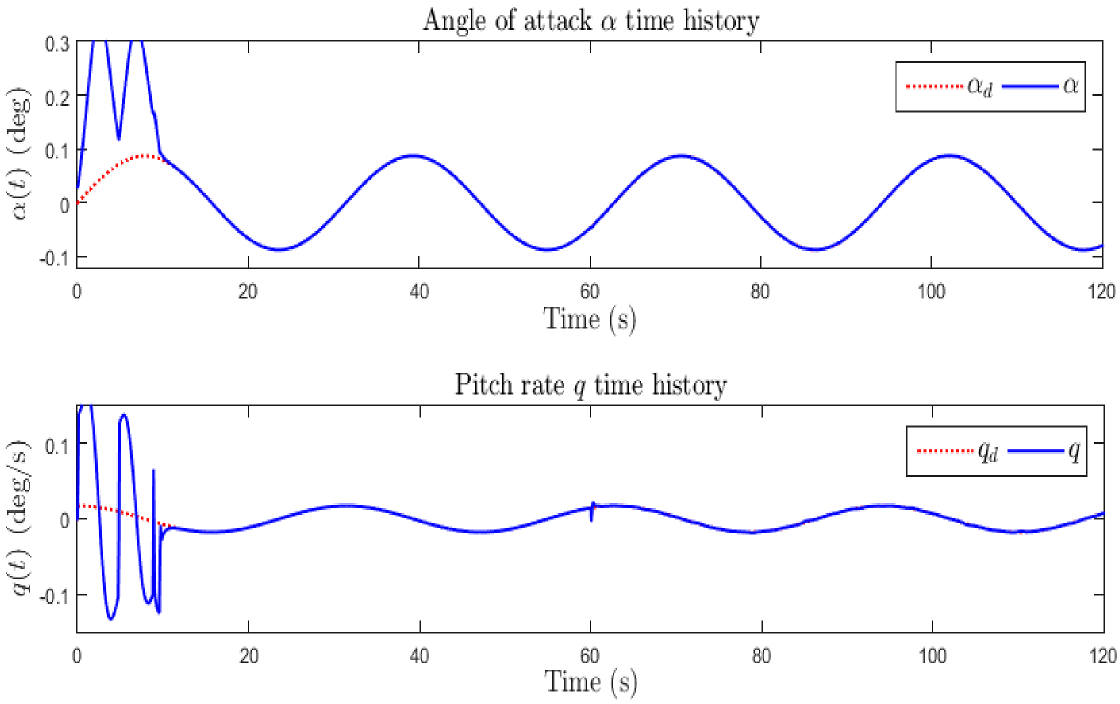

Figure 4.11: Angle of attack and pitch rate without Nussbaum (scenario 1, $\rho(0) = -1$)

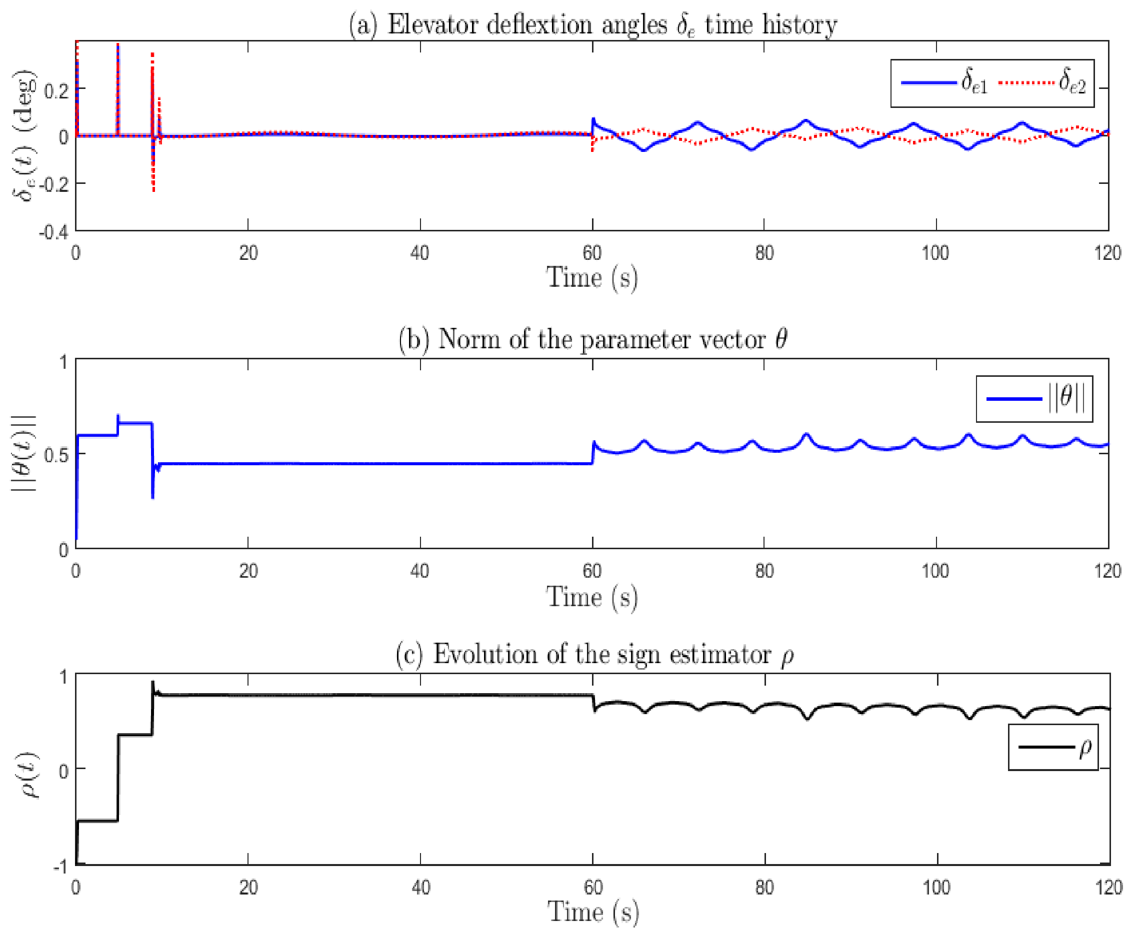

Figure 4.12: Control signals and parameters (scenario 1, $\rho(0) = -1$)

Second failure scenario: in this scenario, we choose $b_1(x) = -1$, $b_2(x) = -2$. Suppose that at the time $t = 40\ s$, the second elevator fails according to the pattern $u_2^f = 0.1x_1 + 0.05u_2 - 0.05\sin(0.5t)$, then at the time $t = 70\ s$ it recovers to normal operation. In addition, we suppose that at the time $t = 90\ s$ the first elevator fails according to the pattern $u_1^f = 0.4u_1^3 + 0.5x_1x_2$. Again, it can be checked that the failure patterns fulfill the prescribed assumptions. The simulation results for $\rho(0) = 1$ are shown in Figure 4.13 for the tracking response and in Figure 4.14 for the control signals and parameters, the simulation results for $\rho(0) = -1$ are shown in Figure 4.15 for the system's response and Figure 4.16 for the control signals and parameters. It can be seen that the tracking and stability performance are maintained despite the presence of actuator failures of the general form. Besides, regardless the sing of $\rho(0)$, the sign of ρ is the same as that of $b_0(x, u_a, t)$ which is negative in this scenario. This confirms the effectiveness of the proposed actuator failure compensation controller in dealing with the problems of non-affine actuator failures and unknown control direction.

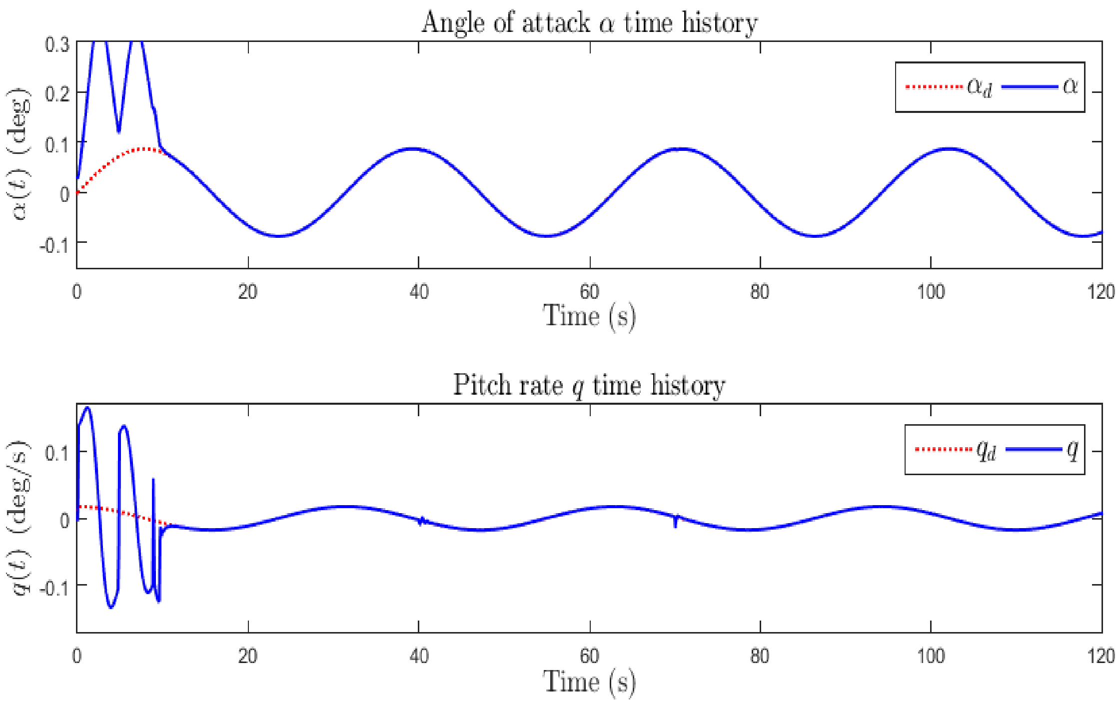

Figure 4.13: Angle of attack and pitch rate without Nussbaum (scenario 2, $\rho(0) = 1$)

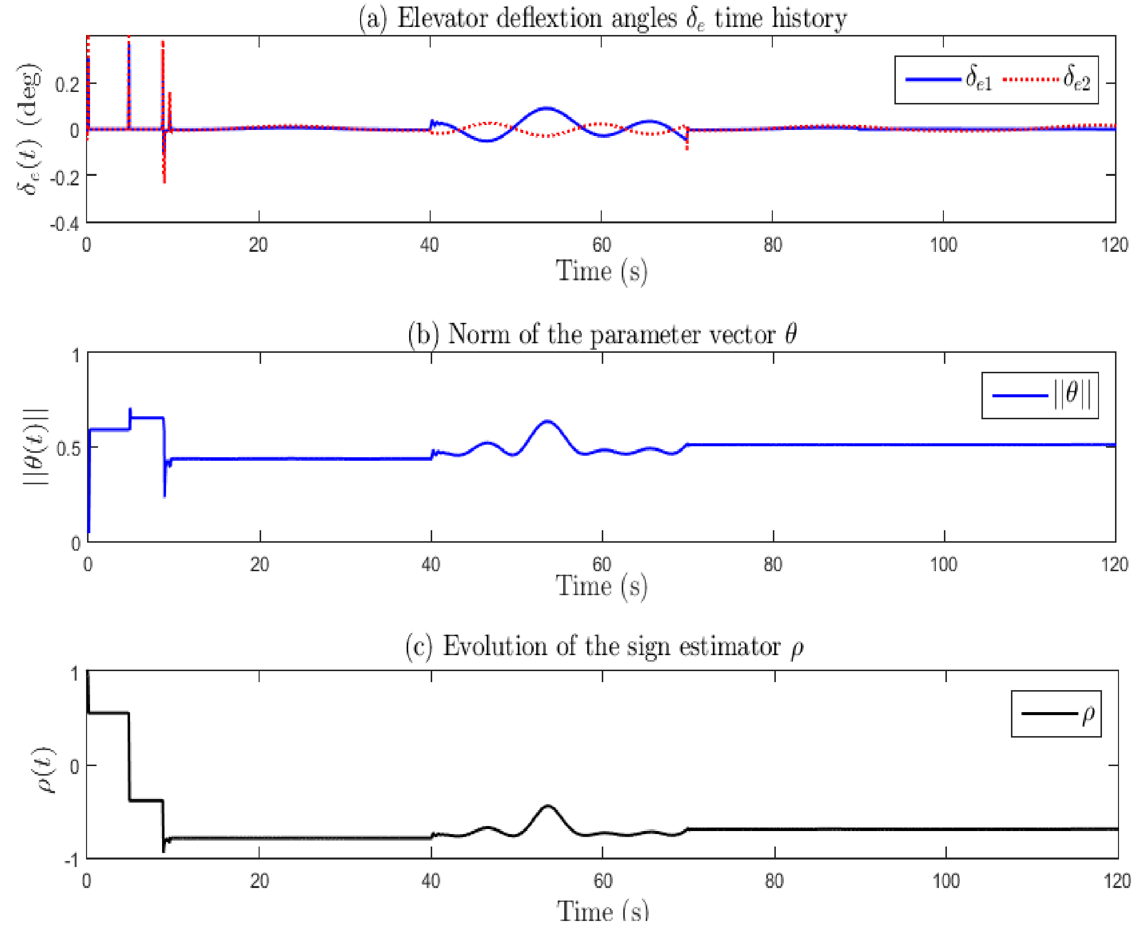

Figure 4.14: Control signals and parameters without Nussbaum (scenario 2, $\rho(0)=1$)

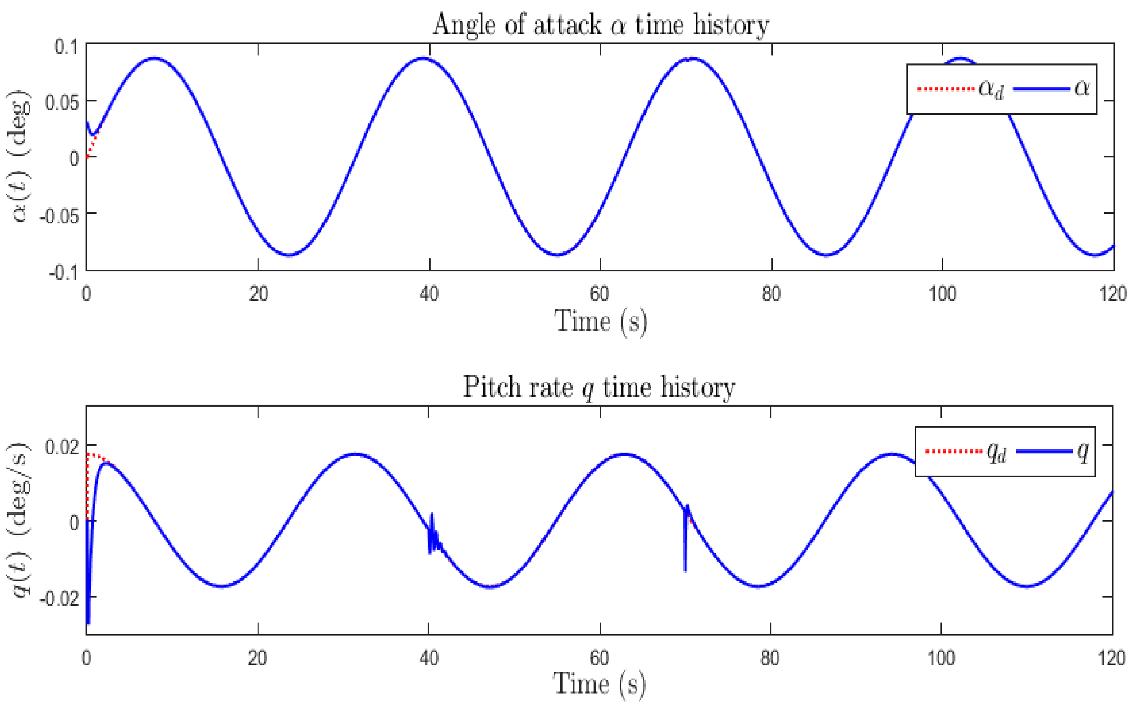

Figure 4.15: Angle of attack and pitch rate without Nussbaum (scenario 2, $\rho(0)=-1$)

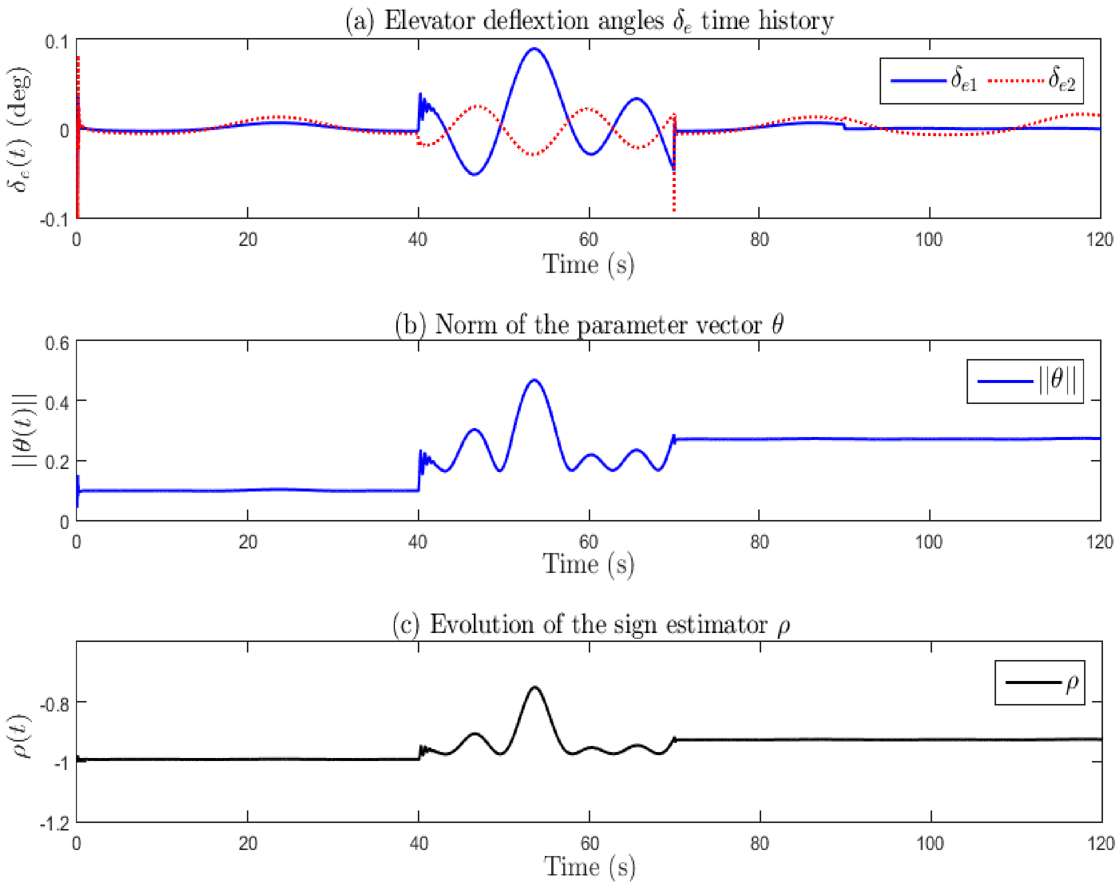

Figure 4.16: Control signals and parameters (scenario 2, $\rho(0) = -1$)

4.5 Conclusion

In this chapter, the problem of adaptive actuator failure compensation control for a class of redundant single-output systems with unknown control direction and subject to generalized (non-affine) actuator failures is investigated. Based on structural assumptions, and using the implicit function theorem the existence of an ideal implicit controller to the problem is proved, then, two adaptive approximation control designs were proposed to construct the ideal implicit controller online. The first design uses the Nussbaum-type function technique while the second design uses another new technique that estimates a sign-like term online. The simulation for both designs is carried out on the dynamic model of the angle of attack in a hypersonic aircraft with failures in elevator segments. Fuzzy logic systems were used as approximators for both designs. The simulation results confirm the theoretical claims for both designs. Notice also that the proposed design can be easily extended to general control affine systems with stable zero dynamics studied in chapter 2. This chapter terminates the thesis, in what follows a general conclusion and perspectives for this doctoral research will be provided.

General Conclusion

The research works presented in this thesis concern the development of adaptive actuator failure compensation controllers for uncertain nonlinear systems with uncertain actuator failures.

The main contexts and directions of the thesis have been shaped based on common challenges encountered during the design of actuator failure compensation controllers for nonlinear systems, these challenges can be summarized in the following points:

1. To achieve resilience and fault tolerance, the system should have redundant actuators; this redundancy should be used in an optimal and efficient way through an intelligent reconfigurable control scheme.
2. Most nonlinear systems are uncertain with strong nonlinearities, therefore adaptive approximation based control methodology is a key solution to deal with such systems.
3. Actuator failures when they occur, they have severe effects on the behavior of the system, besides they are usually uncertain, i.e. it is not known which actuators have failed, when and how they have failed.
4. Actuator failed can have complicated patterns that are hard to describe by simple models. While in some cases, actuator failure can be described by simple patterns (partial loss of effectiveness or bias), there are situations where the failures can be time-varying, state-dependent, and in general, they can have non-affine models.
5. Actuator failure can be incipient or abrupt, while incipient failures exhibit a smooth time behavior, abrupt failures can cause parameter jumps. This point should be carefully addressed when designing control laws.
6. The problem of actuator failure compensation control is more critical and relevant for safety critical systems, which are required to achieve high missions such as aircraft systems, spacecraft systems, manipulators working is critical environments or handling hazardous materials.

The objective was to design actuator failure compensation controllers by taking all these key issues in consideration. Therefore, the methodology followed in this doctoral thesis is summarized as follows:

The definitions of faults, failures and their classifications have been reviewed in chapter 1 with particular emphasis on actuator failures. Then a state of the art on existing passive and active actuator failure compensation strategies from the research literature was provided. Different models describing different types of actuator failures with different levels of complexity were also given. Then a brief review of related works on the adaptive actuator failure compensation control for both linear and nonlinear systems was provided.

Chapter 2 has been devoted to the design of direct adaptive actuator failure compensation controllers for a class of nonlinear multi-input single output systems (MISO) with stable zero dynamics. In this chapter, only partial or total loss of effectiveness failure types was considered. Two designs were proposed in this regard. A first design assumes no parameterization of the actuator failures, for which an adaptive controller was provided, fuzzy logic systems (FLS) were used for the approximation of the ideal controller and a simulation study on the model of the angle of attack dynamics of a hypersonic aircraft with failures of elevator segments was provided. In a second step, another design assumes that the actuator failures are in a completely parameterized form is developed, this design was developed around a plant controller that accounts for the plant dynamics with effective actuators, the second term is an actuator failure estimation and compensation term which accounts for the failure effects, this second design was used with a linear regressor and applied to control the wing rock oscillations in the presence of aileron segments failures. Note that while the first design is simpler in terms of derivation and implementation, the second term has the advantage of providing a rough estimate of the failure values which can be seen as a failure alarm.

In Chapter 3, the problem of actuator failure compensation control for a class of redundant multi-input multi-output (MIMO) nonlinear systems was investigated. The compensable actuator failures set was extended to affine models that can represent a larger set of failures. Some new assumptions on the control gain matrix were introduced, these assumptions become less restrictive for the case of multivariable interconnected systems. Then an integrated approximation based adaptive controller was derived. Numerical simulations are carried out on a robot manipulator with a redundant actuation system, which falls in the general class of systems considered, in this case, fuzzy logic systems (FLS) were used as function approximators, the second simulation was carried out on a flexible spacecraft with redundant reaction wheels which falls in the particular case of interconnected systems, in this case, a new form of the regressor vector was introduced to approximate the ideal controller.

In Chapter 4, the problem of actuator failure compensation for a class of redundant multi-input single output (MISO) nonlinear systems with unknown control directions was investigated. In addition to the difficulty encountered when the control direction is unknown, another difficulty was introduced in this chapter as we further extended the class of compensable actuator failure to the very general time-varying and state-dependent models which are non-affine. The problem of unknown control direction was solved using two approaches, in the first approach, the Nussbaum- type function technique was used to derivate the adaptive controller while in the second approach a new technique that directly gives an information on the control gain sign is used. In addition, is has been seen that the presence of non-affine failures transforms the dynamic system to a non-affine model, this problem was then solved based on the implicit function theorem to prove the existence of an ideal controller, then this implicit controller was approximated online using an adaptive controller. The simulation for both designs was carried out on the dynamic model of the angle of attack in a hypersonic aircraft with failures of redundant elevator segments.

The investigations carried out throughout this thesis have proved the efficiency of adaptive approximation based control in dealing with the problem of adaptive actuation failure compensation for uncertain nonlinear systems. It was shown that many levels of complexity can be solved using this methodology. The superiority of adaptive control was also put into value as the proposed designs did not require fault diagnosis and isolation (FDI) blocks which makes them computationally effective as FDI based controller require some latent time for fault diagnosis and isolation, which can limit their real-time implementation.

The adaptive actuator failure compensation control proposed in this thesis are valuable from the theoretical and practical point of view. They provide a theoretical framework from which many adaptive actuator failure compensation designs can be developed, and they apply directly to several practical systems such as aircraft spacecraft and robot manipulator. Notice however that a key assumption is made throughout the thesis, it was assumed that the actuators are ideal, i.e. they can provide an invite response. In practical situations, actuator nonlinearities, amplitude and rate saturation constraints are present. The proposed control mechanisms will need to be tailed and tweaked to deal with such constraints. This point is not addressed in this thesis and is suggested for future research. As another suggestion for future research, we suggest the investigation of the class of redundant non-affine systems (multivariable and single variable), the challenge, in this case, can be envisaged in setting mathematical assumptions on the system and the actuator failures that can define the redundancy and the set of compensable failures. The extension of the study to other models (such as parametric strict) can be investigated in a way similar to that present throughout this thesis. Besides, the assumption on the plant and the actuator failures may be regarded as conservatives or restrictive, so it would be of interest to see more relaxed assumptions, mainly on the control gain matrix for the case of multivariable systems. in all the proposed designs the actuation scheme was fixed, it would be interesting to investigate the case of adaptive actuation schemes, i.e. the control allocation coefficients are updated online to cope with changes and actuator failures. Notice also that for the design based on Nussbaum-type function, the approximation error was assumed zero which is the main drawback, as a future task, we seek to remedy this drawback via the use of special types of Nussbaum functions, such as restricted input functions Last but not least, we can integrate state observers in the proposed designs to reconstruct the states in case they are not available for feedback.

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ملخص:

تهدف أعمال البحث المقدمة في هذه الأطروحة إلي تصميم قوانين التحكم في الأنظمة اللاخطية الغير معرفة بدقة من أجل ضمان تسيير واستغلال فعالين لتكرار المشغلات الموجودة في هذه الأنظمة في حالة ظهور أعطال في هذه المشغلات أثناء عمل الأنظمة. أعتمدنا في هذه الأطروحة بشكل أساسي على تقنية التحكم التلاؤمي التقريبي. عدة قوانين تحكم تلاؤمي تم اقتراحها بهذا الصدد من أجل حل عدة اشكاليات. الإشكالية الأولي تخص الأنظمة متعددة المذاخل أحادية المخارج مع وجود ضياع جرئي أو كلي في فعالية المشعلات. الحالة الثانية تخص الأنظمة متعددة المذاخل والمخارج مع وجود أعطال في المشغلات ذات نموذج تألفي لكنها غير معرفة. أما الحالة الثالثة فهي تخص الأنظمة متعددة المذاحل أحادية المخارج مع وجود أعطال في المشعلات ذات نمودج عام (غير تألفي). كل قوانين التحمكم التي تم اقتراحها تم إثباث فعاليتها نظريا بالإعتماد على نظرية ليابونوف كما تمت محاكاتها على عدة أنظمة ديناميكية متل ديناميكية الطائرة، الروبوت والمركبة الفضائية.

كلمات مفتاحية: التحكم المسامح للأعطال، أعطال المشغلات، تكرار المشغلات، التحكم التلاؤمي، الأنظمة اللاخطية، إستقرار ليابونوف.

Abstract:

The research works presented in this thesis aim to design control laws for redundant uncertain nonlinear systems to ensure an efficient management of the existing redundancy when uncertain actuator failures occur during the course of operation. The approximation based adaptive control methodology is mainly considered. Different adaptive actuator failure compensation control designs were developed for different problems. The first problem is the actuator failure compensation for uncertain redundant single variable systems in the present of partial or total actuator loss of effectiveness. The second problem is the actuator failure compensation for uncertain redundancy multivariable systems in the presence of uncertain affine actuator failures. The third problem is the actuator failure compensation for redundancy single variable systems in the presence of generalized (non-affine) actuator failures and unknown control directions. The effectiveness of the proposed controller is proved theoretically using Lyapunov theory and through numerical simulation on several systems such as aircraft systems, redundant manipulators and spacecraft systems.

Keywords: fault tolerant control, actuator failures, redundant actuators, adaptive control, nonlinear systems, Lyapunov stability.

Résumé :

Les travaux de recherche présentés dans cette thèse ont pour objectif la conception des lois de commande pour les systèmes non linéaires incertains afin d’assurer une gestion éfficace de la redondance lorsque des défauts actionneurs apparaissent durant le fonctionnement. Ici, la méthodologie de la commande adaptative utilisant les approximateurs de fonctions est adoptée. Plusieurs contrôleurs adaptives sont proposés pour différents problèmes. Le premier problème concerne les systèmes mono-variables redondants avec des défauts actionneurs de type perte d’efficacité (totale ou partielle). Le second problème est pour les systèmes multi-variables redondants avec des défauts actionneurs qui sont modélisée par un modèle affine. Le troisième problème et pour les systèmes mono-variables redondants avec des défauts actionneurs généralisée (non affines) et un gain de commande de signe inconnue. L’efficacité des contrôleurs développés et prouvé théoriquement par la méthode de Lyapunov et validé par simulation numérique sur plusieurs systèmes tels que la dynamique de l’avion, les robots manipulateurs, et les vaisseaux spatiaux.

Mots clés : commande tolérante aux défauts, défauts actionneurs, redondance d’actionneurs, commande adaptative, systèmes non linéaires, stabilité de Lyapunov.